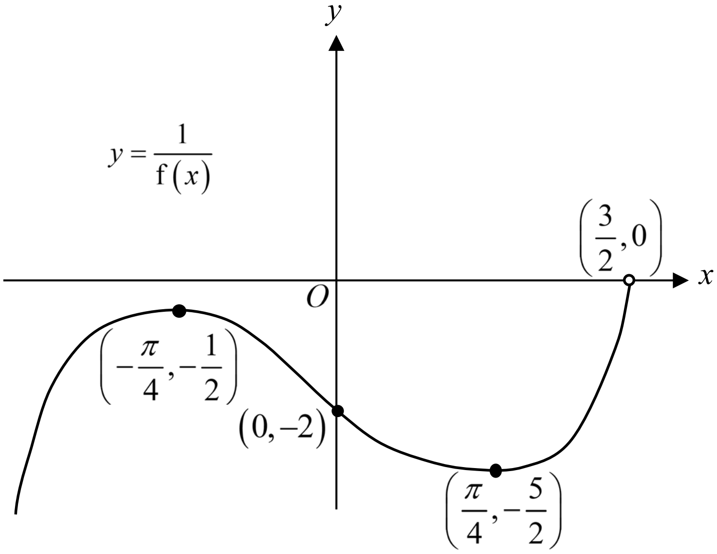
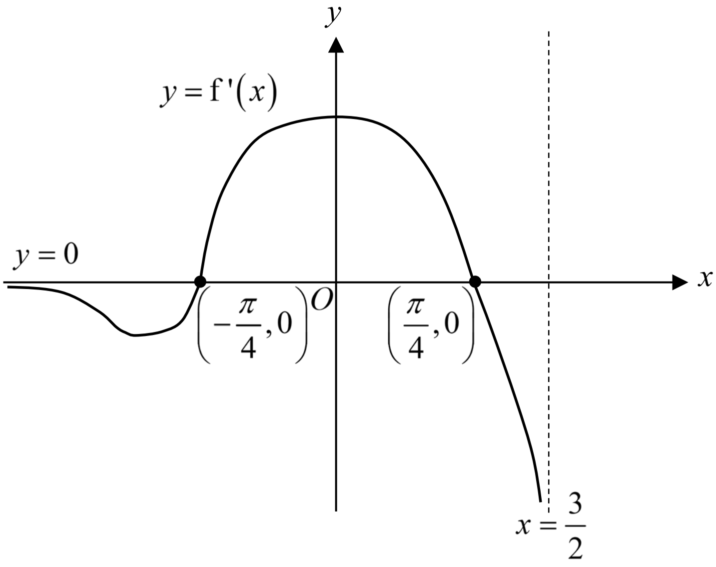
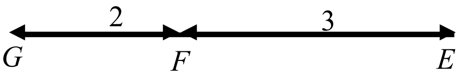
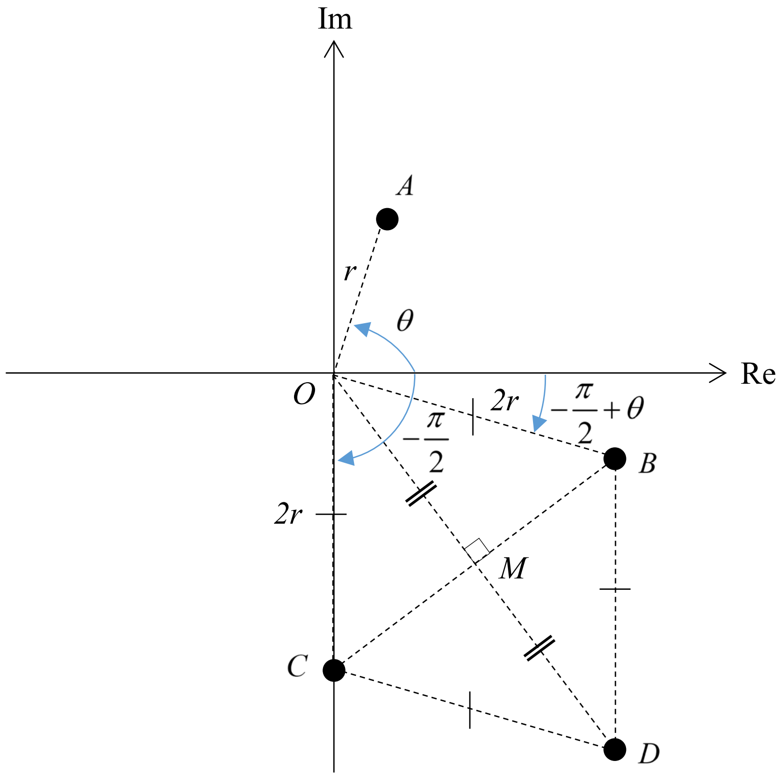
2020 HCI H2 Mathematics Preliminary Examination P2 Solution

