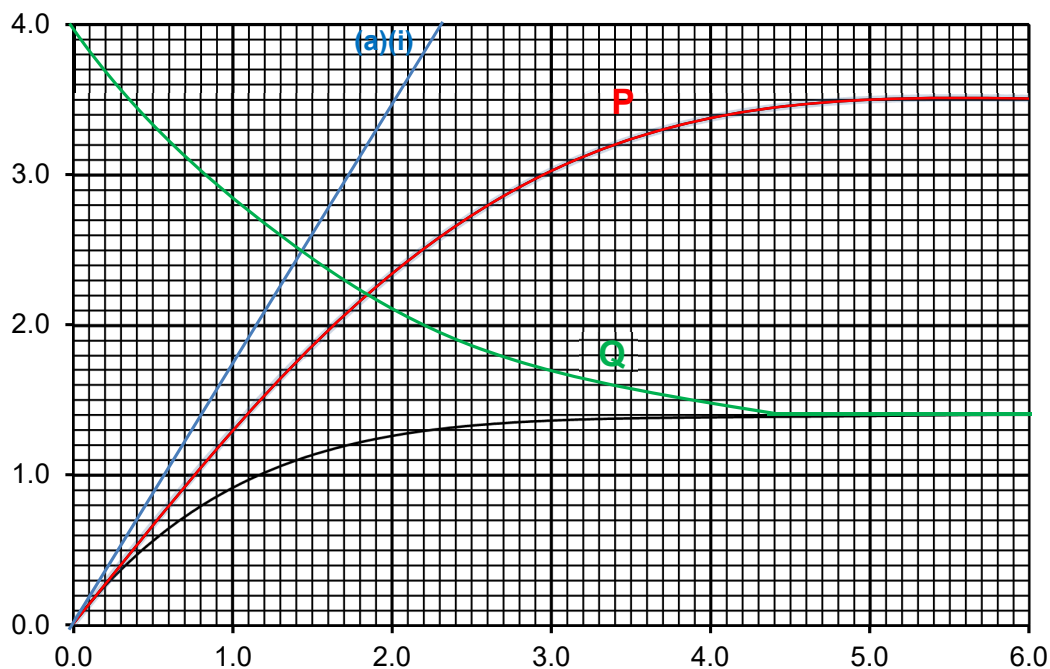


Paper 3 – Solutions

Section A

1



- (a) (i) (The graph is a curve. This implies there is non-constant acceleration due to effect of air resistance. Only at $t = 0$ is the acceleration equal to g)

Draw tangent at $t = 0$

$$\text{acceleration} = \text{gradient} = \frac{4.00}{2.30} = 1.74 \text{ m s}^{-2}$$

- (ii) The Moon's surface has no atmosphere (very negligible compared to Earth). There is no air resistance outside the container.

Read off from tangent (as this is the v - t graph for no air resistance):

$$v = 1.4 \text{ m s}^{-1}, t = \underline{0.80 \text{ s}}$$

or use equation: $v = u + at$

(substitute value of a from (a)(i), $v = 1.4$, $u = 0$)

- (b) (i) At terminal velocity, resistive force F reaches its maximum value.

$$\begin{aligned} F_{\text{max}} &= mg \\ &= 0.0027(1.74) \\ &= 4.70 \times 10^{-3} \text{ N} \end{aligned}$$

- (ii) $kv_T = mg$ (v_T is terminal velocity on Moon)

$$\begin{aligned} k(1.40) &= 4.70 \times 10^{-3} \\ k &= 3.36 \times 10^{-3} \text{ kg s}^{-1} \end{aligned}$$

(iii) $kv_T' = mg$ (v_T' is terminal velocity on Earth)
 $3.36 \times 10^{-3} v_T' = 0.0027(9.81)$
 $v_T' = 7.89 \text{ m s}^{-1}$

- (iv) 1. Total mass is 2.5 x initial mass.

$$kv_T = mg$$

$$v_T = \frac{mg}{k} = \frac{2.5(4.70 \times 10^{-3})}{3.36 \times 10^{-3}} = 3.50 \text{ m s}^{-1}$$

