Name _____Solutions_____________ ( )	Class 4 ______

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

24 August 2023
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to three significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Marks	3	4	4	3	4	7	8	6	11	11	9	8	8	4

Table of Penalties		Qn. No.		
Presentation	–1			
Accuracy/ Units	–1		**Parent's/ Guardian's Signature**	**90**

This question paper consists of 20 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $k = 2\sqrt{2} - \sqrt{3}$, **without using the calculator**, express $3k - \dfrac{2}{k}$ in the form $\dfrac{a\sqrt{2} - b\sqrt{3}}{c}$, where a, b and c are integers. [3]

$k = 2\sqrt{2} - \sqrt{3}$ $3k - \dfrac{2}{k} = 3\left(2\sqrt{2} - \sqrt{3}\right) - \dfrac{2}{2\sqrt{2} - \sqrt{3}}$ $= 6\sqrt{2} - 3\sqrt{3} - \dfrac{2\left(2\sqrt{2} + \sqrt{3}\right)}{8 - 3}$ $= 6\sqrt{2} - 3\sqrt{3} - \dfrac{4}{5}\sqrt{2} - \dfrac{2}{5}\sqrt{3}$ $= \dfrac{26\sqrt{2} - 17\sqrt{3}}{5}$ $= \dfrac{26}{5}\sqrt{2} - \dfrac{17}{5}\sqrt{3}$	

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2 The straight line $y = kx + 20$ intersects the curve $3y = 2kx^2 - 21$ at the points A and B whose x-coordinates are –3 and 4.5 respectively. Find the value of k. [4]

$3(kx + 20) = 2kx^2 - 21$ $2kx^2 - 3kx - 81 = 0$ $2kx^2 - 3kx - 81 = x^2 - 1.5x - \dfrac{81}{2k} = 0$ -3 and 4.5 are solutions $\left(x + 3\right)\left(x - 4.5\right) = 0$ $x^2 - 1.5x - 13.5 = 0$ by comparison $-\dfrac{81}{2k} = -13.5$ $k = 3$	

3 Express $-3x^2+12x-4$ in the form $a(x-h)^2+k$, where a, h and k are integers.

Hence state the coordinates of the turning point of the curve $y=-3x^2+12x-4$. [4]

$-3x^2+12x-4=-3(x^2-4x)-4$ $-3(x^2-4x+2^2-2^2)-4=-3(x-2)^2+8$ Turning point (2, 8)	

4 Integrate $\tan^2 2x$ with respect to x. [3]

$\int \tan^2 2x \, dx = \int \sec^2 2x - 1 \, dx$ $= \dfrac{\tan 2x}{2} - x + c$ (c is a constant)	

5 Express $\dfrac{6x^2-5x+5}{(x-1)\ (x^2+2)}$ in partial fractions. [4]

$\dfrac{6x^2-5x+5}{(x-1)(x^2+2)} = \dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+2}$ $6x^2-5x+5 \quad = A(x^2+2)+(Bx+C)(x-1)$ Sub $x=1$, $6=3A$ $A=2$ Sub $x=0$, $5=2A-C$ $C=-1$ Sub $x=2$, $2B=8$ $B=4$ $\therefore \dfrac{6x^2-5x+5}{(x-1)(x^2+2)} = \dfrac{2}{x-1}+\dfrac{4x-1}{x^2+2}$	

6 $f(x) = x^{2n} - (p+1)x^2 + p$, where n and p are positive integers.

(a) Show that $(x+1)$ is a factor of $f(x)$ for all values of p. [2]

$f(-1) = (-1)^{2n} - (p+1)(-1)^2 + p$ $= 1 - (p+1)(1) + p$ $= 0$ Therefore $(x + 1)$ is a factor	

(b) Given $p = 4$,

(i) find the value of n for which $(x-2)$ is a factor, [2]

$f(2) = 0$ $0 = 2^{2n} - (4+1)(2)^2 + 4$ $0 = 2^{2n} - 16$ $n = 2$	

(ii) hence, solve $f(x) = 0$. [3]

$x^4 - 5x^2 + 4 = (x+1)(x-2)(x^2 + ax + b)$ by observation $b = -2$ substitute $x = 1$, $1 - 5 + 4 = 2(-1)(1 + a - 2)$ $a = 1$ $(x+1)(x-2)(x^2 + x - 2) = 0$ $(x+1)(x-2)(x-1)(x+2) = 0$ $x = -1, 1, -2, 2$	

7 For $0 \le x \le \pi$, $f(x) = 3\sin nx$, where n is a positive integer, and $g(x) = 4\cos 2x + 1$.

(i) Given that $\frac{\pi}{6}$ satisfies the equation $f(x) = g(x)$, show that smallest value for $n = 3$.

