JC1 H2 Mathematics (9758)
Term 4 Revision Topical Quick Check
Chapter 9 Maclaurin Series

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Section 1: Using Maclaurin's expansion (MF27):

$$\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\ldots+\frac{x^n}{n!}\mathrm{f}^{(n)}(0)+\ldots$$

Steps for use of Maclaurin's expansion

1) Through multiple implicit differentiation, obtain equations involving $\frac{\mathrm{d}y}{\mathrm{d}x},\frac{\mathrm{d}^2y}{\mathrm{d}x^2},\frac{\mathrm{d}^3y}{\mathrm{d}x^3},\ldots$

2) Substitute $x=0$ to obtain y. Subsequently, substitute $x=0$ and y to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$.

   Continue with the substitutions to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2},\frac{\mathrm{d}^3y}{\mathrm{d}x^3},\ldots$.

3) Substitute $y,\frac{\mathrm{d}y}{\mathrm{d}x},\frac{\mathrm{d}^2y}{\mathrm{d}x^2},\ldots$ into $\mathrm{f}(x)=\mathrm{f}(0)+x\mathrm{f}'(0)+\frac{x^2}{2!}\mathrm{f}''(0)+\ldots+\frac{x^n}{n!}\mathrm{f}^{(n)}(0)+\ldots$

   [Note: $\mathrm{f}(0)$ refers to y when $x=0$, $\mathrm{f}'(0)$ refers to $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=0$, …]

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Example 1: 2012 YJC/1/8

Objectives:

1. Finding Maclaurin Series using Differentiation (Implicit Differentiation).
2. Use of previous result and standard series.

It is given that $y=\ln\left(1+\tan^{-1}2x\right)$. Show that

(i) $\left(1+4x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-y}$, [2]

(ii) $\left(1+4x^2\right)\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]+8x\frac{\mathrm{d}y}{\mathrm{d}x}=0$. [2]

Hence find the Maclaurin series for y, up to and including the term in x^2. [3]

Deduce the Maclaurin series for $y=\ln\left(\frac{1+\tan^{-1}2x}{1-x}\right)$, up to and including the term in x^2. [3]

1	**Solution**
(i)	$y=\ln\left(1+\tan^{-1}2x\right)$ $e^{y}=1+\tan^{-1}2x$ Diff. wrt. x: $e^{y}\frac{dy}{dx}=\frac{1}{1+4x^{2}}(2)$ $\left(1+4x^{2}\right)\frac{dy}{dx}=2e^{-y}$ (shown) **Learning Point:** Simplify the equation so that it is easier to differentiate
(ii)	$\left(1+4x^{2}\right)\frac{dy}{dx}=2e^{-y}$ Diff. wrt. x: $\left(1+4x^{2}\right)\frac{d^{2}y}{dx^{2}}+8x\frac{dy}{dx}=-2e^{-y}\frac{dy}{dx}$ $\left(1+4x^{2}\right)\frac{d^{2}y}{dx^{2}}+8x\frac{dy}{dx}=-\left(1+4x^{2}\right)\left(\frac{dy}{dx}\right)^{2}$ $\left(1+4x^{2}\right)\frac{d^{2}y}{dx^{2}}+8x\frac{dy}{dx}+\left(1+4x^{2}\right)\left(\frac{dy}{dx}\right)^{2}=0$ $\left(1+4x^{2}\right)\left[\frac{d^{2}y}{dx^{2}}+\left(\frac{dy}{dx}\right)^{2}\right]+8x\frac{dy}{dx}=0$ (shown) Carry out implicit differentiation on the given result
	When $x = 0$, $y=0,\ \frac{dy}{dx}=2,\ \frac{d^{2}y}{dx^{2}}=-4$ $\therefore y=2x-2x^{2}+\ldots$ (*) Use Maclaurin's expansion formula in MF27: $f(x)=f(0)+xf'(0)+\frac{x^{2}}{2!}f''(0)+\ldots+\frac{x^{n}}{n!}f^{(n)}(0)+\ldots$
	$\ln\left(\frac{1+\tan^{-1}2x}{1-x}\right)$ $=\ln\left(1+\tan^{-1}2x\right)-\ln\left(1-x\right)$ $\approx\left(2x-2x^{2}\right)-\left(-x-\frac{1}{2}x^{2}\right)$ $=3x-\frac{3}{2}x^{2}$ **'Deduce'**: Relate to Maclaurin's series for $\ln\left(1+\tan^{-1}2x\right)$ found earlier For $\ln\left(1+\tan^{-1}2x\right)$: from (*) For $\ln\left(1-x\right)$: Use standard series for $\ln\left(1+x\right)$ in MF27, and replace x by $-x$ $\ln(1+x)=x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\ldots+\frac{(-1)^{r+1}x^{r}}{r}+\ldots$

Let's Try Now!

