Name | Reg. No | Class

MAYFLOWER SECONDARY SCHOOL

4E5N

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 [80 marks]

PRELIMINARY EXAMINATION

14 September 2020

2 hours

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** the questions.

Write your answers on the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

Brand / Model of Calculator

For Examiner's Use

This question paper consists of **16** printed pages, including the cover page.

Setter: Ms Shen Sirui

Vetter: Mr Nara

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 The roots of the quadratic equation $x^2 - 2x + 5 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Show that $\alpha^3 + \beta^3 = -22$. [1]

(iii) Hence, find a quadratic equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$. [4]

i	$\alpha + \beta = -\frac{-2}{1} = 2$ $\alpha\beta = \frac{5}{1} = 5$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 2^2 - 2(5)$ $= -6$	M1 M1 A1
ii	$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= 2(-6 - 5)$ $= -22$	B1 for working
iii	$\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$ $= \frac{-22}{5^2}$ $= -\frac{22}{25}$ $\frac{\alpha}{\beta^2}\left(\frac{\beta}{\alpha^2}\right) = \frac{1}{\alpha\beta} = \frac{1}{5}$ New equation: $x^2 - (-\frac{22}{25})x + \frac{1}{5} = 0$ $x^2 + \frac{22}{25}x + \frac{1}{5} = 0$	 M1 (ecf from i) M1 (ecf from i) M1 (ecf from i) A1 (accept any equivalent form)

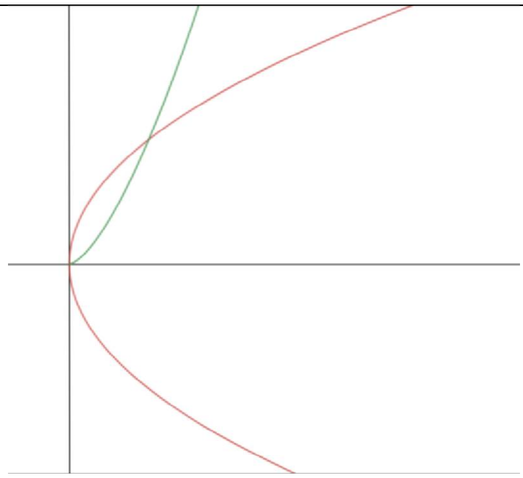
2 **(i)** An equilateral triangle has side $2(\sqrt{3}-1)$ cm. Find the exact value of the area of the triangle in the form $a\sqrt{b}+c$, where a, b and c are integers. [3]

(ii) A triangular prism has the equilateral triangle in **(i)** as its uniform cross-section. Given that the volume of the prism is $12(\sqrt{3}-1)$ cm^3, show that the height of the prism can be expressed in the form $p+2\sqrt{q}$, where p and q are integers. [4]

i	Area of triangle $= \frac{1}{2}\times 2(\sqrt{3}-1)\times 2(\sqrt{3}-1)\sin 60$ $= \frac{1}{2}(4)(3-2\sqrt{3}+1)\left(\frac{\sqrt{3}}{2}\right)$ $= \sqrt{3}(4-2\sqrt{3})$ $= 4\sqrt{3}-6$	M1 M1 for $3-2\sqrt{3}+1$ or $\left(\frac{\sqrt{3}}{2}\right)$ A1
ii	Vol of prism = Area of triangle × height Height $= \frac{12(\sqrt{3}-1)}{4\sqrt{3}-6}$ $= \frac{6\sqrt{3}-6}{2\sqrt{3}-3}\times\frac{2\sqrt{3}+3}{2\sqrt{3}+3}$ $= \frac{(6\sqrt{3}-6)(2\sqrt{3}+3)}{(2\sqrt{3})^2-3^2}$ $= \frac{36+18\sqrt{3}-12\sqrt{3}-18}{12-9}$ $= \frac{18+6\sqrt{3}}{3}$ $= 6+2\sqrt{3}$	Vol. of prism = Cross-sectional area × height M1 for $\times\frac{2\sqrt{3}+3}{2\sqrt{3}+3}$ (ecf from i) M1 for $36+18\sqrt{3}-12\sqrt{3}-18$ (ecf from i) M1 for $18+6\sqrt{3}$ (ecf from i) A1