Initial gradient is the same, and graph always above the original (but below the tangent) – B1

Curve reaches terminal velocity 3.5 m s^{-1} – B1

(time it reaches terminal velocity is later – not marking pt)

2. Starts from 4.0 m s^{-1}

Curve must show speed decreasing to v_T – B1

- 2 (a) (i) The gravitational field strength at a point in space is defined as the gravitational force experienced per unit mass at that point.
- (ii) Gravitational field strength is a vector since it is defined using gravitational force which is a vector.
- (iii) Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Since gravitational field strength g is defined as the gravitational force acting per unit mass:

$$g = \frac{F_G}{m} = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$$

(b) (i)

$$\frac{g_{\text{Sun}}}{g_{\text{Jupiter}}} = \frac{\left(\frac{GM_{\text{Sun}}}{r_{\text{Sun}}^2}\right)}{\left(\frac{GM_{\text{Jupiter}}}{r_{\text{Jupiter}}^2}\right)} = \frac{M_{\text{Sun}}}{M_{\text{Jupiter}}} \left(\frac{r_{\text{Jupiter}}}{r_{\text{Sun}}}\right)^2$$

$$= \left(\frac{1.99 \times 10^{30}}{1.90 \times 10^{27}}\right) \left(\frac{7.14 \times 10^7}{7.79 \times 10^{11}}\right)^2$$

$$= 8.80 \times 10^{-6}$$

- (ii) Since the gravitation field strength due to the Sun on the surface of Jupiter is only $\sim 10^{-6}$ ($< 0.001\%$) that of the gravitational field strength on the surface of Jupiter due to its mass, it can be neglected.

- (c) (i) For a moon in circular orbit about its planet, the centripetal force is provided by the gravitational force acting on the moon by the planet:

$$\frac{GM_p M_m}{R^2} = M_m R \omega^2$$

Since $T = \frac{2\pi}{\omega}$, we have $\omega = \frac{2\pi}{T}$

$$GM_p = R^3 \omega^2 = R^3 \left(\frac{2\pi}{T} \right)^2$$

Rearranging, we have:

$$T^2 = \frac{4\pi^2}{GM_p} R^3$$

Since M_p is the mass of the planet, $\frac{4\pi^2}{GM_p}$ is a constant

$$\therefore T^2 = KR^3 \text{ where } K = \frac{4\pi^2}{GM_p}$$

(ii) Since $T^2 \propto R^3$

$$\frac{T_E}{T_I} = \sqrt{\left(\frac{R_E}{R_I} \right)^3} = \sqrt{\left(\frac{R_E}{R_I} \right)^3} = \sqrt{\left(\frac{6.71 \times 10^8}{4.22 \times 10^9} \right)^3} = 2.01 \approx 2$$

$$\frac{T_G}{T_I} = \sqrt{\left(\frac{R_G}{R_I} \right)^3} = \sqrt{\left(\frac{R_G}{R_I} \right)^3} = \sqrt{\left(\frac{1.07 \times 10^9}{4.22 \times 10^8} \right)^3} = 4.04 \approx 4$$

Hence, the ratio of the orbital period of Io: Europa: Ganymede is 1: 2: 4

3 (a) Since the test-tube is in equilibrium, $F_{net} = (M + m)g - \rho(AH)g = 0$

$$(M + m)g = \rho(AH)g$$

$$H = \frac{(M + m)}{\rho A}$$

(b) (i) $-\rho Agy$

(ii) $-\rho Agy = (M + m)a$

$$a = -\frac{\rho Ag}{M + m} y$$

(c) By comparing with $a = -\omega^2 x$,

$$\omega^2 = \frac{\rho Ag}{M + m}$$

$$T = \frac{2\pi}{\omega}$$

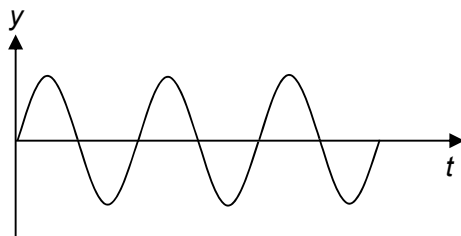
$$= 2\pi \sqrt{\frac{M + m}{\rho Ag}}$$

$$= 2\pi \sqrt{\frac{0.012 + 0.025}{1000 \times 6.0 \times 10^{-4} \times 9.81}}$$

$$= 0.498 \approx 0.50 \text{ s}$$

- (d) As the test-tube oscillates, it experiences drag force exerted by the water. This results in light damping and energy is gradually lost as heat.