1 **EJC Promo 9758/2022/Q2**

It is given that $y = \mathrm{e}^{2x}\cos x$.

(a) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\dfrac{\mathrm{d}y}{\mathrm{d}x} - 5y$. [3]

(b) Find the Maclaurin series for y up to the term in x^2. [2]

(c) Hence, show that the Maclaurin series for $\mathrm{e}^x\sqrt{\cos x}$ is $1 + x + \dfrac{1}{4}x^2$, up to the term in x^2. [2]

(a)

$y = \mathrm{e}^{2x}\cos x$

Differentiate wrt x:

Differentiate using Product Rule

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}(-\sin x) + (\cos x)2\mathrm{e}^{2x}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\mathrm{e}^{2x}\sin x + 2y \text{ ------- (1)}$$

Rewrite your equation in terms of y using the substitution $y = \mathrm{e}^{2x}\cos x$ given in the question.

Differentiate wrt x:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\mathrm{e}^{2x}(\cos x) - \sin x(2\mathrm{e}^{2x}) + 2\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -y - 2\left(2y - \frac{\mathrm{d}y}{\mathrm{d}x}\right) + 2\frac{\mathrm{d}y}{\mathrm{d}x}$$

(From (1): $\mathrm{e}^{2x}(\sin x) = 2y - \dfrac{\mathrm{d}y}{\mathrm{d}x}$)

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\frac{\mathrm{d}y}{\mathrm{d}x} - 5y \text{ (shown)}$$

(b)

When $x = 0$,

$$y = y = \mathrm{e}^{2(0)}\cos 0 = 1,$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\mathrm{e}^{0}\sin 0 + 2(1) = 2,$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4(2) - 5(1) = 3$$

Thus

Concept: Use Maclaurin Theorem to find the Maclaurin Series required

➔ $\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \dfrac{x^2}{2!}\mathrm{f}''(0) + \cdots + \dfrac{x^n}{n!}\mathrm{f}^{(n)}(0) + \cdots$ (in MF 27)

$$y = 1 + 2x + \frac{3}{2!}x^2 + \ldots$$

$$= 1 + 2x + \frac{3}{2}x^2 + \ldots$$

(c)

$$e^{x}\sqrt{\cos x}=\left(e^{2x}\cos x\right)^{\frac{1}{2}}$$

$$=\left(1+2x+\frac{3}{2}x^{2}+\ldots\right)^{\frac{1}{2}}$$

Rewrite the expression such that the original expression $e^{2x}\cos x$ from **(a)** appears

See $\left(1+2x+\frac{3}{2}x^{2}\ldots\right)^{\frac{1}{2}}$ as $\left(1+\left(2x+\frac{3}{2}x^{2}\ldots\right)\right)^{\frac{1}{2}}$ and use the Binomial Expansion in MF27

$$(1+x)^{n}=1+nx+\frac{n(n-1)}{2!}x^{2}+\ldots+\frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^{r}+\ldots$$

$$=1+\frac{1}{2}\left(2x+\frac{3}{2}x^{2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(2x+\frac{3}{2}x^{2}\right)^{2}+\ldots$$

$$=1+x+\frac{3}{4}x^{2}-\frac{1}{8}\left(4x^{2}+\ldots\right)$$

$$=1+x+\frac{1}{4}x^{2}+\ldots$$

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Section 2: Using Standard Series

Series expansion	Validity range
$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$	$\lvert x \rvert < 1$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots$	all x
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots$	all x
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots$	all x
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots$	$-1 < x \leq 1$

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Example 3: 2011 NYJC Prelim/P1/3

Objectives:
1. Use of standard series (MF27) to find Maclaurin Series.
2. Use of Binomial Theorem when denominator is a polynomial (usually linear or quadratic).

