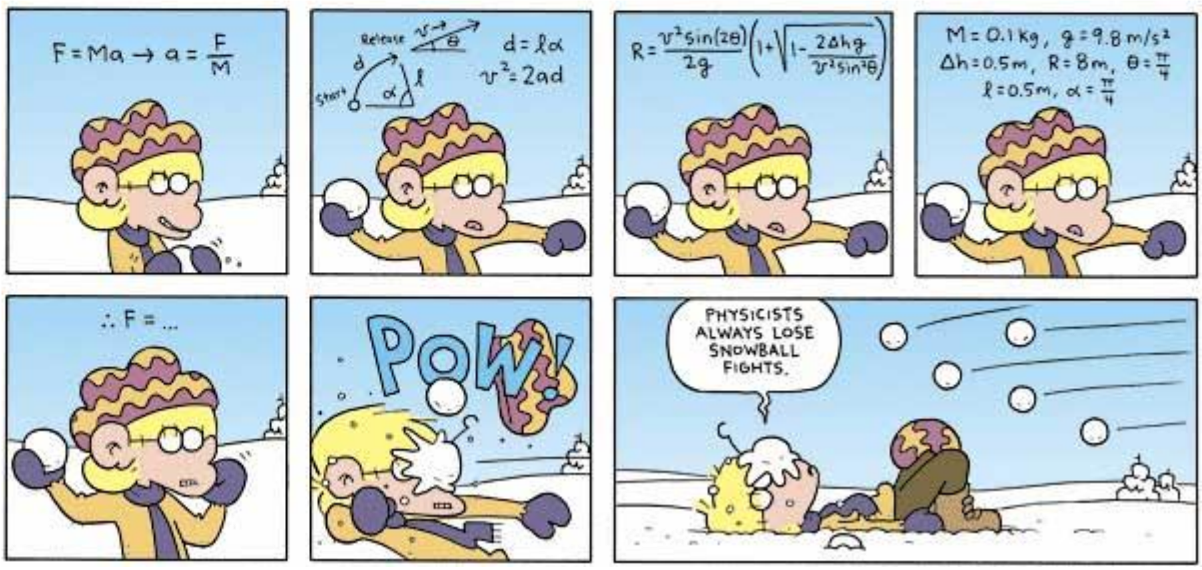
H2 Topic 02 – Kinematics



Kinematics is often referred to as the "geometry of motion" as the values of position, displacement, velocity and acceleration are worked on mathematically without considering the *type* of force causing the motion. We shall talk about how forces act on bodies in the topic of Dynamics.

Content

- Rectilinear motion
- Non-linear motion

Learning Outcomes

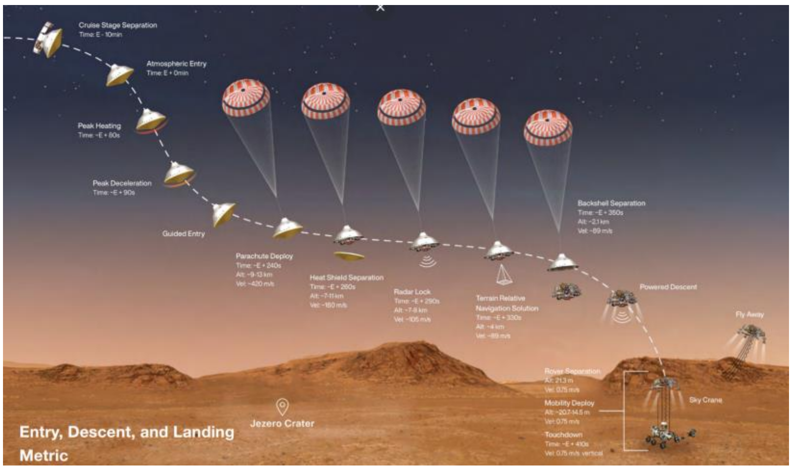
Candidates should be able to:

(a) show an understanding of and use the terms distance, displacement, speed, velocity and acceleration
(b) use graphical methods to represent distance, displacement, speed, velocity and acceleration
(c) identify and use the physical quantities from the gradients of displacement-time graphs and areas under and gradients of velocity-time graphs, including cases of non-uniform acceleration
(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line
(e) solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance
(f) describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance
(g) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

2.0 Introduction

Kinematics describe the motion of points, bodies and systems of bodies without considering the forces that cause them to move.

Before we can fully appreciate the motion, we need a common vocabulary to help establish an understanding of the science behind describing motion.