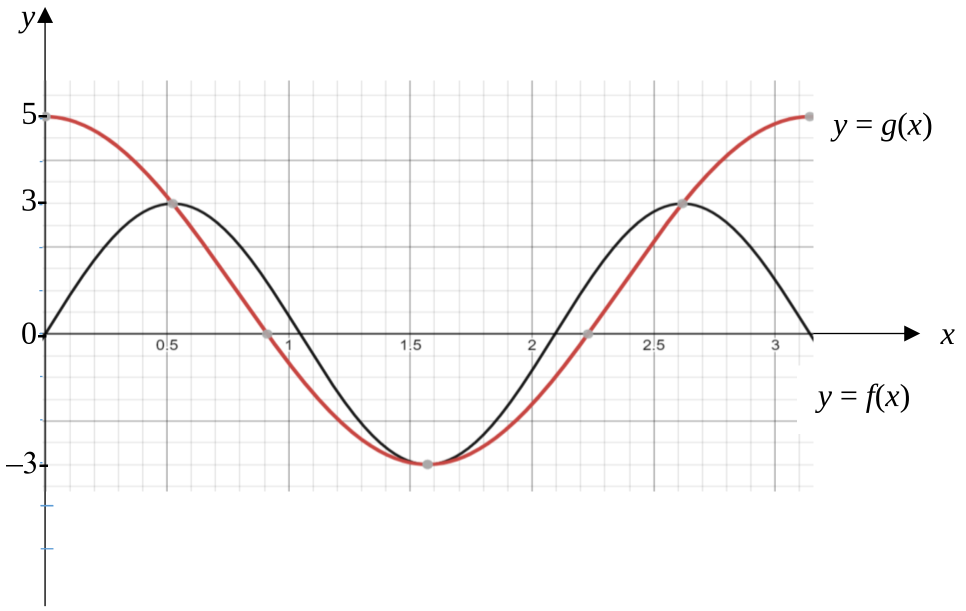
[2]

$3\sin n\left(\frac{\pi}{6}\right) = 4\cos 2\left(\frac{\pi}{6}\right) + 1$ $\sin n\left(\frac{\pi}{6}\right) = 1$ smallest $n = 3$	

(ii) State the amplitude of $g(x)$. [1]

Amplitude = 4	

(iii) Sketch, on the axes below, the graphs of $y = f(x)$ and $y = g(x)$. [4]

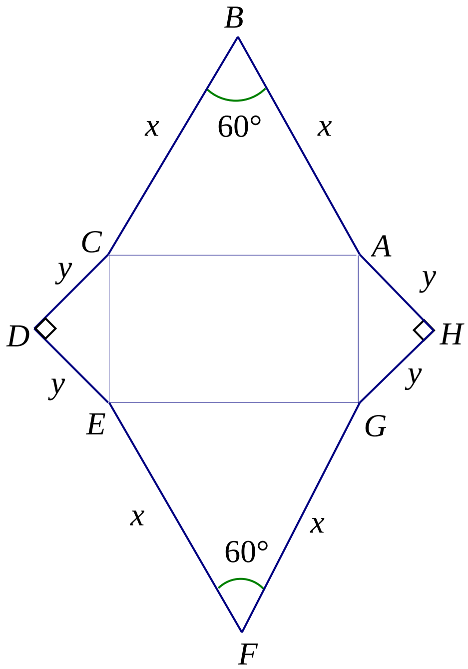


$f(x)$ $g(x)$	

(iv) State, in terms of π, the other roots of the equation $f(x) = g(x)$ for $0 \le x \le \pi$. [1]

$\frac{\pi}{2}, \frac{5\pi}{6}$	

8 A piece of wire, 100 cm in length, is bent to form the figure as shown.



Given that angle ABC = angle $EFG = 60°$, angle CDE = angle $GHA = 90°$,

$AB = BC = EF = FG = x$ cm and $CD = DE = GH = HA = y$ cm.

