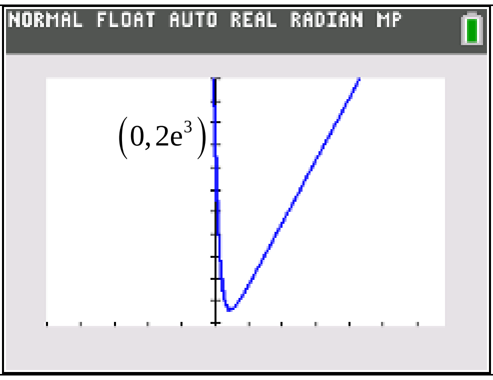
2022 NYJC J2 H2 Mathematics Prelim 8865/1 Marking Guide

Q1	**Suggested Answers**
	For $\left(3-4k\right)x^2+2x-k<0$, $3-4k<0$ and $2^2-4\left(3-4k\right)\left(-k\right)<0$ $k>\frac{3}{4}$ and $4+12k-16k^2<0$ $k>\frac{3}{4}$ and $4k^2-3k-1>0$ $k>\frac{3}{4}$ and $\left(4k+1\right)\left(k-1\right)>0$ $k>\frac{3}{4}$ and $k<-\frac{1}{4}$ or $k>1$ Combining solution on a number line, $k>1$

Q2	**Suggested Answers**
(i)	$y=ax^3+bx^2+cx+d$ At $\left(0,2\right)$, $d=2$
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3ax^2+2bx+c$ Since $\left(0,2\right)$ is a maximum point, $c=0$ $\left(-5,-\frac{119}{3}\right)$ is a minimum point, $3a\left(-5\right)^2+2b\left(-5\right)+c=0$ $75a-10b=0$ ----------(1) $-\frac{119}{3}=a\left(-5\right)^3+b\left(-5\right)^2+2$ $125a-25b=\frac{125}{3}$ -------(2) Using GC, $a=-\frac{2}{3}$, $b=-5$ Hence $y=-\frac{2}{3}x^3-5x^2+2$

Q3	**Suggested Answers**
(i)	$\ln\left(\frac{\mathrm{e}^{\sqrt{x}}}{(1-2x)^3}\right)=\sqrt{x}\ln\mathrm{e}-3\ln(1-2x)=\sqrt{x}-3\ln(1-2x)$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\frac{\mathrm{e}^{\sqrt{x}}}{(1-2x)^3}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}-3\ln(1-2x)\right)$ $=\frac{1}{2}x^{-\frac{1}{2}}-3\left(\frac{-2}{1-2x}\right)$ $=\frac{1}{2\sqrt{x}}+\frac{6}{1-2x}$

(ii)	$\int_{-1}^{e}\left(2\sqrt{x}-\frac{3}{\sqrt{x}}\right)^2 \mathrm{d}x$ $=\int_{-1}^{e}\left(4x-12+\frac{9}{x}\right)\mathrm{d}x$ $=\left[2x^2-12x+9\ln\lvert x\rvert\right]_{-1}^{e}$ $=2e^2-12e+9\ln e-\left(2+12+9\ln\lvert -1\rvert\right)$ $=2e^2-12e+9-14$ $=2e^2-12e-5$

Q4	**Suggested Answers**
(i)	$y=14x+2e^{3-7x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=14-14e^{3-7x}$ $0=14-14e^{3-7x}$ $1=e^{3-7x}$ $\ln 1=3-7x$ $x=\frac{3}{7}$ At $x=\frac{3}{7}$, $y=14\left(\frac{3}{7}\right)+2e^{3-7\left(\frac{3}{7}\right)}=6+2=8$ Coordinates of turning point: $\left(\frac{3}{7},8\right)$
(ii)	
(iii)	$x=1\Rightarrow y=14+2e^{-4}$ & $\frac{\mathrm{d}y}{\mathrm{d}x}=14-14e^{-4}$ Equation of the tangent: $y-\left(14+2e^{-4}\right)=\left(14-14e^{-4}\right)(x-1)$ $y=\left(14-14e^{-4}\right)x-14+14e^{-4}+14+2e^{-4}$ $y=\left(14-14e^{-4}\right)x+16e^{-4}$
(iv)	Area required $=\int_0^1 14x+2e^{3-7x}\mathrm{d}x-\frac{1}{2}(1)\left(16e^{-4}+14+2e^{-4}\right)$

	$= \left[7x^2 - \frac{2e^{3-7x}}{7}\right]_0^1 - 7.1648$ $= 5.57$ (3 s.f.)
	Alternative: Area required $= \int_0^1 14x + 2e^{3-7x}\,dx - \int_0^1 \left(14 - 14e^{-4}\right)x + 16e^{-4}\,dx$ $= 5.57$ (3 s.f.)