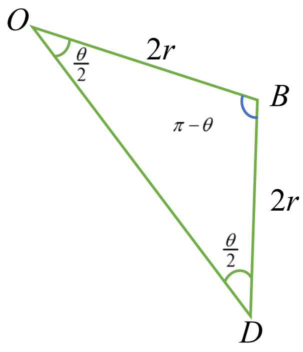
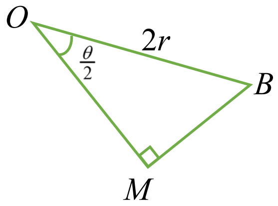
No	Suggested Solution
1(i)	
1(ii)	
2(a)	$\overrightarrow{OP}\bullet\left(\overrightarrow{OP}+\overrightarrow{OQ}+\overrightarrow{OR}\right)$ $=\overrightarrow{OP}\bullet\left[\overrightarrow{OP}+\left(\overrightarrow{OP}+\overrightarrow{PQ}\right)+2\overrightarrow{PQ}\right]$ $=\overrightarrow{OP}\bullet\left[2\overrightarrow{OP}+3\overrightarrow{PQ}\right]$ $=2\left\|\overrightarrow{OP}\right\|^2+3(5)^2\cos 60°$ $=2(5)^2+3(5)^2\left(\frac{1}{2}\right)$ =87.5
	Alternatively,

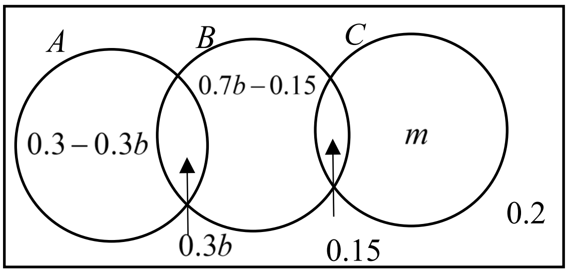
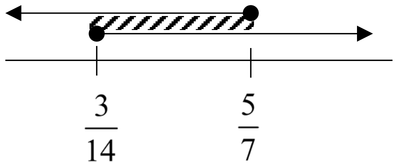
No	**Suggested Solution**
	$\measuredangle POQ = 30°$ $\measuredangle QOR = 30°$ $\measuredangle POR = 60°$ $\overrightarrow{OP}\bullet\overrightarrow{OP} = 5^2$ $\overrightarrow{OP}\bullet\overrightarrow{OQ} = 5\times\sqrt{10^2-5^2}\cos 30° = 25\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = \frac{75}{2}$ $\overrightarrow{OP}\bullet\overrightarrow{OR} = 5\times 10\cos 60° = 25$ $\overrightarrow{OP}\bullet\left(\overrightarrow{OP}+\overrightarrow{OQ}+\overrightarrow{OR}\right)$ $= \overrightarrow{OP}\bullet\overrightarrow{OP}+\overrightarrow{OP}\bullet\overrightarrow{OQ}+\overrightarrow{OP}\bullet\overrightarrow{OR}$ $= 87.5$
2(b)	$2\overrightarrow{OE}-5\overrightarrow{OF}+3\overrightarrow{OG}=\underset{\sim}{0}$ $2\overrightarrow{OE}-2\overrightarrow{OF}-3\overrightarrow{OF}+3\overrightarrow{OG}=\underset{\sim}{0}$ $2\overrightarrow{OE}-2\overrightarrow{OF}=3\overrightarrow{OF}-3\overrightarrow{OG}$ $2\overrightarrow{FE}=3\overrightarrow{GF}$ Since $\overrightarrow{FE}//\overrightarrow{FG}$ and F is the common point, E, F and G are collinear.  $EF : EG = 3{:}5$ Since they are parallel, $\overrightarrow{EF}\times\overrightarrow{EG}=\underset{\sim}{0}$
3(i)	$\mathrm{f}(r+1)-\mathrm{f}(r) = (r-1)2^r-(r-2)2^{r-1}$ $= 2^{r-1}(2(r-1)-r+2)$ $= r2^{r-1}$ $= 2^r\left(\frac{1}{2}r\right)$ (shown) $a=\frac{1}{2}$