3 **(a)** On the same axes, sketch the curves $y^2 = 6x$ and $y = x^{\frac{3}{2}}$, for $x \geq 0$. [2]

(b) Solve $\log_3(2x - 17) = \log_9 81 - \log_3 x$. [4]

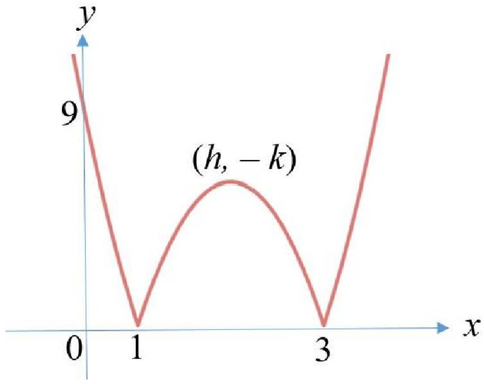
a		B1 for $y^2 = 6x$ parabola B1 for $y = x^{\frac{3}{2}}$ sub $x=0$, $y=0$ $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ As $x \uparrow$, $\frac{dy}{dx} \uparrow$ So, graph is
b	$\log_3(2x - 17) = \log_9 81 - \log_3 x$ $\log_3(2x - 17) + \log_3 x = 2$ $\log_3 x(2x - 17) = 2$ $2x^2 - 17x = 9$ $2x^2 - 17x - 9 = 0$ $(2x + 1)(x - 9) = 0$ $x = -\frac{1}{2}$ (rej), $x = 9$	 M1 for $\log_9 81 = 2$ M1 for $\log_3 x(2x - 17)$ M1 A1

4 **(i)** Prove that $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$. [4]

(ii) Hence, solve the equation $\frac{\cos 2\theta}{1+\sin 2\theta} + \frac{1+\sin 2\theta}{\cos 2\theta} = \tan^2 2\theta - 2$ for $0 \le \theta \le 6$. [5]

i	$LHS = \dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta}$ $= \dfrac{\cos^2\theta + (1+\sin\theta)^2}{\cos\theta(1+\sin\theta)}$ $= \dfrac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$ $= \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$ $= \dfrac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$ $= \dfrac{2}{\cos\theta}$ $= 2\sec\theta = RHS$ $= 2\sec 2\theta$ (from i)	M1 for combining fractions M1 for $2+2\sin\theta$ M1 for factorised numerator and denominator A1
ii	$\dfrac{\cos 2\theta}{1+\sin 2\theta} + \dfrac{1+\sin 2\theta}{\cos 2\theta} = \tan^2 2\theta - 2$ $2\sec 2\theta = \tan^2 2\theta - 2$ $2\sec 2\theta = \sec^2 2\theta - 1 - 2$ $\sec^2 2\theta - 2\sec 2\theta - 3 = 0$ $(\sec 2\theta - 3)(\sec 2\theta + 1) = 0$ $\sec 2\theta = 3$ $\qquad$ $\sec 2\theta = -1$ $\cos 2\theta = \dfrac{1}{3}$ $\qquad$ $\cos 2\theta = -1$ Basic $\angle = 1.2309$ $2\theta = 1.2309, 2\pi - 1.2309,$ $1.2309 + 2\pi, 2\pi - 1.2309 + 2\pi$ $\qquad$ $2\theta = \pi, \pi + 2\pi$ $\theta = 0.61545, 2.5261, 3.7570, 5.6677$ $\qquad$ $\theta = 1.5707, 4.7123$ $\theta = 0.615, 2.53, 3.76, 5.67$ $\qquad$ $\theta = 1.57, 4.71$ (3sf)	M1 for $\sec^2 2\theta - 1$ M1 for factorised form or formula (-1 mark if not shown) M1 for Basic $\angle = 1.2309$ A1 for $0.615, 2.53, 3.76, 5.67$ A1 for $1.57, 4.71$

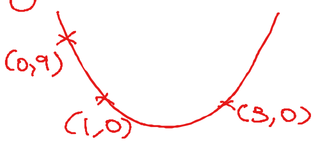
5 The diagram shows part of the curve $y = |a(x-h)^2 + k|$, where $a > 0$. The curve touches the x-axis at $(1,0)$ and $(3,0)$ and has a stationary point at $(h, -k)$. The curve intersects the y-axis at 9.