By using standard series expansions, find the Maclaurin series for $f(x) = \ln\left(e^x \cos 2x\right)$ up to and including the term in x^2. [3]

Given that the first two non-zero terms in the Maclaurin series for $f(x)$ are equal to the first two non-zero terms in the series expansion of $\frac{2x}{a-bx}$, find *a* and *b*, where *a* and *b* are constants. [4]

3 Solution

Use $\ln ab = \ln a + \ln b$

$$f(x) = \ln\left(e^x \cos 2x\right)$$
$$= \ln e^x + \ln\left(\cos 2x\right)$$
$$= x + \ln\left(1 - \frac{(2x)^2}{2} + \ldots\right)$$
$$= x + \ln\left(1 - 2x^2 + \ldots\right)$$
$$= x - 2x^2 + \ldots$$

For $\cos 2x$: Use standard series for $\cos x$ in MF27, and replace x by $2x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots$$

For $\ln\left(1-2x^2\right)$: Use standard series for $\ln(1+x)$ in MF27, and replace x by $-2x^2$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots$$

$$\frac{2x}{a-bx} = 2x(a-bx)^{-1}$$
$$= 2xa^{-1}\left(1 - \frac{b}{a}x\right)^{-1}$$
$$= \frac{2}{a}x\left(1 - \frac{b}{a}x\right)^{-1}$$
$$\approx \frac{2}{a}x\left(1 + \frac{b}{a}x\right)$$

[Expand till x^2 so as to compare the terms to find a and b]

Make first term of the expansion 1 and use the standard series in MF27

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$$

$$\frac{2x}{a-bx} \approx \frac{2}{a}x + \frac{2b}{a^2}x^2 = x - 2x^2$$

Comparing like terms: $\frac{2}{a} = 1 \quad \Rightarrow \quad a = 2$

$\frac{2b}{a^2} = -2 \quad \Rightarrow \quad b = -a^2 = -4$

Let's Try Now!

2 MI PU2 P2 Promo 9758/2022/Q6(modified)

It is given that $y=\ln(\cos x)$.

(i) Find the Maclaurin series for $\ln(\cos x)$ up to and including the term in x^4. [2]

(ii) Hence, by substituting $x=\frac{\pi}{3}$, show that $\ln 2 \approx \frac{\pi^2}{18}+\frac{\pi^4}{972}$. [2]

Q2	MI PU2 P2 Promo 9758/2022/Q6(modified)
(i)	$\ln(\cos x)=\ln\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots\right)$ Use the Standard Series of $\cos x$ in MF27 $=\ln\left(1+\left(-\frac{x^2}{2!}+\frac{x^4}{4!}\right)+\ldots\right)$ $=-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{\left(-\frac{x^2}{2!}+\frac{x^4}{4!}\right)^2}{2!}+\ldots$ Use the Standard Series of $\ln(1+x)$ and replace x by $-\frac{x^2}{2!}+\frac{x^4}{4!}$ $=-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^4}{8}$ $=-\frac{x^2}{2}-\frac{x^4}{12}+\ldots$
(ii)	Substitute $x=\frac{\pi}{3}$ into the Maclaurin series in (i): $\ln\left(\cos\frac{\pi}{3}\right)=-\frac{\left(\frac{\pi}{3}\right)^2}{2}-\frac{\left(\frac{\pi}{3}\right)^4}{12}+\ldots$ Subst $x=\frac{\pi}{3}$ into both LHS and RHS of the Maclaurin Series. $\ln\left(\frac{1}{2}\right)\approx-\frac{\pi^2}{18}-\frac{\pi^4}{972}$ $\ln\left(2^{-1}\right)\approx-\frac{\pi^2}{18}-\frac{\pi^4}{972}$ $-\ln(2)\approx-\frac{\pi^2}{18}-\frac{\pi^4}{972}$ $\ln(2)\approx\frac{\pi^2}{18}+\frac{\pi^4}{972}$

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Section 4: The Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n$$

where n is a **POSITIVE INTEGER** and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

When n is a **NEGATIVE INTEGER OR A FRACTION**, the binomial expansion of $(1+x)^n$
is an infinite series which is **valid only for** $|x|<1$.

The following formula is given in the formulae list (MF27):

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots \text{ where } |x|<1$$

Note: The coefficient of x^r must be written as $\frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$

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Example 4

Objectives:

1. Use of Binomial Theorem when denominator is a polynomial (usually linear or quadratic).
2. Validity range of Binomial Theorem (MF27)

Expand $\frac{1-x}{2+x}$ in ascending powers of x up to and including the term in x^2 and state the set of values of x for which the expansion is valid. [6]