(a) Show that the area of the figure, P cm^2, is given by

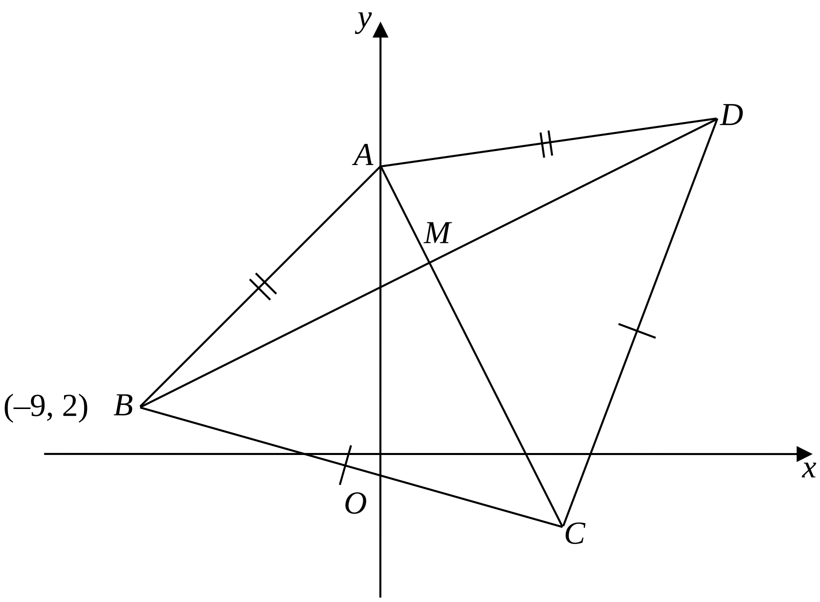
$$P = \left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x^2 + (25\sqrt{2} - 50)x + 625.$$ [4]

$4(x+y)=100 \Rightarrow$ $x+y=25$ $y=25-x$ $CE=\sqrt{y^2+y^2}=\sqrt{2}y$ $P=$ 2 × Area of ΔABC + 2 × Area of ΔCDE + Area of rectangle $ACEG$ $= x^2 \sin 60° + y^2 + x(\sqrt{2}y)$ $= \frac{\sqrt{3}}{2}x^2 + (25-x)^2 + \sqrt{2}x(25-x)$ $= \frac{\sqrt{3}}{2}x^2 + 625 - 50x + x^2 + 25\sqrt{2}x - \sqrt{2}x^2$ $= \left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x^2 + (25\sqrt{2} - 50)x + 625$	

(b) Find the value of x for which P has a stationary value. [2]

$\frac{\mathrm{d}P}{\mathrm{d}x} = 2\left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x + 25\sqrt{2} - 50$ $\frac{dP}{dx} = 0$ $2\left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)x = 50 - 25\sqrt{2}$ $x = \frac{50 - 25\sqrt{2}}{2\left(\frac{\sqrt{3}}{2} + 1 - \sqrt{2}\right)} = 16.2\,(3\text{ s.f.})$	

9



The diagram shows a kite *ABCD* with $AB = AD$ and $CB = CD$.

The diagonals intersect at *M*. The point *A* lies on the *y*-axis, the point *B* is (–9, 2) and the equation of *AC* is $2x + y = 9$.

(i) State the coordinates of *A*. [1]

A (0, 9)	

(ii) Find the equation of *BD*. [2]

Gradient of *AC* = –2 Gradient of *BD* = $\frac{1}{2}$ Equation *BD* : $y-2=\frac{1}{2}(x-(-9))$ $y=\frac{1}{2}x+6\frac{1}{2}$	

(iii) Find the coordinates of *M* and of *D*. [4]

By solving simultaneous equations $y = \frac{1}{2}x + 6\frac{1}{2}$ and $2x + y = 9$ $9 - 2x = \frac{1}{2}x + 6\frac{1}{2}$ $x = 1$ $y = 7$ $M\ (1,7)$ *M* is mid-point of *BD* $\frac{-9 + x_D}{2} = 1, \frac{2 + y_D}{2} = 7$ $D\ (11,12)$	

Given further that the area of the triangle *ABD* is $\frac{1}{4}$ of the area of the triangle *CBD*, find

(iv) the coordinates of *C*, [2]

$MC = 4\ AM$ $x_C = 1 + 4(1) = 5, y_C = 7 - 4(2) = -1$ $C(5,-1)$	

(v) the area of the kite *ABCD*. [2]

$\frac{1}{2}\begin{vmatrix} 0 & -9 & 5 & 11 & 0 \\ 9 & 2 & -1 & 12 & 9 \end{vmatrix}$ $= 125$ units^2	

10 **(a)** Find in radians, the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$. [2]

Principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is in the 4th quadrant Since $\tan\left(\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$, The principal value is $-\frac{\pi}{6}$ or –0.524.	