No	**Suggested Solution**
3(ii)	$\sum_{r=1}^{n} 2^r\left(\frac{1}{2}r\right)=\sum_{r=1}^{n}\left[\mathrm{f}(r+1)-\mathrm{f}(r)\right]$ $\frac{1}{2}\sum_{r=1}^{n} r2^r=\cancel{\mathrm{f}(2)}-\mathrm{f}(1)$ $+\cancel{\mathrm{f}(3)}-\cancel{\mathrm{f}(2)}$ $+\ldots$ $+\cancel{\mathrm{f}(n)}-\cancel{\mathrm{f}(n-1)}$ $+\mathrm{f}(n+1)-\cancel{\mathrm{f}(n)}$ $=\mathrm{f}(n+1)-\mathrm{f}(1)$ $=(n-1)2^n-(-1)2^0$ $=(n-1)2^n+1$ $\sum_{r=1}^{n} r2^r=2\left[(n-1)2^n+1\right]$ $=(n-1)2^{n+1}+2$
3(iii)	$\sum_{r=3}^{n}(r+2)2^r=\sum_{r=3}^{n} r2^r+\sum_{r=3}^{n} 2^{r+1}$ $=\sum_{r=1}^{n} r2^r-\sum_{r=1}^{2} r2^r+\frac{2^4(2^{n-2}-1)}{2-1}$ $=((n-1)2^{n+1}+2)-10+(2^{n+2}-16)$ $=(n+1)2^{n+1}-24$
	Alternatively, $\sum_{r=3}^{n}(r+2)2^r$ $=\sum_{r-2=3}^{r-2=n} r2^{r-2}$ $=\sum_{r=5}^{r=n+2} r\left(\frac{2^r}{2^2}\right)$ $=\frac{1}{4}\sum_{r=5}^{n+2} r2^r$ $=\frac{1}{4}\left[\sum_{r=1}^{n+2} r2^r-\sum_{r=1}^{4} r2^r\right]$ $=\frac{1}{4}\left[(n+1)2^{n+3}+2-(3(2^5)+2)\right]$ $=\frac{1}{4}\left[(n+1)2^{n+3}-96\right]$ $=(n+1)2^{n+1}-24$
4(i)	$y=x-\cos^{-1}x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{1}{\sqrt{1-x^2}}=1+\left(1-x^2\right)^{-\frac{1}{2}}$

No	Suggested Solution
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x\left(1-x^2\right)^{-\frac{3}{2}}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$ Since $\frac{\mathrm{d}y}{\mathrm{d}x} - 1 = \left(1-x^2\right)^{-\frac{1}{2}}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{x}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^{-3}}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^3 x$ (shown)
	$y = x - \cos^{-1} x$ $x - y = \cos^{-1} x$ $\cos(x-y) = x$ Differentiating throughout w.r.t. x, $-\left(1-\frac{\mathrm{d}y}{\mathrm{d}x}\right)\sin(x-y) = 1$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)\sin(x-y) = 1$ Differentiating throughout w.r.t. x, $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\sin(x-y) + \left[\left(1-\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)\right]\cos(x-y) = 0$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\sin(x-y) - \left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^2\cos(x-y) = 0$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left[\frac{1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)}\right] - \left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^2 x = 0$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]\left[\frac{1}{\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)}\right] = \left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^2 x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^3 x$ (shown)

No	**Suggested Solution**
4(ii)	When $x=0$, $y=-\frac{\pi}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$
4(iii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^3 x$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^3+x\left[3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}-1\right)^2\right]$ When $x=0$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=1$ Series expansion is $y=-\frac{\pi}{2}+(2)x+\left(\frac{0}{2!}\right)x^2+\left(\frac{1}{3!}\right)x^3+\ldots$ $y=-\frac{\pi}{2}+2x+\frac{x^3}{6}+\ldots$
4(iv)	$\frac{\mathrm{d}y}{\mathrm{d}x}=1+\frac{1}{\sqrt{1-x^2}}=\frac{\mathrm{d}}{\mathrm{d}x}\left[-\frac{\pi}{2}+2x+\frac{x^3}{6}+\ldots\right]=2+\frac{x^2}{2}+\ldots$
5(i)	 Note:

No	**Suggested Solution**
	$\lvert b\rvert = \lvert -2\mathrm{i}a\rvert$ $= \lvert -2\mathrm{i}\rvert\lvert a\rvert = 2r$ $\arg(b) = \arg(-2\mathrm{i}a)$ $= \arg(-2\mathrm{i}) + \arg(a) = -\dfrac{\pi}{2} + \theta$
5(ii)	$\measuredangle AOB = \dfrac{\pi}{2}$ (by previous part) $\measuredangle BOC = \theta$
5(iii)	Since the length of adjacent sides of *OBDC* are equal, it is a rhombus.   *OD* bisects $\measuredangle BOC$. $\cos\measuredangle COM = \dfrac{OM}{2r}$ $OM = 2r\cos\dfrac{\theta}{2}$ $\lvert b+c\rvert = OD = 2OM = 4r\cos\dfrac{\theta}{2}$ $\arg(b+c) = -\dfrac{\pi}{2} + \measuredangle COM$ $= -\dfrac{\pi}{2} + \dfrac{\theta}{2}$ $b + c = 4r\cos\dfrac{\theta}{2}\mathrm{e}^{\mathrm{i}\left(\frac{\theta}{2}-\frac{\pi}{2}\right)}$
6(a)	$\mathrm{P}(A'\cap B) = \mathrm{P}(B) - \mathrm{P}(A\cap B)$ $= \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ $= \mathrm{P}(B) - [\mathrm{P}(A)\times\mathrm{P}(B)](*)$ $= \mathrm{P}(B)[1-\mathrm{P}(A)]$ $= \mathrm{P}(A')\times\mathrm{P}(B)$

No	Suggested Solution
	Alternative $\mathrm{P}(A'\cap B)=\mathrm{P}(A\cup B)-\mathrm{P}(A)$ $=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)-\mathrm{P}(A)$ $=\mathrm{P}(B)-\left[\mathrm{P}(A)\times\mathrm{P}(B)\right](*)$ $=\mathrm{P}(B)\left[1-\mathrm{P}(A)\right]$ $=\mathrm{P}(A')\times\mathrm{P}(B)$
6(b)	 $0.7b-0.15\geq 0\Rightarrow b\geq\frac{3}{14}$ ------(1) When $m=0$, $0.3+0.7b\leq 0.8$ $0.7b\leq 0.5\Rightarrow b\leq\frac{5}{7}$ Or Use $0.5-0.7b\geq 0$  $\frac{3}{14}\leq b\leq\frac{5}{7}\Rightarrow\frac{9}{140}\leq 0.3b\leq\frac{3}{14}$ $\frac{9}{140}\leq\mathrm{P}(A\cap B)\leq\frac{3}{14}$
7(a)	Required probability $=\dfrac{\frac{13!}{13}\times{}^{13}C_2\times 2!}{\frac{15!}{15}}$ $=\frac{6}{7}$ or 0.857 (3 s.f.) **Alternative** Total numbers of ways = $(15-1)!=14!$ Number of ways leaders seated together

No	**Suggested Solution**
	$=(14-1)!\times 2=13!\times 2$ Probability $=1-\dfrac{13!\times 2}{14!}=1-\dfrac{1}{7}=\dfrac{6}{7}$
7(b)	Let X be the event that the 3 students from the same class are seated separately. Let Y be the event that both leaders are seated separately. Required probability $=\dfrac{\mathrm{P}(X\cap Y)}{\mathrm{P}(Y)}$ $=\dfrac{\mathrm{P}(X)-\mathrm{P}(X\cap Y')}{P(Y)}$ $=\dfrac{\left(\dfrac{\dfrac{12!}{12}\times{}^{12}C_3\times 3!-\dfrac{11!}{11}\times 2!\times{}^{11}C_3\times 3!}{\dfrac{15!}{15}}\right)}{6/7}$ $=\dfrac{95}{156}$ or 0.609 (3 s.f.)
7(c)	Required probability $=\dfrac{{}^{13}C_6}{{}^{15}C_7}$ $=\dfrac{4}{15}$ or 0.267 (3 s.f)
8(i)	$\mathrm{P}(\text{match}\mid(\text{sum}<5))=\dfrac{\mathrm{P}(\text{match}\cap(\text{sum}<5))}{\mathrm{P}(\text{sum}<5)}$ $=\dfrac{\mathrm{P}(\{1,1\})}{\mathrm{P}(\{1,1\}\text{ or }\{1,2\}\text{ or }\{2,1\}\text{ or }\{1,3\}\text{ or }\{3,1\})}$ $=\dfrac{\frac{2}{9}\times\frac{1}{8}}{\underbrace{\left(\frac{2}{9}\times\frac{1}{8}\right)}_{\{1,1\}}+\underbrace{2\left(\frac{2}{9}\times\frac{1}{8}\right)}_{\{1,2\}\text{ or }\{2,1\}}+\underbrace{2\left(\frac{2}{9}\times\frac{2}{8}\right)}_{\{1,3\}\text{ or }\{3,1\}}}$ $=\dfrac{\frac{1}{36}}{\frac{7}{36}}$ $=\frac{1}{7}$ <u>Method 1</u> $\mathrm{P}(\text{match})=\mathrm{P}(\{1,1\}\text{ or }\{3,3\}\text{ or }\{5,5\})=3\left(\frac{2}{9}\times\frac{1}{8}\right)=\frac{1}{12}$ From above, $\mathrm{P}(\text{match}\mid(\text{sum}<5))=\frac{1}{7}$ Since $\mathrm{P}(\text{match}\mid(\text{sum}<5))=\frac{1}{7}\neq\frac{1}{12}=\mathrm{P}(\text{match})$, $\therefore$ the 2 events are not independent.