(e) (i)



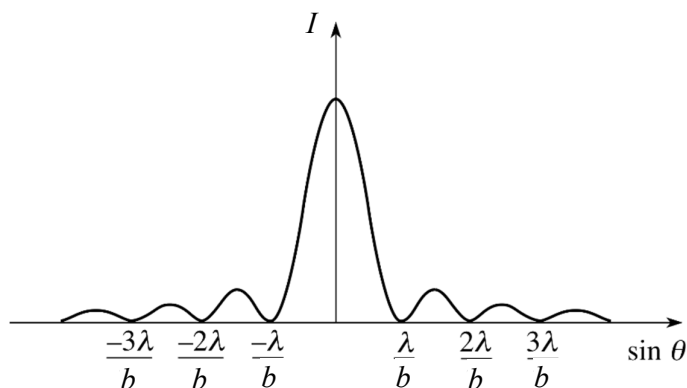
- (ii) The amplitude of oscillation is small because the frequency of the driving force (the waves) is too low (0.30 Hz) compared to the natural frequency of the test-tube (2 Hz).

The amplitude of the oscillations can be increased by adding ball bearings to the test-tube to decrease the natural frequency so that it is closer to the frequency of the driving force.

- 4 (a) (i) The principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.
- (ii) According to Huygen's principle, all the points on the wavefronts that pass through the single slit are individual sources of circular wavelets which interfere with one another by the principle of superposition to form the interference pattern.
- (b) (i) For single slit diffraction, $b \sin \theta = m\lambda$

$$\text{Positions of minima, } \sin \theta = \frac{\lambda}{b} m = \frac{600 \times 10^{-9}}{0.30 \times 10^{-3}} m = (2.0 \times 10^{-3}) m$$

$$\text{where } \frac{\lambda}{b} = 2.0 \times 10^{-3}$$



- (ii) limiting angle of resolution for the single slit, $\theta_{\min} \approx \frac{\lambda}{b} = 2.0 \times 10^{-3} \text{ rad}$

$$\theta \approx \frac{s}{r} = \frac{1.0 \times 10^{-3}}{0.25} = 4.0 \times 10^{-3} \text{ rad}$$

Since $\theta > \theta_{\min}$, the interference patterns due to the two point sources of light can be resolved.

- (c) (i) Position of first minima of the single slit diffraction envelope is given by:
 $b \sin \theta = \lambda$ ----- (1)
 where b is the slit width

Positions of maxima of the double slit interference pattern is given by:
 $a \sin \theta = n \lambda$ ----- (2)
 where a is the slit separation

Where the first minima of the single slit diffraction envelope coincide with a double slit maxima, θ is the same in equations (1) and (2).

$$\frac{(2)}{(1)}: n = \frac{a}{b} = \frac{1.2}{0.30} = 4$$

Hence the 4th orders will be missing from the double slit interference pattern.

Number of maxima in the central region / within the diffraction envelope
 $= 3 + 3 + 1 = 7$
 (3 maxima on either side of the principle axis and the central maximum)

(ii)

$$\begin{aligned} \tan \theta &= \frac{12 \times 10^{-3}}{2} \times \frac{1}{D} \\ D &\approx \frac{12 \times 10^{-3}}{2\theta} \\ &= \frac{12 \times 10^{-3}}{2(2.0 \times 10^{-3})} \\ &= 3.0 \text{ m} \end{aligned}$$