(i) Explain why $h = 2$. [1]

(ii) Find the value of a and of k. [3]

(iii) State the set of values of m for which the line $y = m$ intersects the curve $y = |a(x-h)^2 + k|$ at 4 distinct points. [1]

i	$x = h$ is the line of symmetry, so $h = \frac{1+3}{2} = 2$.	B1 accept h is exactly in between 1 and 3
* ii	For $y = a(x-h)^2 + k$, Sub $(0,9)$, $9 = a(0-2)^2 + k$ $9 = 4a + k$ – – – (1) Sub $(1,0)$, $0 = a(1-2)^2 + k$ $0 = a + k$ – – – (2) $(1)-(2)$: $3a = 9$ $a = 3$ $k = -3$ Since $a > 0$, $y = a(x-h)^2 + k$ is 	M1 for sub (0,9) to form equation M1 for sub (1,0) or (3,0) into $y = a(x-h)^2 + k$ to form equation A1
iii	$0 < m < 3$	B1

6 **(i)** Find the equation of the line which is perpendicular to the line $4x - 3y - 1 = 0$ at the point $(4, 5)$. [2]

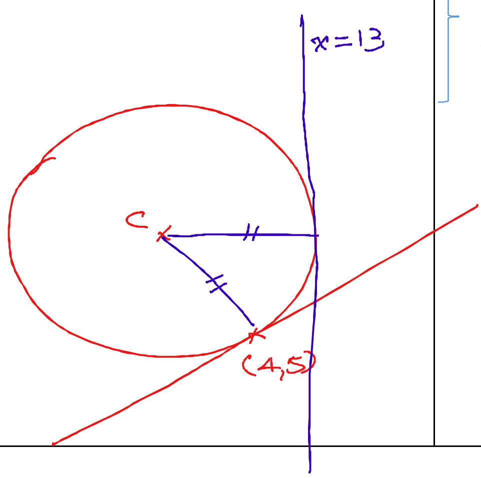
The line $x = 13$ is a tangent to the circle C and the equation of the tangent to C at $(4, 5)$ is $4x - 3y - 1 = 0$.

(ii) The x-coordinate of the centre of C is a, where $a > 0$. By writing the y-coordinate of the centre of C in terms of a, show that $a^2 + 24a - 256 = 0$. [4]

(iii) Find the equation of C. [3]

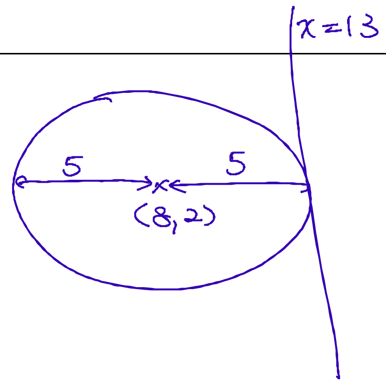
(iv) Determine whether the point $(10, 8)$ lies inside, on or outside C. [1]

(v) Find the range of values of p such that the line $x = p$ does not meet C. [1]

i	$4x - 3y - 1 = 0$ $m_{perpendicular} = -\frac{1}{\frac{4}{3}} = -\frac{3}{4}$ Sub $(4,5), 5 = -\frac{3}{4}(4) + c$ $c = 8$ Perpendicular line: $y = -\frac{3}{4}x + 8$	M1 for $m_{perpendicular}$ A1
ii	Centre: $y = -\frac{3}{4}a + 8$ centre lies on perpendicular line to tangent Distance bet centre & $(4,5) = 13 - a$ $\sqrt{(a-4)^2 + \left(-\frac{3}{4}a + 8 - 5\right)^2} = 13 - a$ $a^2 - 8a + 16 + \left(3 - \frac{3}{4}a\right)^2 = (13 - a)^2$ $a^2 - 8a + 16 + 9 - \frac{9}{2}a + \frac{9}{16}a^2 = 169 - 26a + a^2$ $\frac{9}{16}a^2 + \frac{27}{2}a - 144 = 0$ $a^2 + 24a - 256 = 0$ 	M1 (ecf from i) M1 (ecf) for $\sqrt{(a-4)^2 + \left(-\frac{3}{4}a + 8 - 5\right)^2}$ M1 (ecf) for $\sqrt{(a-4)^2 + \left(-\frac{3}{4}a + 8 - 5\right)^2} = 13 - a$ A1