<table>
<tr><th>No</th><th>Suggested Solution</th></tr>
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<u>Method 2</u>

From above, $\text{P}(\text{sum} < 5) = \frac{7}{36}$ and $\text{P}(\text{match} \cap (\text{sum} < 5)) = \frac{1}{36}$

$\text{P}(\text{match}) = \text{P}\left(\{1,1\} \text{ or } \{3,3\} \text{ or } \{5,5\}\right) = 3\left(\frac{2}{9} \times \frac{1}{8}\right) = \frac{1}{12}$

Since $\text{P}(\text{match} \cap (\text{sum} < 5)) = \frac{1}{36} \neq (\frac{7}{36})(\frac{1}{12}) = \text{P}(\text{match})\text{P}(\text{sum} < 5)$,

$\therefore$ the 2 events are not independent.

</td></tr>
<tr><td>8(ii)</td><td>

Let n be least no. of rounds such that
$\text{P}(\text{match with at most } n \text{ rounds}) \geq 0.75$

<u>Method A</u>:
$\text{P}(\text{match with at most } n \text{ rounds}) \geq 0.75$
$\text{P}(\{\text{match round } 1\} \text{or} \{\text{match round } 2\} \text{or} \ldots \text{or} \{\text{match round } n\}) \geq 0.75$

$$\frac{1}{12} + \left(\frac{11}{12}\right)\left(\frac{1}{12}\right) + \left(\frac{11}{12}\right)^2\left(\frac{1}{12}\right) + \ldots + \left(\frac{11}{12}\right)^{n-1}\left(\frac{1}{12}\right) \geq 0.75$$

$$\frac{\frac{1}{12}\left[1 - \left(\frac{11}{12}\right)^n\right]}{1 - \frac{11}{12}} \geq 0.75$$

Sum of n terms of a GP with $a = \frac{1}{12}$, $r = \frac{11}{12}$

$$\left(\frac{11}{12}\right)^n \leq 0.25$$

<u>Method B</u>: (method of complementation)
$\text{P}(\text{match with at most } n \text{ rounds}) \geq 0.75$
$1 - \text{P}(\text{no match in all } n \text{ rounds}) \geq 0.75$
$\text{P}(\text{no match in all } n \text{ rounds}) \leq 0.25$

$$\left(\frac{11}{12}\right)^n \leq 0.25$$

To solve $\left(\frac{11}{12}\right)^n \leq 0.25$,

<u>Method 1</u>:

$$n \geq \frac{\ln(0.25)}{\ln\left(\frac{11}{12}\right)}$$

$\therefore\ n \geq 15.93$
Least $n = 16$

<u>Method 2</u>:

<table>
<tr><th>n</th><th>$\left(\frac{11}{12}\right)^n$</th></tr>
<tr><td>15</td><td>$0.271 > 0.25$</td></tr>
<tr><td>16</td><td>$0.249 < 0.25$</td></tr>
<tr><td>17</td><td>$0.228 < 0.25$</td></tr>
</table>

Using GC, least $n = 16$

</td></tr>
<tr><td>8(iii)</td><td>

<u>Method 1</u>:
$\text{P}(\text{match on } 3^{\text{rd}} \text{ draw})$

$= \text{P}\left(\{\text{same, different, same}\} \text{ or } \{\text{different, same, same}\}\right)$

For 3 cases of 1 or 3 or 5

$= 3\left(\frac{2}{9} \times \frac{7}{8} \times \frac{1}{7}\right) + 3\left(\frac{7}{9} \times \frac{2}{8} \times \frac{1}{7}\right)$