(b) Given $\sin 3\theta + \sin\theta = 2\sin 2\theta\cos\theta$. Prove that

(i) $\cos 3\theta + \cos\theta = 2\cos 2\theta\cos\theta$, [3]

$\cos 3\theta+\cos\theta=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta+\cos\theta$ $=\left(1-2\sin^2\theta\right)\cos\theta-2\sin^2\theta\cos\theta+\cos\theta$ $=2\cos\theta-4\sin^2\theta\cos\theta$ $=2\cos\theta(1-2\sin^2\theta)$ $=2\cos 2\theta\cos\theta$	

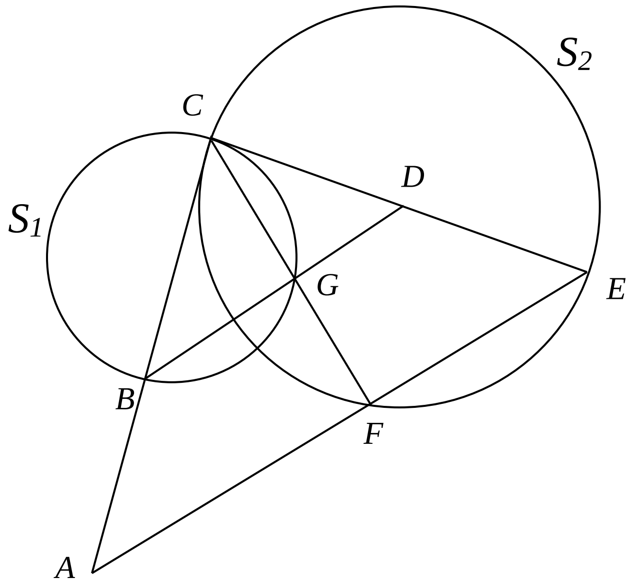
(ii) $\dfrac{\sin\theta-\sin 2\theta+\sin 3\theta}{\cos\theta-\cos 2\theta+\cos 3\theta}=\tan 2\theta$. [3]

$\frac{\sin\theta-\sin 2\theta+\sin 3\theta}{\cos\theta-\cos 2\theta+\cos 3\theta}=\frac{\sin 3\theta+\sin\theta-\sin 2\theta}{\cos 3\theta+\cos\theta-\cos 2\theta}$ $=\frac{2\sin 2\theta\cos\theta-\sin 2\theta}{2\cos 2\theta\cos\theta-\cos 2\theta}$ $=\frac{\sin 2\theta\left(2\cos\theta-1\right)}{\cos 2\theta\left(2\cos\theta-1\right)}$ $=\tan 2\theta$	

(c) Hence solve the equation $\dfrac{\cos\theta - \cos 2\theta + \cos 3\theta}{\sin\theta - \sin 2\theta + \sin 3\theta} = -\dfrac{1}{2}$ for $0 \le \theta \le \pi$. [3]

$\dfrac{\sin\theta - \sin 2\theta + \sin 3\theta}{\cos\theta - \cos 2\theta + \cos 3\theta} = -2$ $\tan 2\theta = -2$ $\alpha = 1.1071$ $2\theta = 2.0345..., 5.1761...$ $\theta = 1.02, 2.59$ (3 s.f.)	

11 In the diagram, not to scale, BC and CE are diameters of the circles, S_1 and S_2, respectively. CE is tangent to S_1 at C, CF and BD meet at G, and G lies on the circumference of S_1. F lies on the circumference of S_2. CB produced and EF produced meet at A.



Show that

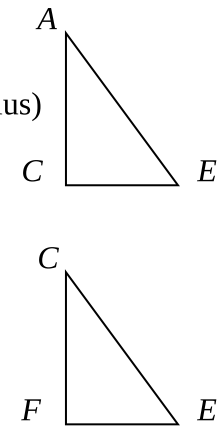
(i) triangles CBG and DCG are similar, [3]

$\angle CGB = 90°$ (Angle in a semi-circle) $\angle CGD = 90°$ (adjacent angles on a straight line) $\therefore \angle CGB = \angle CGD$ $\angle CBG = \angle DCG$ (Alternate Segment Theorem/ Tangent-Chord Theorem) Therefore, by AA pty, triangles CBG and DCG are similar	