Q5	**Suggested Answers**
(i)	$h = 100 - 4r$ Volume of container, $V = \pi r^2 h$ $= \pi r^2 (100 - 4r)$ $= 4\pi\left(25r^2 - r^3\right)$ $\frac{dV}{dr} = 4\pi\left(50r - 3r^2\right)$ For maximum volume, $\frac{dV}{dr} = 0$ $4\pi\left(50r - 3r^2\right) = 0 \Rightarrow 4\pi r(50 - 3r) = 0$ $r = 0$ (rejected) or $r = \frac{50}{3}$

r	$\left(\frac{50}{3}\right)^-$ E.g. 16.65	$\frac{50}{3}$	$\left(\frac{50}{3}\right)^+$ E.g. 16.67
$\frac{dV}{dr}$	$10.4615 > 0$	0	$-2.0948 < 0$
Slope	/	–	\

	Hence $r = \frac{50}{3}$ gives a maximum volume of 29088.82 cm^3
	Alternative method $\frac{d^2V}{dr^2} = 4\pi(50 - 6r)$ When $r = \frac{50}{3}$, $\frac{d^2V}{dr^2} = -200\pi \Rightarrow r = \frac{50}{3}$ gives maximum volume of 29088.82 cm^3
(ii)	Amount of discarded metal $= 100(40) - 2\pi r^2 - 40h$ $= 4000 - 2\pi r^2 - 40(100 - 4r)$ $= 160r - 2\pi r^2$ $= 2\left(80r - \pi r^2\right)$ Alternative method Amount of discarded metal $= 2\left[(2r)(40) - \pi r^2\right]$ $= 2\left(80r - \pi r^2\right)$

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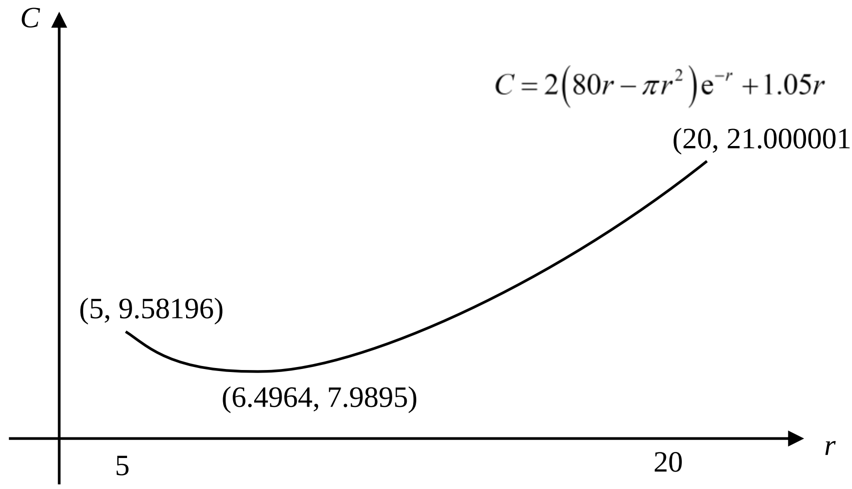
$C = De^{-r} + 1.05r = 2\left(80r - \pi r^2\right)e^{-r} + 1.05r$



From the sketch, the least cost to discard the waste metal from a container is $7.99.
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Maximum volume occurs when $r = \frac{50}{3} = 16.667 \neq 6.4964$

Hence, the supervisor's claim is false.
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When $r = \frac{50}{3}$, $C = 2\left[80\left(\frac{50}{3}\right) - \pi\left(\frac{50}{3}\right)^2\right]e^{-\frac{50}{3}} + 1.05\left(\frac{50}{3}\right) = \$17.50$
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<tr><th>Q6</th><th>Suggested Answers</th></tr>
<tr><td>(i)</td><td>

<u>**Case 1: 5 boys, 0 girls:**</u> No. of ways $= {}^{10}C_5 = 252$

<u>**Case 2: 4 boys, 1 girls:**</u> No. of ways $= \left({}^{10}C_4\right)\left({}^{15}C_1\right) = 3150$

<u>**Case 3: 3 boys, 2 girls:**</u> No. of ways $= \left({}^{10}C_3\right)\left({}^{15}C_2\right) = 12600$