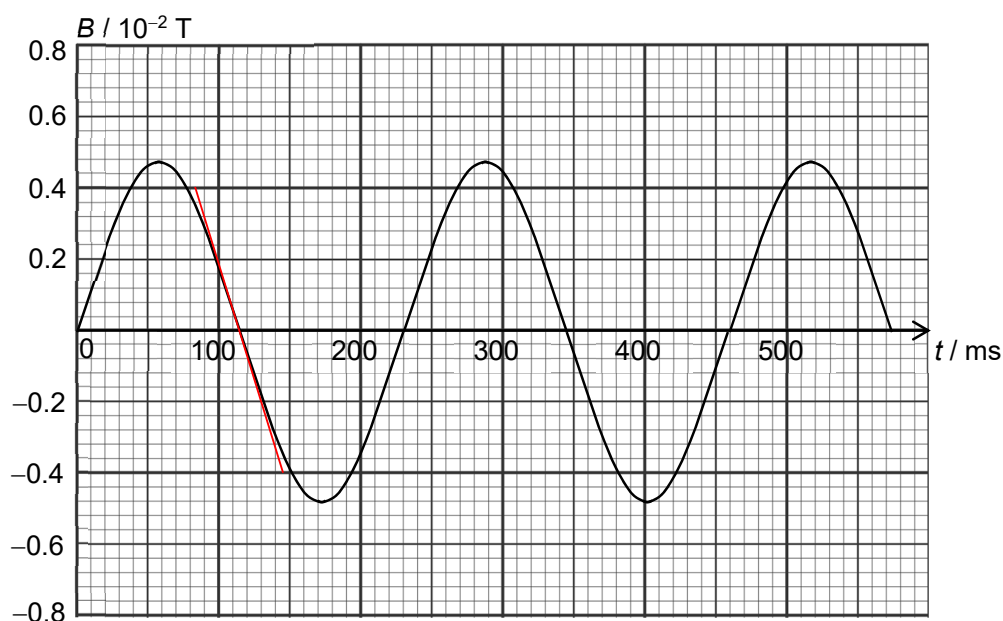
- 5 (a) Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.

- (b) (i) When magnetic field increases in the positive direction, induced current decreases in the negative direction. When magnetic field reaches maximum, induced current becomes zero. As magnetic field decreases in the positive direction, induced current increases in the positive direction until it reaches a minimum value when $B = 0$.

This corresponds to Lenz's law as induced current flows in a direction to produce effects that opposes the change producing it. Hence sensitive ammeter deflects in opposite directions.

- (ii) 1. Accept $t = 115, 230, 345$ or $450 \times 10^{-3} \text{ s}$

2.



Using $(85 \times 10^{-3}, 0.40 \times 10^{-2})$ and $(145 \times 10^{-3}, -0.40 \times 10^{-2})$,

$$\left| \frac{dB}{dt} \right|_{\max} = \frac{(-0.40 \times 10^{-2} - (0.40 \times 10^{-2}))}{(145 \times 10^{-3} - 85 \times 10^{-3})} = 0.1333$$

$$i_{\max} = \frac{E_{\max}}{R} = \frac{AN}{R} \left| \frac{dB}{dt} \right|_{\max} = \frac{4.2 \times 10^{-4} \times 400 \times 0.1333}{5.0}$$