$= \frac{1}{6}$

1 or 3 or 5 to be repeated

Numbers other than the one to be repeated

Same 1 or 3 or 5 drawn earlier

Remaining numbers not drawn

Either 1 or 3 or 5 drawn

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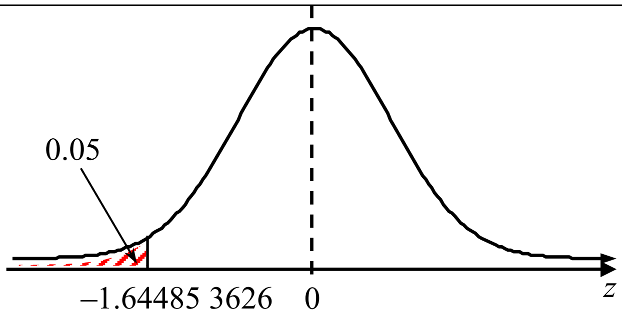
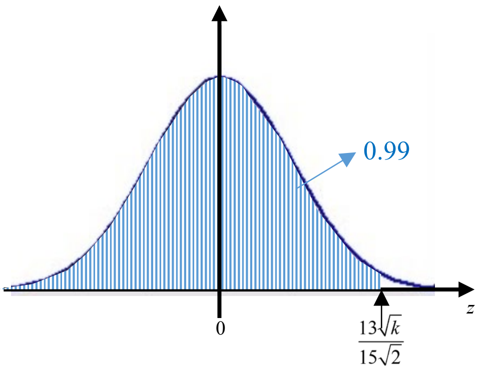
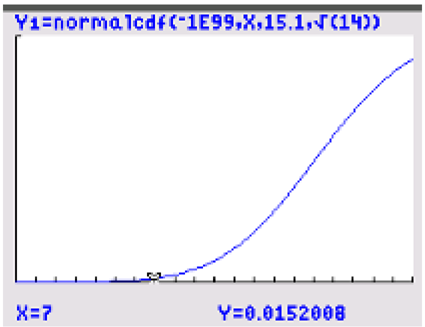
</table>

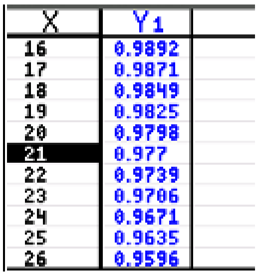
No	Suggested Solution
	Method 2: P(match on 3rd draw) $=\text{P}\left(\{\text{same, different, same}\}\text{ or }\{\text{different, same, same}\}\right)$ Either 1 or 3 or 5 drawn $=\left(\frac{6}{9}\times\frac{7}{8}\times\frac{1}{7}\right)+\left(\frac{3}{9}\times\frac{6}{8}\times\frac{1}{7}+\frac{6}{9}\times\frac{4}{8}\times\frac{1}{7}\right)$ $=\frac{1}{6}$ Remaining 1 or 3 or 5 not drawn previously and repeated Remaining numbers not drawn 1 or 3 or 5 drawn Same 1 or 3 or 5 drawn previously Same 1 or 3 or 5 drawn earlier 1 or 3 or 5 to be repeated 2 or 4 or 6 drawn Method 3: P(match on 3rd draw) $=\text{P(match in 3 draws)}-\text{P(match on 2nd draw)}$ Choose 1 or 3 or 5 to repeat $=3$ ways If 1 is chosen to repeat, choose one other number: 2, 4, 6, 3(2 times), 5(2 times) Total $=7$ ways $=\frac{3\times7}{{}^9C_3}-\frac{1}{12}$ From (i) Choose 3 out of 9 numbers $=\frac{1}{6}$
9(i)	Each bottle of hand sanitiser produced by Cleanser **has an equal chance of being chosen.** The event of one bottle of hand sanitiser being chosen is **independent** of the event of any other bottle of hand sanitiser being chosen.
9(ii)	The probability of **wrongly** concluding that the **population** mean volume is less than 15 ml when the population mean is 15 ml is 0.05.
9(iii)	$s^2=\frac{80}{79}(2)=\frac{160}{79}=2.03$ (3 s.f.)
9(iv)	Let X be the volume of a bottle of hand sanitiser, μ be the population mean volume, and σ be the population standard deviation of the volume. $\text{H}_0:\mu=15$ $\text{H}_1:\mu<15$ Since $n=80$ is large **by Central Limit Theorem,** $\bar{X}\sim\text{N}\left(15,\frac{\sigma^2}{80}\right)$ **approximately** **Test Statistic:** $Z=\frac{\bar{X}-\mu}{\sqrt{\frac{S^2}{n}}}\sim\text{N}(0,1)$ **approximately**