Total no. of ways

$= 252 + 3150 + 12600$

$= 16002$
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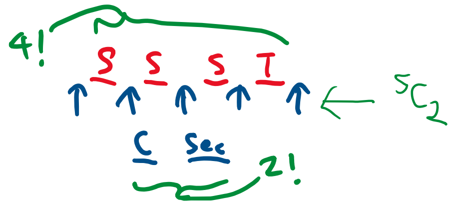

${}^5C_2$ ways to insert the Chairperson & Secretary such that there are exactly 2 students between them

$2!$ ways to arrange between Chairperson & Secretary

$4!$ ways to arrange the other 3 students and the Civics tutor

Total no. of ways

$= \left({}^5C_2\right)(2!)(4!)$

$= 480$
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<tr><td>(ii)</td><td>

<u>**Alternative: Complement method**</u>

$6! - (5!)(2!) = 480$
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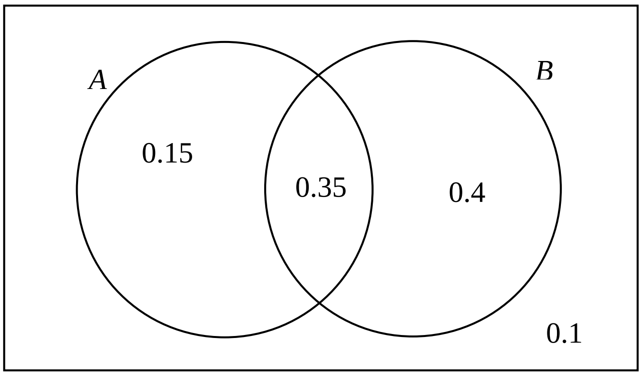
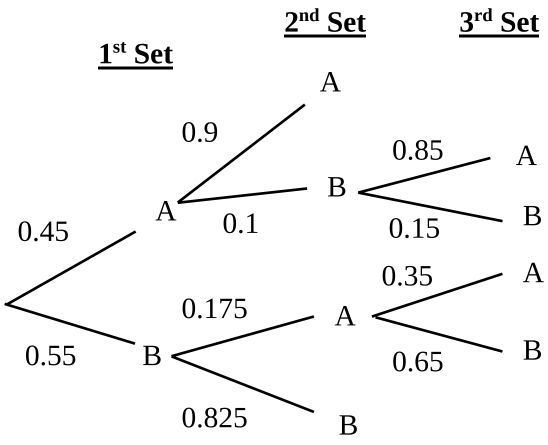
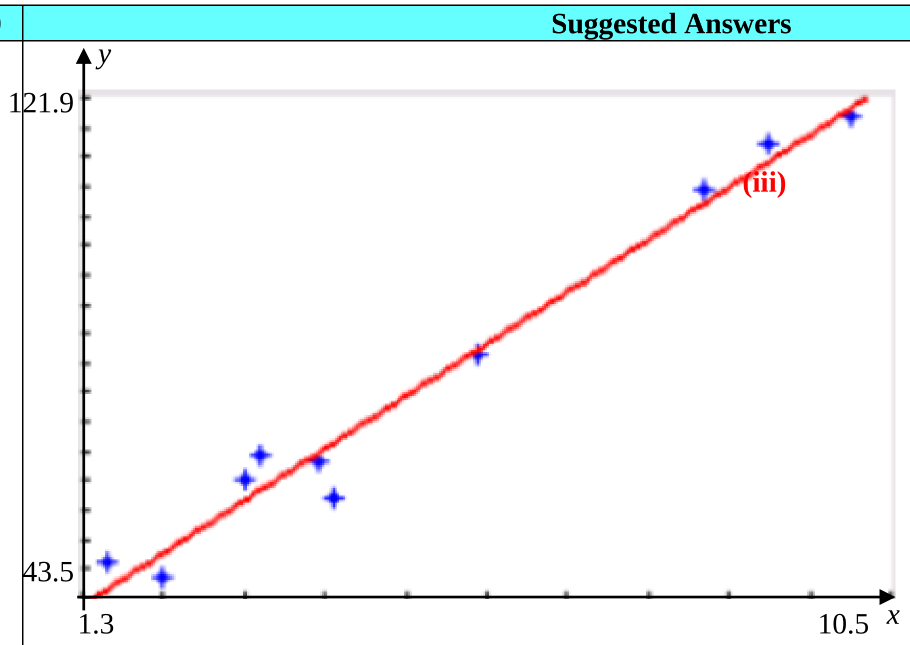
Q7	**Suggested Answers**
(i)	Let X be the number of ripe apples, out of n. $X \sim \text{B}(12, p)$ $\text{Var}(X) = 12p(1-p) = 1.53$ $p = 0.15$ (rejected as $0.5 < p < 1$) or $p = 0.85$
(ii)	$\therefore X \sim \text{B}(12, 0.85)$ $\text{P}(X \geq 9)$ $= 1 - \text{P}(X \leq 8)$ $= 0.90779$ (5 s.f.) $= 0.908$ (3 s.f.)
(iii)	Let Y be the number of boxes with at least 75% of the apples being ripe, out of 20. $Y \sim \text{B}(20, 0.90779)$ $\text{P}(13 < Y \leq 18) = \text{P}(Y \leq 18) - \text{P}(Y \leq 13)$ $= 0.56062$ (5 s.f.) $= 0.561$ (3 s.f.)
(iv)	Let T be the number of apples that are not ripe, out of 240 apples. $T \sim \text{B}(240, 0.15)$ $\text{P}(T \leq 30)$ $= 0.15995$ (5 s.f.) $= 0.160$ (3 s.f.)
	<u>Alternative</u> Let T be the number of apples that are ripe, out of 240 apples. $T \sim \text{B}(240, 0.85)$ $\text{P}(T \geq 210)$ $= 1 - \text{P}(T \leq 209)$ $= 0.15995$ (5 s.f.) $= 0.160$ (3 s.f.)