4	**Solution**
	$\frac{1-x}{2+x}=(1-x)\times 2^{-1}\left(1+\frac{x}{2}\right)^{-1}$ $=(1-x)\times\frac{1}{2}\left(1+\frac{x}{2}\right)^{-1}$ $=\left(\frac{1}{2}-\frac{x}{2}\right)\left(1-\frac{x}{2}+\left(\frac{x}{2}\right)^2+...\right)$ $=\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x+\frac{1}{4}x^2+\frac{1}{8}x^2+...$ $=\frac{1}{2}-\frac{3}{4}x+\frac{3}{8}x^2+...$ Express as a product of two terms. Factorise ½ such that we can use the standard series for $(1+x)^n$ where $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+...\quad ,\lvert x\rvert<1$ Expand till x^2 term
	Range of validity of *x*: $\left\lvert\frac{x}{2}\right\rvert<1\Rightarrow -2<x<2$ Set of values of *x*: $\{x\in\mathbb{R}:-2<x<2\}$ **Note:** Qn asks for Set Notation

Let's Try Now!

3 ASRJC Promo 9758/2022/Q1

By finding the expansion of $(1+3x)^{-1}$ or otherwise, find the expansion of $\frac{\sqrt{1+2x}}{1+3x}$ in ascending powers of *x*, up to and including the term in x^2. Find the range of values of *x* for which the expansion is valid. [4]

Q3	ASRJC Promo 9758/2022/Q1
	Rewrite the denominator as $(1+3x)^{-1}$ and use the Standard Series in MF27. $\frac{\sqrt{1+2x}}{1+3x}=(1+2x)^{\frac{1}{2}}(1+3x)^{-1}$ $=\left(1+\left(\frac{1}{2}\right)(2x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(2x)^2+...\right)\left(1+(-1)(3x)+\frac{(-1)(-2)}{2!}(3x)^2...\right)$ $=\left(1+x-\frac{1}{2}x^2+...\right)(1-3x+9x^2...)$ $=1-3x+x-\frac{1}{2}x^2+9x^2-3x^2+...$ $=1-2x+\frac{11}{2}x^2+...$ Valid for $\lvert 2x\rvert<1$ and $\lvert 3x\rvert<1$ $-\frac{1}{2}<x<\frac{1}{2}$ and $-\frac{1}{3}<x<\frac{1}{3}$ $\therefore -\frac{1}{3}<x<\frac{1}{3}$

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Section 3: Small Angle Approximation for Trigonometric Functions

For small x,
(a) $\sin x \approx x$
(b) $\cos x \approx 1-\frac{1}{2}x^2$
(c) $\tan x \approx x$

Note: x must be in radian

Useful Pre-requisite knowledge:
- ✓ Sine rule
- ✓ Cosine rule
- ✓ Trigonometric Ratio
- ✓ Trigonometric Identities in MF27

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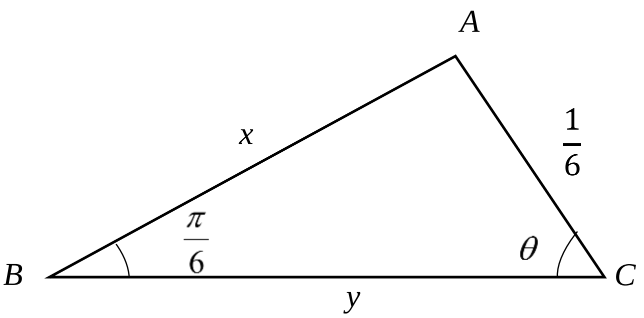
Example 5: 2016 HCI/I/10a

Objectives:
1. Use of small angle approximation to approximate trigonometric functions.
2. Use of Binomial Theorem when denominator is a polynomial (usually linear or quadratic).



In the triangle ABC, $AB = x$, $BC = y$, $AC = \frac{1}{6}$, angle $ABC = \frac{\pi}{6}$ radians and angle $ACB = \theta$ radians (see diagram).

(i) Show that $\frac{x}{y} = \frac{2\sin\theta}{\cos\theta + \sqrt{3}\sin\theta}$. [3]

(ii) Given that θ is sufficiently small, express $\frac{x}{y}$ as a cubic polynomial in θ. [3]

5	Solution
(i)	$\frac{\sin\theta}{x} = \frac{\sin\left(\pi - \frac{\pi}{6} - \theta\right)}{y}$ **Use sine rule** $\frac{x}{y} = \frac{\sin\theta}{\sin\left(\frac{5\pi}{6} - \theta\right)}$