(ii) lines BGD and AFE are parallel, [2]

$\angle CFE = 90°$ (Angle in a semi-circle) Since $\angle CGD = 90°$ as well, lines BGD and AFE are parallel by means of corresponding angles	

(iii) $CE^2 = AE \times EF$. [4]

In triangles CEF and AEC, $\angle EFC = 90°$ (Angle in a semicircle) $\angle ECA = 90°$ (Tangent perpendicular to radius) Therefore $\angle EFC = \angle ECA$. $\angle CEF = \angle AEC$ (Common angle) Triangles CEF and AEC are similar (AA). Comparing triangles AEC and CEF, $\frac{CE}{AE} = \frac{FE}{CE}$ $CE \times CE = FE \times AE$ $CE^2 = AE \times EF$ 	

12 Solve the following equations:

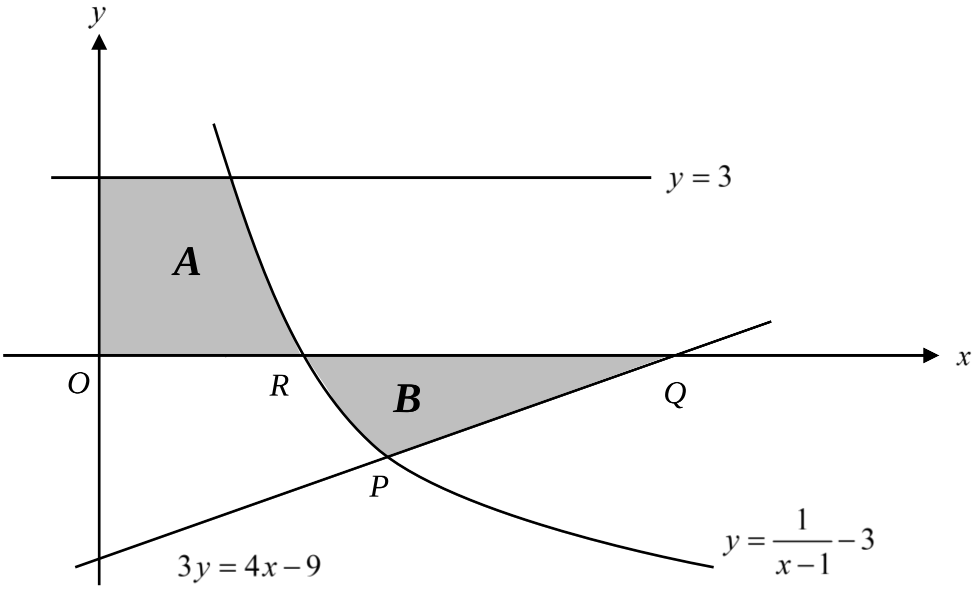
(a) $\log_3(2x-1)-\log_{\sqrt{3}}3=\log_2 4$ [4]

$\log_3(2x-1)-\log_{\sqrt{3}}3=\log_2 4$ $\log_3(2x-1)-\dfrac{\log_3 3}{\log_3\sqrt{3}}=\log_2 4$ $\log_3(2x-1)-\dfrac{1}{\log_3 3^{\frac{1}{2}}}=\log_2 2^2$ $\log_3(2x-1)-\dfrac{1}{\frac{1}{2}\log_3 3}=2\log_2 2$ $\log_3(2x-1)-2=2$ $\log_3(2x-1)=4$ $2x-1=3^4$ $x=\dfrac{3^4+1}{2}=41$	

(b) $\dfrac{1}{\sqrt{25^{2x+1}}}+\dfrac{10}{5^{x+2}}=3$. [4]

$\dfrac{1}{\sqrt{25^{2x+1}}}+\dfrac{10}{5^{x+2}}=3$ $\left(25^{2x+1}\right)^{-\frac{1}{2}}+10\left(5^{-x-2}\right)=3$ $\left(5^{2}\right)^{-x-\frac{1}{2}}+10\left(5^{-x}\right)\left(5^{-2}\right)=3$ $5^{-2x-1}+\dfrac{10}{25}\left(5^{-x}\right)=3$ $\left(5^{-2x}\right)\left(5^{-1}\right)+\dfrac{2}{5}\left(5^{-x}\right)=3$ $5^{-2x}+2\left(5^{-x}\right)=15$ $\left(5^{-x}\right)^{2}+2\left(5^{-x}\right)=15$ Let u = 5^{x} $15u^{2}-2u-1=0$ $(3u-1)(5u+1)=0$ $u=\dfrac{1}{3}$ or $u=-\dfrac{1}{5}$ $5^{x}=\dfrac{1}{3}$ or $5^{x}=-\dfrac{1}{5}$(N.A.) $x=\dfrac{\ln\frac{1}{3}}{\ln 5}=-0.683$	