Q8	**Suggested Answers**
(i)	$\text{P}(B \mid A) = 0.7 \Rightarrow \frac{\text{P}(A \cap B)}{\text{P}(A)} = 0.7$ $\Rightarrow \text{P}(A) = \frac{0.35}{0.7} = 0.5$ $\text{P}(B) = \text{P}(A \cup B) - \text{P}(A) + \text{P}(A \cap B) = 0.9 - 0.5 + 0.35 = 0.75$
(ii)	$\text{P}(A' \cap B')$ represents the probability that both A and B do not occur. $\text{P}(A' \cap B') = 1 - \text{P}(A \cup B) = 1 - 0.9 = 0.1$
(iii)	$\text{P}(B \mid A) = 0.7 \neq \text{P}(B)$ $\therefore$ A and B are not independent. <u>Alternative:</u> $\text{P}(A)\text{P}(B) = (0.5)(0.75) = 0.375 \neq \text{P}(A \cap B)$ $\therefore$ A and B are not independent.

(iv)	

Q9	**Suggested Answers**
(i)	
(ii)	P(Ariana wins the match) $=(0.45)(0.9)+(0.45)(0.1)(0.85)+(0.55)(0.175)(0.35)$ $=0.47694$ (5 s.f.) $=0.477$ (3 s.f.)
(iii)	P(Ariana lost the second set|Ariana lost the match) $=\dfrac{\text{P(Ariana lost the second set and lost the match)}}{\text{P(Ariana lost the match)}}$ $=\dfrac{(0.45)(0.1)(0.15)+(0.55)(0.825)}{1-0.47694}$ $=0.88039$ (5 s.f.) $=0.880$ (3 s.f.)

Q10	**Suggested Answers**
(i)	
(ii)	Using GC, the product moment correlation coefficient, r = 0.98762, i.e. 0.988 Since r is close to 1, it suggests that as the years of working experience of an employee, x, increases, the yearly salary, y, increases in a strong linear correlation. This is also consistent with the scatter diagram which shows that as the years of working experience increase, the yearly salary also increases in a linear trend
(iii)	Using GC, equation of the regression line is $y = 8.8670x + 30.223$, i.e. $y = 8.87x + 30.2$
(iv)	When $x = 6$, $y = 8.8670(6) + 30.223 = 83.425$ Hence yearly salary of an employee with 6 years of working experience is $83425 Estimate is reliable as $x = 6$ lies within the data range $1.3 \le x \le 10.5$ and r is close to 1.
(v)	Value of r is not changed as scaling does not affect the relationship between the variables.