	$\frac{x}{y}=\frac{\sin\theta}{\sin\frac{5\pi}{6}\cos\theta-\sin\theta\cos\frac{5\pi}{6}}$ **Refer to MF27 for sin (*A*-*B*)** $\frac{x}{y}=\frac{\sin\theta}{\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta}=\frac{2\sin\theta}{\cos\theta+\sqrt{3}\sin\theta}$ (shown) **Note:** Question asks for a **<u>cubic polynomial</u>**, so you cannot just use small angle approximation $\sin\theta\approx\theta$, you must use the Maclaurin's expansion for $\sin\theta$ to obtain up to θ^3.
(ii)	$\frac{x}{y}=\frac{2\sin\theta}{\cos\theta+\sqrt{3}\sin\theta}$ $\frac{x}{y}=\frac{2\left(\theta-\frac{\theta^3}{3!}+...\right)}{1-\frac{\theta^2}{2}+\sqrt{3}\left(\theta-\frac{\theta^3}{3!}+...\right)}$ $\frac{x}{y}\approx2\left(\theta-\frac{\theta^3}{3!}\right)\left(1+\left(\sqrt{3}\left(\theta-\frac{\theta^3}{3!}\right)-\frac{\theta^2}{2}\right)\right)^{-1}$ $\frac{x}{y}\approx2\left(\theta-\frac{\theta^3}{3!}\right)\left(1+(-1)\left(\sqrt{3}\theta-\frac{\theta^2}{2}\right)+\frac{(-1)(-2)}{2!}\left(\sqrt{3}\theta-\frac{\theta^2}{2}\right)^2\right)$ Expand till θ^2 term $\frac{x}{y}\approx2\left(\theta-\frac{\theta^3}{3!}\right)\left(1-\sqrt{3}\theta+\frac{\theta^2}{2}+3\theta^2\right)$ $\frac{x}{y}\approx2\theta-2\sqrt{3}\theta^2+\frac{20}{3}\theta^3$ Expand till θ^2 term

Let's Try Now!

4 MI PU2 P2 Promo 9758/2022/Q2

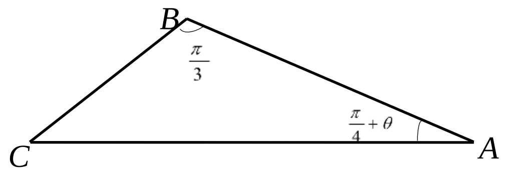
In triangle ABC, angle A is $\left(\frac{\pi}{4}+\theta\right)$ radians and angle B is $\frac{1}{3}\pi$ radians. Show that when θ is sufficiently small for terms in θ^3 and higher powers of θ to be neglected,

$$\frac{AC}{BC} \approx \frac{\sqrt{6}}{2}\left(1-\theta+k\theta^2\right)$$

where k is a constant to be found. [6]

Using sine rule,

$$\frac{AC}{\sin B} = \frac{BC}{\sin A} \Rightarrow \frac{AC}{BC} = \frac{\sin B}{\sin A}$$



$$\frac{AC}{BC} = \frac{\sin\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{4}+\theta\right)}$$

Use addition formula from MF27 to expand your denominator

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{\sqrt{2}}\left(\cos\theta+\sin\theta\right)}$$

$$= \frac{\sqrt{6}}{2}\left(\cos\theta+\sin\theta\right)^{-1}$$

Therefore,

$$\frac{AC}{BC} \approx \frac{\sqrt{6}}{2}\left(1-\frac{\theta^2}{2}+\theta\right)^{-1}$$

$$= \frac{\sqrt{6}}{2}\left(1-\left(\frac{\theta^2}{2}-\theta\right)\right)^{-1}$$

$$= \frac{\sqrt{6}}{2}\left(1+\left(\frac{\theta^2}{2}-\theta\right)+\left(\frac{\theta^2}{2}-\theta\right)^2+\ldots\right)$$

$$\approx \frac{\sqrt{6}}{2}\left(1-\theta+\frac{\theta^2}{2}+\theta^2\right)$$

$$= \frac{\sqrt{6}}{2}\left(1-\theta+\frac{3}{2}\theta^2\right) \qquad \Rightarrow k = \frac{3}{2}$$

For small x,
(a) $\sin x \approx x$
(b) $\cos x \approx 1-\frac{1}{2}x^2$

Answer Key

No.	Year	JC	Answers
1	2022	EJC	(b) $y \approx 1+2x+\frac{3}{2}x^2$
2	2022	MI	(i) $\ln(\cos x) = -\frac{x^2}{2}-\frac{x^4}{12}+\ldots$
3	2022	ASRJC	$1-2x+\frac{11}{2}x^2+\ldots$ $-\frac{1}{3}<x<\frac{1}{3}$
4	2022	MI	$k=\frac{3}{2}$