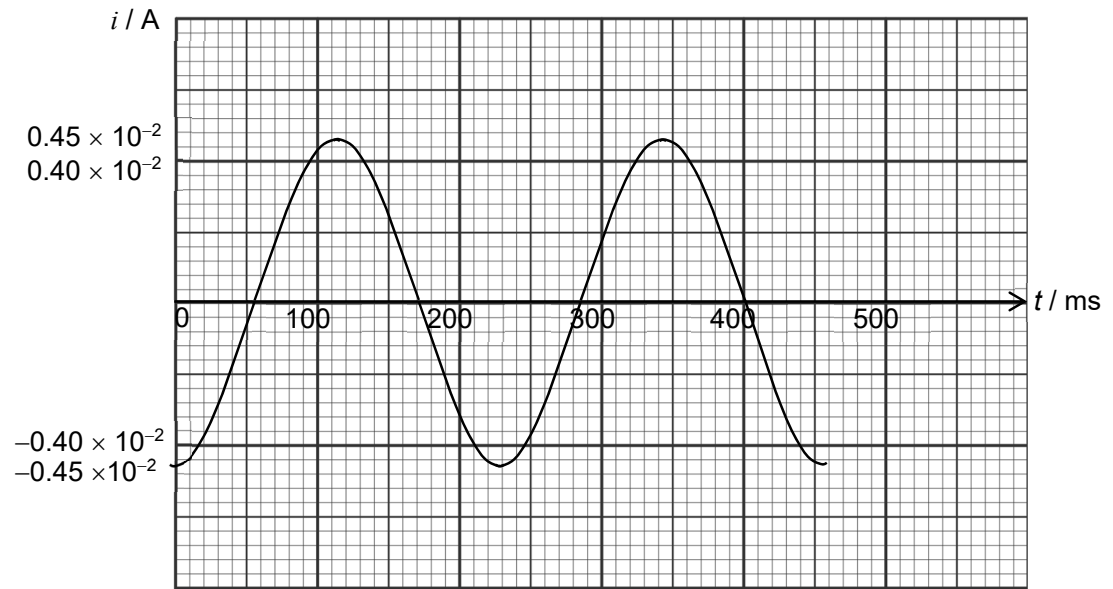
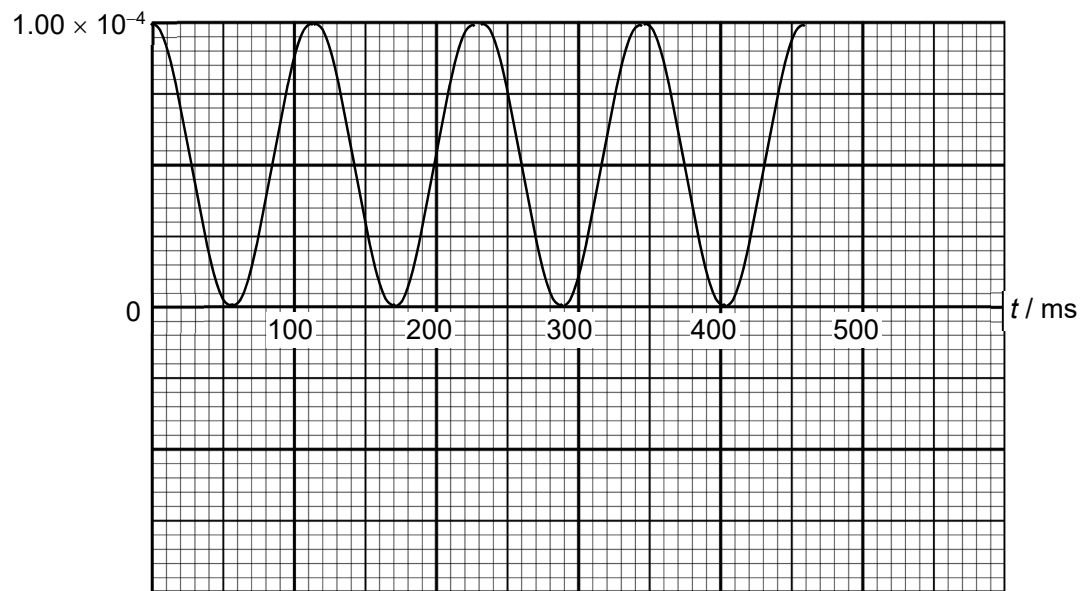
$$\therefore i_{\max} = 4.5 \times 10^{-3} \text{ A}$$

3.

$$\langle P \rangle = \frac{P_o}{2} = \frac{i_{\max}^2 R}{2} = \frac{(4.479 \times 10^{-3})^2 \times 5.0}{2}$$

$$\therefore \langle P \rangle = 5.0 \times 10^{-5} \text{ W}$$

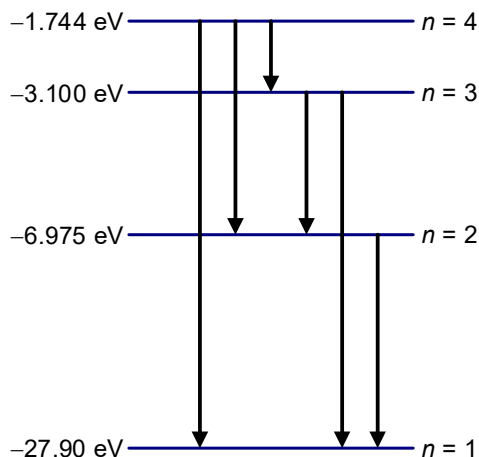
(iii)