iii	$a^2 + 24a - 256 = 0$ $(a-8)(a+32) = 0$ $a = 8, \quad a = -32$ (rej) Radius of $C = 13 - 8 = 5$ units Centre of $C = \left(8, -\frac{3}{4}(8) + 8\right) = (8, 2)$ Equation of C: $(x-8)^2 + (y-2)^2 = 25$	M1 for $a = 8$ M1 for either radius or centre (ecf from i) A1 (ignore subsequent working)
iv	Distance from centre $= \sqrt{(10-8)^2 + (8-2)^2} = 6.3245$ Since the distance of (10, 8) from the centre is > radius, (10, 8) lies outside the circle.	B1
v	$p < 3, p > 13$	B1

v)



7 A radioactive substance is known to decay with time such that its mass, M grams, after t hours is given by $M = M_0 e^{-kt}$, where k is a positive constant. The radioactive substance has an initial mass of 150 grams. After 30 hours, its mass decreased to 90 grams.

(i) Find the mass of the substance after 100 hours. [3]

(ii) Explain why the mass of the substance can never reach zero gram. [1]

i	Sub $t = 0, M = 150$, $150 = M_0$ Sub $t = 30, M = 90$, $90 = 150e^{-k(30)}$ $e^{-k(30)} = 0.6$ $k = -\frac{\ln 0.6}{30} = 0.017027$ Sub $t = 100$, $M = 150e^{-0.017027(100)}$ $M = 27.328$ $M = 27.3$ g (3 sf)	 M1 M1 A1
ii	Since $e^{-kt} > 0, M_0e^{-kt} > 0$. Hence, $M > 0$ for all values of t and mass can never reach zero gram.	B1

8 **(a)** **(i)** Write down, and simplify, the first three terms in the expansion of $(1-2x)^7$ in ascending powers of x. [2]

(ii) The first three terms in the expansion, in ascending powers of x, of $(p+qx)(1-2x)^7$ are $3+rx+182x^2$. Find the value of p, of q and of r. [4]

(b) Given that the term independent of x in the binomial expansion of $(x^2+\frac{k}{x})^9$ is 5376, find the value of the positive constant k. [4]

ai	$(1-2x)^7 = 1^7 + \binom{7}{1}(1^6)(-2x)^1 + \binom{7}{2}(1^5)(-2x)^2 + \cdots$ $= 1 - 14x + 84x^2 + \cdots$	M1 for either 2nd or 3rd term A1
aii	$(p+qx)(1-2x)^7 = (p+qx)(1-14x+84x^2+\cdots)$ $= p - 14px + qx + 84px^2 - 14qx^2 + \cdots$ By comparing coefficients, $p = 3$ $84p - 14q = 182$ $84(3) - 14q = 182$ $q = 5$ $-14p + q = r$ $r = -14(3) + 5 = -37$	M1 for $-14px + qx + 84px^2 - 14qx^2$ (ecf from i) A1 A1 A1
b	$\left(x^2 + \frac{k}{x}\right)^9$ $T_{r+1} = \binom{9}{r}(x^2)^{9-r}\left(\frac{k}{x}\right)^r$ For x^0, $(x^2)^{9-r}(x^{-1})^r = x^0$ $2(9-r) - r = 0$ $r = 6$ $\binom{9}{6}(x^2)^{9-6}\left(\frac{k}{x}\right)^6 = 5376$ $84k^6 = 5376$ $k^6 = 64$ $k = 2, \quad -2(\text{rej})$	M1 for general term M1 for $r = 6$ M1 for equation A1 (ignore if students did not write $k = -2$)

9 The equation of a curve is $y = -2x^3 + gx + h$, where g and h are constants. The curve has a stationary point at $(-2, -16)$.

