RAFFLES INSTITUTION
H2 Mathematics (9758)
2024 Year 6

2024 Year 6 Timed Practice Revision Practice Paper 2
(Source: 2020 Year 6 Term 3 Timed Practice)

The solution will be released in Ivy on 3 June (Mon)

Total Marks: 100 **Duration: 3h**

****** Note that Qn 1 (Complex Numbers) will NOT be examined in the coming Timed Practice. So, you should complete this paper within 2 hrs 51 mins.**

Section A: Pure Mathematics (60 marks)

1 The complex number z is given by $x+\mathrm{i}y$, where x and y are real numbers.

(i) Express y in terms of x if $\arg\left(z^2\right)=-\frac{\pi}{2}$. [2]

(ii) State the values of x and y if $\mathrm{Re}\left(z\right)>0$ and $\left|z\right|=2$. [1]

Using these values of x and y, find the smallest positive integer n for which $\frac{z^*}{z^n}$ is a negative real number. [2]

2 **(a)** By writing $\frac{2}{r\left(r^2-1\right)}$ in partial fractions, find an expression for $\sum_{r=2}^{n}\frac{2}{r\left(r^2-1\right)}$. [3]

(b) A geometric series has first term a and common ratio r, where a and r are non-zero and $r\neq 1$. The 3rd and 9th terms of the series are 448 and 7 respectively. Given also that the sum of the first n terms is 1197, find the values of a, r and n. [4]

3 The functions f and g are defined by

$$\mathrm{f}: x\mapsto \sqrt{16-4x},\ x\in\mathbb{R},\ x\leq 4,$$

$$\mathrm{g}: x\mapsto x^2,\ x\in\mathbb{R}.$$

(i) Sketch on the same diagram, the graphs of f and f^{-1}, giving the coordinates of all points of intersection. [4]

(ii) Explain why the composite function fg does not exist. [1]

(iii) Find gf in similar form and state its range. [2]

4 A curve C has equation $y = k(x+1) + \dfrac{4}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$, where $0 < k < 2$ is a constant.

(i) Sketch C, labelling clearly the axial intercept(s), the coordinates of turning points and equations of the asymptotes. [4]

The graph of C is transformed by a reflection in the x-axis, followed by a translation of 1 unit in the positive x-direction, followed by a stretch with scale factor $\dfrac{1}{2}$ parallel to the y-axis.

(ii) Find the equation of the resulting curve in the form $y = \mathrm{f}(x)$. [3]

5 The curve C has equation $y = \mathrm{e}^{\left(x-\frac{1}{2}\right)^2}$.

(i) Sketch C, labelling clearly the coordinates of the axial intercept(s) and turning point(s), if any. [2]

(ii) Show that the equation of the tangent to C at the point where $x = p$ can be expressed as

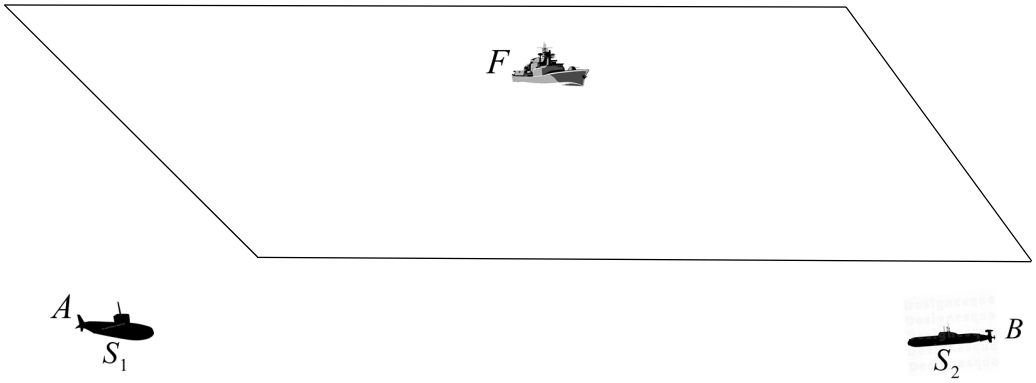
$$(2p-1)x - \mathrm{e}^{-\left(p-\frac{1}{2}\right)^2} y = 2p^2 - p - 1.$$

Hence find the equations of the tangents to C which passes through the origin. [4]

(iii) The straight line $y = mx$ intersects C at two distinct points.

State the range of values of m. [2]

6 A frigate is stationed at position $F(1,2,0)$. Two submarines S_1 and S_2 are under the sea surface. Submarine S_1 is at position $A(-2,-1,-1)$ and travelling in a path parallel to vector $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. An enemy submarine S_2 is detected at position $B(3,2,-2)$ travelling in a path parallel to vector $-2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.



(i) Determine if the paths of the submarines will intersect each other. [3]

(ii) The enemy submarine S_2 will launch a torpedo at the frigate when it is at a point P in its path that is closest to F. Find the co-ordinates of P. [4]

(iii) Find a cartesian equation of the plane π that contains F and the path of S_2.