No	Suggested Solution
	 Since H_0 is not rejected $\dfrac{\bar{x}-15}{\sqrt{\dfrac{160}{80(79)}}}>-1.644853626$ $\bar{x}>14.738$ $\therefore \bar{x}>14.74$ (2 d. p)
9(v)	$\bar{x}=14.9125$ (exact) Or $\bar{x}=\dfrac{1193}{80}$ $s^2=(1.31393)^2=1.726\approx 1.73$ Or $s^2=\dfrac{10911}{6320}$
9(vi)	Since s^2 from Germsfree is smaller, hence the test statistic value from the test for Germsfree will be more negative. Therefore $p_1>p_2$.
10(i)	$\dfrac{1+2+3+4}{k}=1\Rightarrow k=10$ (shown)
10(ii)	$\{1,1\}$ $\text{P}(W=1)=(0.1)^2=\dfrac{1}{100}=0.01$ $\{1,2\},\{2,1\}$ $\text{P}(W=1.5)=2(0.1)(0.2)=\dfrac{1}{25}=0.04$ $\{1,3\},\{2,2\},\{3,1\}$ $\text{P}(W=2)=2(0.1)(0.3)+(0.2)^2=\dfrac{1}{10}=0.1$ $\{2,3\},\{3,2\},\{1,4\},\{4,1\}$ $\text{P}(W=2.5)=2(0.1)(0.4)+2(0.2)(0.3)=\dfrac{1}{5}=0.2$ $\{3,3\},\{2,4\},\{4,2\}$ $\text{P}(W=3)=2(0.2)(0.4)+(0.3)^2=\dfrac{1}{4}=0.25$

No	**Suggested Solution**
	$\{3,4\},\{4,3\}$ $\mathrm{P}(W=3.5)=2(0.3)(0.4)=\frac{6}{25}=0.24$ $\{4,4\}$ $\mathrm{P}(W=4)=(0.4)^2=\frac{4}{25}=0.16$ $\begin{array}{cccccccc} w & 1 & 1.5 & 2 & 2.5 & 3 & 3.5 & 4 \\ \mathrm{P}(W=w) & \frac{1}{100} & \frac{1}{25} & \frac{1}{10} & \frac{1}{5} & \frac{1}{4} & \frac{6}{25} & \frac{4}{25} \end{array}$
10(iii)	Mean of the numbers on the balls = $1(0.1)+2(0.2)+3(0.3)+4(0.4)=3$ $\mathrm{E}(W)=0.01+1.5(0.04)+2(0.1)+2.5(0.2)$ $+3(0.25)+3.5(0.24)+4(0.16)=3$ Since $\mathrm{E}(W)=$ Mean of numbers on the balls, therefore W is an unbiased estimator of the population mean. **OR** $\mathrm{E}(W)=\mathrm{E}\left(\frac{X_1+X_2}{2}\right)=\mathrm{E}(X)=\mu$
10(iv)	$\mathrm{P}(W>2.50)=0.25+0.24+0.16=0.65$ Let X be the number of games where Pat earns more than \$2.50 out of 5 games $X\sim\mathrm{B}(5,0.65)$ $\mathrm{P}(X\geq 3)=1-\mathrm{P}(X\leq 2)=0.7648\approx 0.765$ (3 s.f.) **OR** $\mathrm{P}(\text{proceed to second round})$ $=\binom{5}{3}\left(\frac{13}{20}\right)^3\left(\frac{17}{20}\right)^2+\binom{5}{4}\left(\frac{13}{20}\right)^4\left(\frac{17}{20}\right)^1+\left(\frac{13}{20}\right)^5$ $=0.765$
10(v)	$3m\left(\frac{2}{m}\right)-\frac{m}{2}\left(\frac{m-2}{m}\right)\geq 0$ $12m-m^2+2m\geq 0$ $14m-m^2\geq 0$ $m(14-m)\geq 0$ Since $m>0$, $14-m\geq 0$ i.e. $m\leq 14$ Since there are at least one white balls