(i) Find the value of g and of h. [4]

(ii) Find the coordinates of the other stationary point of the curve and determine the nature of this stationary point. [4]

i	$y = -2x^3 + gx + h$ Sub $(-2,-16)$, $-2(-2)^3 + g(-2) + h = -16$ $16 - 2g + h = -16$ $h = 2g - 32$ curve has stationary value at $(-2, -16)$. So, $\frac{dy}{dx} = 0$ when $x = -2$ $\frac{dy}{dx} = -6x^2 + g$ Sub $x = -2, \frac{dy}{dx} = 0,$ $-6(-2)^2 + g = 0$ $g = 24$ $h = 16$	M1 for sub $(-2,-16)$ M1 for $\frac{dy}{dx}$ M1 for sub $x = -2, \frac{dy}{dx} = 0$ A1
ii	$y = -2x^3 + 24x + 16$ $\frac{dy}{dx} = -6x^2 + 24$ $\frac{dy}{dx} = 0,$ $-6x^2 + 24 = 0$ $x = 2,$ -2 $y = 48$ $\frac{d^2y}{dx^2} = -12x$ Sub $x = 2,$ $\frac{d^2y}{dx^2} = -24 < 0$ (2, 48) is max point	M1 for $x = 2$ (ecf from i) M1 for $y = 48$ (ecf from i) M1 for 1st or 2nd derivative test (ecf from i) A1

10 The polynomial $\mathrm{f}(x) = x^3 + ax^2 + bx - 18$, where a and b are constants, is exactly divisible by $x - 2$. Given that $\mathrm{f}(x)$ leaves a remainder of -30 when divided by $x + 1$,

(i) find the value of a and of b, [4]

(ii) determine, showing all necessary working, the number of real roots of the equation $\mathrm{f}(x) = 0$. [3]

i	$f(2) = 0$ $2^3 + a(2^2) + 2b - 18 = 0$ $4a + 2b = 10$ $b = 5 - 2a ---(1)$ $f(-1) = -30$ $(-1)^3 + a(-1)^2 - b - 18 = -30$ $a - b = -11$ $b = a + 11 ---(2)$ $(1) = (2)$: $5 - 2a = a + 11$ $a = -2$ $b = 9$	 M1 M1 A1 A1
ii	$x^3 - 2x^2 + 9x - 18 = 0$ $x^2 + 9$ $x - 2\sqrt{x^3 - 2x^2 + 9x - 18}$ $-(x^3 - 2x^2)$ $9x - 18$ $-(9x - 18)$ 0 $(x - 2)(x^2 + 9) = 0$ $x = 2, \quad x^2 = -9$ OR: $b^2 - 4ac = 0^2 - 4(1)(9)$ $= -36 < 0$ No real roots for $x^2 + 9 = 0$. Since $x^2 \geq 0, \ x^2 = -9$ is undefined. Hence, there is only 1 real root for $\mathrm{f}(x) = 0$.	 M1 for long division (ecf from i) M1 for $(x^2 + 9)$ A1 (accept $b^2 - 4ac < 0$)

11 **(i)** Given that $y = (x+3)\sqrt{2x-3}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{kx}{\sqrt{2x-3}}$. [3]

(ii) Hence, find $\int \frac{x}{\sqrt{2x-3}} dx$. [2]

i	$y = (x+3)\sqrt{2x-3}$ $\frac{dy}{dx} = 1(2x-3)^{\frac{1}{2}} + (x+3)\left(\frac{1}{2}\right)(2x-3)^{-\frac{1}{2}}(2)$ $= (2x-3)^{\frac{1}{2}} + (x+3)(2x-3)^{-\frac{1}{2}}$ $= (2x-3)^{-\frac{1}{2}}(2x-3+x+3)$ $= \frac{3x}{\sqrt{2x-3}}$	M1 for $1(2x-3)^{\frac{1}{2}}$ M1 for $(x+3)\left(\frac{1}{2}\right)(2x-3)^{-\frac{1}{2}}(2)$ A1
ii	$\int \frac{3x}{\sqrt{2x-3}} dx = (x+3)\sqrt{2x-3} + c_1$ $\int \frac{x}{\sqrt{2x-3}} dx = \frac{1}{3}\left[(x+3)\sqrt{2x-3} + c_1\right]$ $= \frac{1}{3}(x+3)\sqrt{2x-3} + c$	M1 (soi) A1 (ignore all constants)