(iv) P / W 

Section B

6 (a) (i)

n	E_n / eV
1	-27.90
2	-6.975
3	-3.100
4	-1.744



(ii)

n	$\Delta E / \text{eV}$
1 \rightarrow 2	20.92
1 \rightarrow 3	24.80
1 \rightarrow 4	26.16
1 \rightarrow 5	26.78

(iii) Refer to diagram above.

(iv) To calculate wavelength, we use

$$\Delta E = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda = \frac{hc}{\Delta E}$$

Longest wavelength

$$\lambda = \frac{hc}{\Delta E_{4 \rightarrow 3}} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.36 \times 1.60 \times 10^{-19}} = 914 \text{ nm}$$

Shortest wavelength

$$\lambda = \frac{hc}{\Delta E_{4 \rightarrow 1}} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{26.16 \times 1.60 \times 10^{-19}} = 47.5 \text{ nm}$$

- (b) (i) Bombarding electrons experience (rapid) deceleration when they are deflected by or collide/interact with the nuclei/atoms in the target metal. During each of these interactions, an electron loses a fraction of its K.E. and emit a photon with energy equals the loss in K.E. Since an electron can lose varying amount of its K.E. (due to different degree of deceleration), emitted X-ray photons will have a continuous range of energy up to the entire K.E. of the bombarding electrons. Hence the continuous spectrum W. The most energetic X-ray photon with the minimum wavelength has energy equals to K.E. of bombarding electron.

- (ii) Maximum energy of electron is equal to energy of photon with shortest wavelength.

$$E_{\max} = \frac{hc}{\lambda_{\min}}$$

Accelerating potential

$$V = \frac{hc}{e\lambda_{\min}} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.60 \times 10^{-19} \times 2.00 \times 10^{-10}} \\ = 6220 \text{ V}$$

- (iii) $L_{\alpha} = Y$
 $L_{\beta} = X$
 $M_{\alpha} = Z$

- (iv) The energy of the bombarding electron is insufficient to knock out an electron from the K-shell of the atom.

- 7 (a) (total count rate $C = C_x + C_y$)

As half-life of X is much longer than Y, after about 8 days, the activity (or count-rate) of Y becomes negligible, hence the total count rate is only given by X.

$$(C \approx C_x)$$

As the count rate of X obeys the exponential decay law

$$C_x = C_{x_0} e^{-\lambda_x t}$$

$$\ln C_x = \ln C_{x_0} - \lambda_x t$$

A graph of $\ln C_x$ against t is a straight line (with gradient $-\lambda_x$ and vertical intercept $\ln C_{x_0}$).

- (b) (i) Extrapolate the straight line portion to find y-intercept, to get the initial count rate of X.

Value of y-intercept = 5.4

$$\ln C_{x_0} = 5.4$$

$$C_{x_0} = e^{5.4}$$

$$= 221 = 220 \text{ counts per minute (2 or 3 s.f.)}$$

$$\text{Slope of the straight line} = -\lambda_x = (3.90 - 4.50) / (14.0 - 8.0) = -0.10 \text{ day}^{-1}$$

OR

Taking coordinates of the extrapolated line

$$\text{Slope of the straight line} = -\lambda_x = (3.90 - 5.40) / (14.0 - 0) = -0.11 \text{ day}^{-1}$$

$$t_{1/2} = \frac{\ln 2}{\lambda_x} = \frac{\ln 2}{0.11}$$

$$= 6.3 \text{ days (or 6.9 days)}$$

- (ii) After 19 days, the count rate is nearly all due to X.

$$\begin{aligned} C_X &= C_{X_0} e^{-\lambda t} \\ &= 220 e^{-0.11(19)} \\ &= 27 \text{ counts per minute} \end{aligned}$$

- (c) (i) The range of beta particles through the human body is short.