13 The sketch shows the graphs of the curve, $y=\dfrac{1}{x-1}-3$, the lines $3y=4x-9$ and $y=3$. The curve and the line $3y=4x-9$ intersect at P. The curve cuts the x-axis at $R\left(\dfrac{4}{3},0\right)$. The line $3y=4x-9$ cuts the x-axis at $Q\left(2\dfrac{1}{4},0\right)$.



The region A is bounded by the curve, $y=\dfrac{1}{x-1}-3$, the line $y=3$ and the y-axis.

The region B is bounded by the curve, the line $3y=4x-9$, and the x-axis.

(i) Verify that the coordinates of P are $\left(\dfrac{3}{2},-1\right)$. [2]

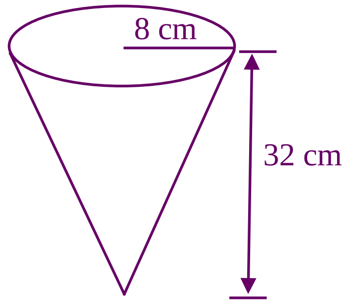
Solve simultaneous, $3\left(\dfrac{1}{x-1}-3\right)=4x-9$ $\dfrac{3}{x-1}=4x$ $4x^2-4x-3=0$ $(2x-3)(2x+1)=0$ $2x-3=0$ or $2x+1=0$ $x=\dfrac{3}{2}$ or $x=-\dfrac{1}{2}$ (N.A.) $x=\dfrac{3}{2}$, $3y=4\left(\dfrac{3}{2}\right)-9$, $y=-1$ $P\left(\dfrac{3}{2},-1\right)$	

(ii) Find the area of A and of B. [6]

Area $A = \int_0^3 \left(\frac{1}{y+3} + 1 \right) dy$ $= \left[\ln(y+3) + y \right]_0^3$ $= \left(\ln(3+3) + 3 \right) - \left(\ln(0+3) + 0 \right)$ $= \ln 6 - \ln 3 + 3$ $= \ln 2 + 3$ $= 3.69 \text{ units}^2$ (3 s.f.) Area $B = \left| \int_{\frac{4}{3}}^{\frac{3}{2}} \left(\frac{1}{x-1} - 3 \right) dx \right| + \frac{1}{2} \times \left(2\frac{1}{4} - \frac{3}{2} \right) \times 1$ $= \left| \left[\ln(x-1) - 3x \right]_{\frac{4}{3}}^{\frac{3}{2}} \right| + \frac{3}{8}$ $= \left| \left(\ln\left(\frac{3}{2} - 1 \right) - 3\left(\frac{3}{2} \right) \right) - \left(\ln\left(\frac{4}{3} - 1 \right) - 3\left(\frac{4}{3} \right) \right) \right| + \frac{3}{8}$ $= \left| \left(\ln\left(\frac{1}{2} \right) - \left(\frac{9}{2} \right) \right) - \left(\ln\left(\frac{1}{3} \right) - 4 \right) \right| + \frac{3}{8}$ $= \left| \left(\ln\left(\frac{3}{2} \right) - \frac{1}{2} \right) \right| + \frac{3}{8}$ $= 0.470 \text{ units}^2$ (3 s.f.)	

14 A vessel is in the shape of a right circular cone.
The radius of cone is 8 cm and the height is 32 cm.
Water is poured into the vessel at a rate of 10 cm^3/s.

Calculate the rate at which the water level is rising when the vessel is $\frac{1}{8}$ full. [4]



Using similar triangles $\frac{r}{h} = \frac{8}{32}$ $r = \frac{1}{4}h$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$ $V = \frac{1}{48}\pi h^3$ $\frac{dV}{dh} = \frac{1}{16}\pi h^2$ $\frac{v}{V} : \left(\frac{h}{H}\right)^3$ $\frac{1}{8} : \left(\frac{h}{32}\right)^3$ $h = 16$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $10 = \frac{1}{16}\pi(16)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = 10 \div 16\pi$ $\frac{dh}{dt} = \frac{5}{8\pi}$ cm/ s or 0.199 cm/ s	

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