Calculate the acute angle between π and the x-y plane. [4]

(iv) A depth charge is a countermeasure used against submarines. The frigate releases a depth charge which descends to the position $Q(1, 2, -k)$ to target S_2. The depth charge is detonated when the distance from Q to P is at a minimum. Find the value of k. [1]

When detonated the depth charge will cause damage within a 0.1 unit radius. Explain whether S_2 will be damaged by the depth charge. [2]

7 A chemical substance Uru starts to form when N units of Vibranium react with M units of Kryptonite, where $M < N$. Let y denote the number of units of Uru formed t hours after the reaction takes place.

The differential equation for the chemical reaction is given by

$$\frac{\mathrm{d}y}{\mathrm{d}t} = k(N-y)(M-y),$$

where $0 \le y \le M$ and k is a positive constant.

(i) By first expressing $\dfrac{1}{(N-y)(M-y)}$ in the form $\dfrac{A}{N-y}+\dfrac{B}{M-y}$, where A and B are constants to be found in terms of N and M, find t in terms of y. [5]

It is given that $\dfrac{M}{N}=\dfrac{3}{4}$.

(ii) Find the time taken to produce $\dfrac{N}{4}$ units of Uru, giving your answer in terms of N and k. [2]

(iii) Express the solution of the differential equation in the form $y = \mathrm{f}(t)$. [2]

Sketch the part of the curve with this equation which is relevant in this context and state what happens to y for large values of t. [3]

Section B: Probability and Statistics (40 marks)

8 Find the number of ways in which the letters of the word INTEGRITY can be arranged if

(i) there are no restrictions, [1]

(ii) any 2 vowels must be separated by exactly 2 consonants, [2]

(iii) no adjacent letters are the same. [3]

9 During training, the time in seconds for a soldier to dismantle a certain type of equipment is a normally distributed continuous random variable T. The standard deviation of T is 3.5 and the expected value of T is 35.1. After a new set of instructional material is introduced, n soldiers are selected at random and the mean time taken for this sample of soldiers to dismantle the equipment is found to be $\bar{t}$ seconds. A test is carried out, at 5% significance level, to determine whether the mean time taken to dismantle the equipment has been reduced.

(i) State the appropriate hypothesis for the test. [1]

(ii) Given that $n = 20$, find the set of values of $\bar{t}$ for which the result of the test would be to reject the null hypothesis. [3]

(iii) Given instead that $\bar{t} = 33.2$ and the result of the test is that the null hypothesis is not rejected. Find the largest possible value of n. [3]

10 A bag contains 3 red balls and n yellow balls, where $n \geq 2$. In a game, Joe removes 2 balls at random from the bag one at a time without replacement. The number of red balls Joe removes from the bag is denoted by T.

(i) Find $\mathrm{P}(T = t)$ for all possible values of t. [2]

(ii) Find $\mathrm{E}(T)$ and $\mathrm{Var}(T)$. [5]

11 For events A and B it is given that $\mathrm{P}(A) = 0.55$, $\mathrm{P}(B) = 0.65$ and $\mathrm{P}(A \cup B) = 0.93$.

Find

(i) $\mathrm{P}(A \cap B)$, [2]

(ii) $\mathrm{P}(A \cup B')$. [2]

(iii) Determine if A and B are independent. [1]

It is further given that $\mathrm{P}(C) = 0.3$ and that events B and C are mutually exclusive.

(iv) Find the greatest and least possible values of $\mathrm{P}(A \cap C)$. [3]

12 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

A local supermarket sells two types of potatoes, Russet potatoes and Holland potatoes. The masses, in grams, of the Russet potatoes have the distribution $\mathrm{N}\left(200, 30^2\right)$ and the masses, in grams, of the Holland potatoes have the distribution $\mathrm{N}\left(150, 24^2\right)$.

(i) Find the probability that the total mass of 3 randomly chosen Holland potatoes is less than twice the mass of a randomly chosen Russet potato. [3]

(ii) The supermarket decides to pack the potatoes into a variety pack. Each variety pack consists of 5 randomly chosen Russet and 4 randomly chosen Holland potatoes. The probability that a randomly chosen variety pack is within k grams of 1600 grams is found to be 0.775. Find k. [3]

(iii) 40 variety packs are randomly selected and are to be donated to needy families. Using the value of k found in **(ii)**, find the probability that at most 5 variety packs are not within k grams of 1600 grams, [2]

(iv) State an assumption needed for your calculations in parts **(i), (ii)** and **(iii)**. [1]

The supermarket recently introduced a new type of potatoes, called New potatoes. The mean mass of New potatoes is 55 grams and standard deviation is 13 grams.

(v) Find an approximate value for the probability that the average mass from a random sample of 100 New potatoes is not more than 58 grams. [3]

***** End of Paper *****