OR

The penetrative power of beta particles is weak.

- (ii) The gamma rays are emitted in all directions (and the detector is small and only measures the rays at a point).

- (iii)
- The physical state of the source (solid, liquid, or in solution) as it affects the way the source is administered into the body.
 - The nuclide used should decay to a stable product (so that it no longer emits radioactive substance longer than it should).
 - If the nuclide is introduced to the human body, it must be able to be removed naturally e.g. in urine, exhaled or excreted.
 - The source should not be carcinogenic or toxic.

- (d) Background count = 15 s^{-1}

Corrected initial count-rate (due to Y alone) = $100 - 15 = 85 \text{ s}^{-1}$

Count-rate after $3 t_{1/2}$ is $85 \times \left(\frac{1}{2}\right)^3 = 10.625 \text{ s}^{-1}$

Assuming emission is in all direction hence count-rate $\propto 1/r^2$

$$\begin{aligned} \frac{C_2}{C_1} &= \left(\frac{r_1}{r_2}\right)^2 \\ \frac{85}{10.625} &= \left(\frac{30}{r}\right)^2 \\ r &= 10.6 \text{ cm} \end{aligned}$$

Note:

The correct answer can also be obtained even if background is not accounted for :

$$C_2 = C_1 \times \left(\frac{1}{2}\right)^3 = \frac{C_1}{8}$$

Hence

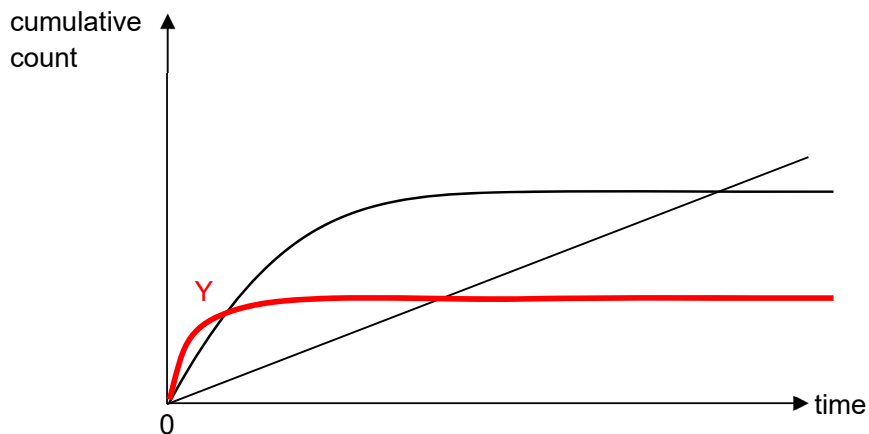
$$\begin{aligned} \frac{C_2}{C_1} &= \left(\frac{r_1}{r_2}\right)^2 \\ \frac{C_1/8}{C_1} &= \left(\frac{30}{r}\right)^2 \\ \frac{1}{8} &= \left(\frac{30}{r}\right)^2 \\ r &= 10.6 \text{ cm} \end{aligned}$$

which is independent of what C_1 is (whether background is considered or not).

This is unique in this context. Students should still be aware that in general, background count needs to be subtracted when accounting for radioactivity of the nuclides.

- (e) (i) Graph B
Since count rate (of background) is constant, the total count increases at a constant rate.

(ii)



*Initial increase is greater than X, reaches plateau sooner than X. Lower final total count.

The following workings are for info only:

X's half-life = 6.9 days

Y's half-life = 20 hours

$$A_0 = \frac{\ln 2}{t_{1/2}} N_0$$

$$\text{For X: } A_0 = \frac{\ln 2}{6.9 \times 24} N_0 = 0.004 N_0$$

$$\text{For Y: } A_0 = \frac{\ln 2}{20} \frac{N_0}{2} = 0.017 N_0$$

$$A = -\frac{dN}{dt}$$

Graph is equivalent to $(N_0 - N)$ against time.

Hence gradient of graph is proportional to A .

Since initial activity of $Y > X$, hence initial gradient of $Y > X$