<table>
<tr><th>No</th><th>Suggested Solution</th></tr>
<tr><td></td><td>$3 \le m \le 14$, where m is an integer</td></tr>
<tr><td>11(i)</td><td>Let X be the running timing of a C1 girl.
$\bar{X} \sim \mathrm{N}\left(14.8, \frac{2}{k}\right)$
$\mathrm{P}\left(0 < \bar{X} \le 15\frac{2}{3}\right) > 0.99$
<table>
<tr><td>k</td><td>$\mathrm{P}\left(0 < \bar{X} \le 15\frac{2}{3}\right)$</td></tr>
<tr><td>14</td><td>0.9881</td></tr>
<tr><td>15</td><td>$0.9912 > 0.99$</td></tr>
</table>
Least value of $k = 15$</td></tr>
<tr><td></td><td>$\mathrm{P}\left(\bar{X} \le 15\frac{2}{3}\right) > 0.99$
$\mathrm{P}\left(Z \le \frac{15\frac{2}{3} - 14.8}{\sqrt{\frac{2}{k}}}\right) > 0.99$
$\mathrm{P}\left(Z \le \frac{13\sqrt{k}}{15\sqrt{2}}\right) > 0.99$

</td></tr>
<tr><td>11(ii)</td><td>Let W be the random variable for the running timing of a girl from a C2 class in the following year.
$W \sim \mathrm{N}\left(\mu, \sigma^2\right)$
$\mathrm{P}\left(W > 17\frac{1}{3}\right) = 0.05 \Rightarrow \mathrm{P}\left(Z > \frac{17\frac{1}{3} - \mu}{\sigma}\right) = 0.05$</td></tr>
</table>

No	Suggested Solution
	$\mathrm{P}\left(W \le 15\frac{2}{3}\right) = 0.7 \Rightarrow \mathrm{P}\left(Z \le \frac{15\frac{2}{3} - \mu}{\sigma}\right) = 0.7$ $\frac{17\frac{1}{3} - \mu}{\sigma} = 1.644853626$ $\Rightarrow \mu + 1.644853626\sigma = 17\frac{1}{3} \quad -----(1)$ $\frac{15\frac{2}{3} - \mu}{\sigma} = 0.5244005101$ $\Rightarrow \mu + 0.5244005101\sigma = 15\frac{2}{3} \quad -----(2)$ Solving equations (1) and (2), $\mu = 14.8866 = 14.9$ (to 3sf) $\sigma = 1.48749 = 1.49$ (to 3sf)
11(iii)	By GC, the distribution graph shows a significant probability of 0.0152 for a running timing of 7 minutes or even lesser, which is highly not possible for a girl running 2.4 km.  Y1=normalcdf(-1E99,X,15.1,√(14)) X=7 Y=0.0152008 *Alternatively,* By GC, $\mathrm{P}(Y \le 8) = 0.028877$ This means that if there are 500 girls in a cohort, there will be around 14 girls who will be running 2.4 km at a timing of 8 minutes or lesser. This is an unrealistic timing for a 2.4 km run.
11(iv)	Let X be the random variable for the running timing of a girl from C1 cohort. $X \sim \mathrm{N}(14.8, 2)$ Let Y be the random variable for the running timing of a girl from C2 cohort. $Y \sim \mathrm{N}(15.1, 1.4)$

No	**Suggested Solution**
	$\mathrm{E}\left(X_1+X_2+X_3+X_4-4Y\right)$ $=4(14.8)-4(15.1)$ $=-1.2$ $\mathrm{Var}\left(X_1+X_2+X_3+X_4-4Y\right)$ $=4(2)+4^2(1.4)$ $=30.4$ $\therefore X_1+X_2+X_3+X_4-4Y \sim \mathrm{N}(-1.2, 30.4)$ $\mathrm{P}\left(\lvert X_1+X_2+X_3+X_4-4Y\rvert<1\right)$ $=\mathrm{P}\left(-1<X_1+X_2+X_3+X_4-4Y<1\right)$ $=0.140589$ $=0.141$ (to 3 sf)
11(v) **(a)**	1. **Every girl** in the C1 class **has equal probability** of **failing/passing** the 2.4 km running test. 2. **The event of a girl failing/passing the 2.4 km test** is **independent** of **another girl failing/passing the 2.4 km test** in the C1 class**.**
11(v) **(b)**	$\mathrm{P}\left(Y>17\frac{1}{3}\right)=0.0295457369$ **<leave answer to at least 5 s.f. or use raw values>** Let A be the number of girls from a C2 class who will fail the test, out of n girls. $A \sim \mathrm{B}(n, 0.0295457369)$ $\mathrm{P}(A<3)=\mathrm{P}(A \le 2)=0.977$ (3 s.f) By GC, X Y1 16 0.9892 17 0.9871 18 0.9849 19 0.9825 20 0.9798 21 0.977 22 0.9739 23 0.9706 24 0.9671 25 0.9635 26 0.9596 $n=21$