Q11	**Suggested Answers**
(i)	Unbiased estimate of population mean, $\overline{x} = \dfrac{141.68}{35}$ $= 4.048$ Unbiased estimate of population variance, $s^2 = \dfrac{1}{34}\left[573.74 - \dfrac{(141.68)^2}{35}\right]$ = 0.0064518 = 0.00645
(ii)	Let X be the random variable denoting the mass of a paperclip and μ the population mean mass. To test $H_0 : \mu = 4$ at 1% level of significance $H_1 : \mu > 4$

	Under H_0, since *n* = 35 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(4, \frac{0.0064518}{35}\right)$ approximately $Z = \frac{\overline{X}-4}{\sqrt{\frac{0.0064518}{35}}} \sim \mathrm{N}(0,1)$ Using a one-tailed test, $\overline{x} = 4.048$ gives *p*-value = 0.00020364 Since *p*-value < 0.01, we reject H_0 and conclude that at 1% level of significance, there is sufficient evidence that the quality control officer's claim that the mass of paperclip is understated is justified.
(iii)	1% level of significance means that there is a probability of 0.01 of wrongly concluding the mean mass of paperclip is understated when it is not.
(iv)	Given $X \sim \mathrm{N}\left(\mu, 0.08^2\right)$ To test $H_0 : \mu = 4$ at 1% level of significance $H_1 : \mu > 4$ Under H_0, $\overline{X} \sim \mathrm{N}\left(4, \frac{0.08^2}{n}\right)$ $Z = \frac{\overline{X}-4}{\frac{0.08}{\sqrt{n}}} \sim \mathrm{N}(0,1)$ Reject H_0 if $z_{cal} \geq 2.3263$ Since the quality control officer's claim is favoured, H_0 is rejected. Hence $z_{cal} \geq 2.3263$ $\frac{4.048-4}{0.08/\sqrt{n}} \geq 2.3263$ $\frac{0.048}{0.08}\sqrt{n} \geq 2.3263$ $\sqrt{n} \geq 3.877167$ $n \geq 15.032$ Hence smallest *n* is 15

Q12	**Suggested Answers**
(i)	Let C denote the preparation time taken by Cassano for a randomly chosen order. $C \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $\mu = \dfrac{10+12.8}{2} = 11.4$ $\mathrm{P}(C < 12.8) = 0.782$ $\Rightarrow \mathrm{P}(Z < \dfrac{12.8-11.4}{\sigma}) = 0.782$ $\Rightarrow \dfrac{1.4}{\sigma} = 0.77897$ $\Rightarrow \sigma = 1.7973$ (5 s.f.) $= 1.80$ (3 s.f.)
(ii)	Let V denote the preparation time taken by Vincenzo for a randomly chosen order. $V \sim \mathrm{N}\left(10.8, 1.2^2\right) \Rightarrow \overline{V} = \dfrac{V_1 + \ldots + V_5}{5} \sim \mathrm{N}\left(10.8, \dfrac{1.2^2}{5}\right)$ $C \sim \mathrm{N}\left(11.4, 1.8^2\right) \Rightarrow \overline{C} = \dfrac{C_1 + \ldots + C_3}{3} \sim \mathrm{N}\left(11.4, \dfrac{1.8^2}{3}\right)$ $\mathrm{E}\left(\overline{V} - \overline{C}\right) = \mathrm{E}\left(\overline{V}\right) - \mathrm{E}\left(\overline{C}\right) = -0.6$ $\mathrm{Var}\left(\overline{V} - \overline{C}\right) = \mathrm{Var}\left(\overline{V}\right) + \mathrm{Var}\left(\overline{C}\right) = 1.368$ $\overline{V} - \overline{C} \sim \mathrm{N}\left(-0.6, 1.368\right)$ $\mathrm{P}\left(\lvert\overline{V} - \overline{C}\rvert < 1.5\right) = \mathrm{P}\left(-1.5 < \overline{V} - \overline{C} < 1.5\right)$ $= 0.74291$ (5 s.f.) $= 0.743$ (3 s.f.)
(iii)	Let I denote the duration of each break, in minutes. $I \sim \mathrm{N}\left(1.5, 0.4^2\right)$ Let $T = V_1 + \ldots + V_5 + C_1 + \ldots + C_5 + I_1 + \ldots + I_9$ $\mathrm{E}(T) = 5(10.8) + 5(11.4) + 9(1.5) = 124.5$ $\mathrm{Var}(T) = 5(1.2^2) + 5(1.8^2) + 9(0.4^2) = 24.84$ $T \sim \mathrm{N}\left(124.5, 24.84\right)$ $\mathrm{P}\left(T > 120\right) = 0.81671$ (5 s.f.) $= 0.817$ (3 s.f.)
(iv)	Required probability $= \left[\mathrm{P}\left(T > 120\right)\right]^2$ $= \left(0.81671\right)^2$ $= 0.66702$ (5 s.f.) $= 0.667$ (3 s.f.)
(v)	The event in (iv) is a subset of the event that a total of more four hours is needed for both chefs to complete 20 orders in the lunch and dinner shifts.