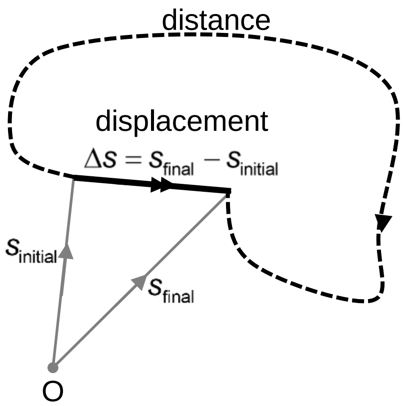


"Perseverance" Mars rover landing. *The intense entry, descent, and landing (EDL) phase begins when the spacecraft reaches the top of the Martian atmosphere, traveling at nearly 20 000 kmph. Many refer to the time it takes to land on Mars as the "seven minutes of terror."*

2.1 Vocabulary of Terms

2.1.1 Displacement and Distance

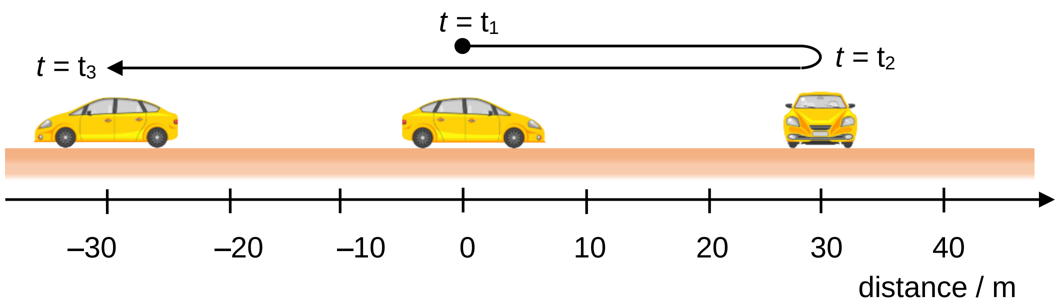
Displacement is	**Distance** is
the straight line distance from the start to the finish point in that direction.	the total length of the actual path travelled between the start and the finish points.



We typically denote position vectors using *s* with respect to an origin. Then the displacement (or change in displacement) can be $\Delta s = s_{\text{final}} - s_{\text{initial}}$.

Example 1

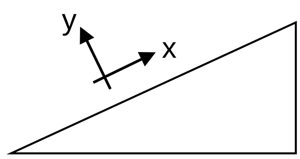
A car travels on a straight horizontal road. It starts moving at time t = t_1 and comes to a stop momentarily at t = t_2. It then reverses and comes to a stop again at t = t_3.



Taking rightwards direction as positive, state the
(a) total distance travelled, and
(b) displacement of the car with respect to origin x = 0.

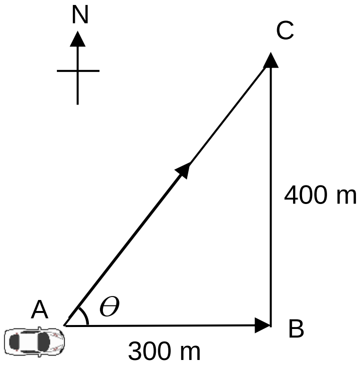
time	distance travelled	displacement from origin
t_1	0 m	0 m
t_2	30 m	+30 m or 30 m to the right
t_3	90 m	–30 m or 30 m to the left

Note: It is quite conventional to take rightwards as positive for horizontal motion and upwards as positive for vertical motion. It is equally conventional to take the direction *normal* and *away* from an inclined surface as positive displacement from the surface. It is always clearer to include a sketch or a phrase of how you define your convention.



Example 2

From point A, a car travelled 300 m towards the east to point B. It then travelled 400 m north to point C. Calculate the total distance it travelled and its displacement from its starting position.



$$\text{distance travelled} = AB + BC = 300 + 400 = 700 \text{ m}$$

$$\text{displacement} = AC = \sqrt{300^2 + 400^2} = 500 \text{ m}$$

$$\tan\theta = \frac{400}{300}$$
$$\theta = 53.1^\circ$$

displacement of car is
[magnitude] 500 m,
[direction] 53.1° north of east (or bearing of 36.9°)

Note: Reminder to give both magnitude and direction for vector quantities such as displacement.

2.1.2 Velocity and Speed

Velocity is

the rate of change of displacement.

Speed is

the rate of change of distance.

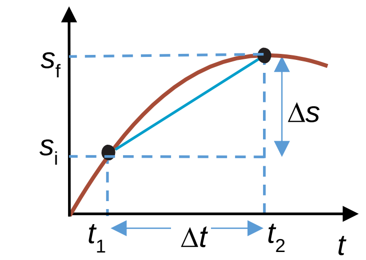
Speed is a scalar quantity and so has magnitude only.

Velocity is a vector quantity and so has magnitude and direction.

$$v = \frac{ds}{dt}$$

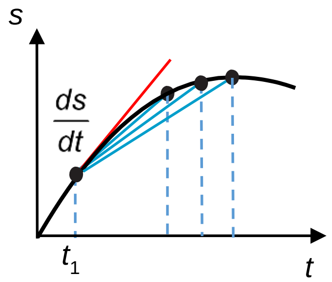
s : displacement (m)
t : time (s)
v : velocity (m s^{-1})

Both have units of m s^{-1}.



$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time}}$$

$$\text{average velocity} \langle v \rangle = \frac{\Delta s}{\Delta t} = \frac{\text{displacement}}{\text{total time}} = \frac{s_{\text{final}} - s_{\text{initial}}}{\Delta t}$$

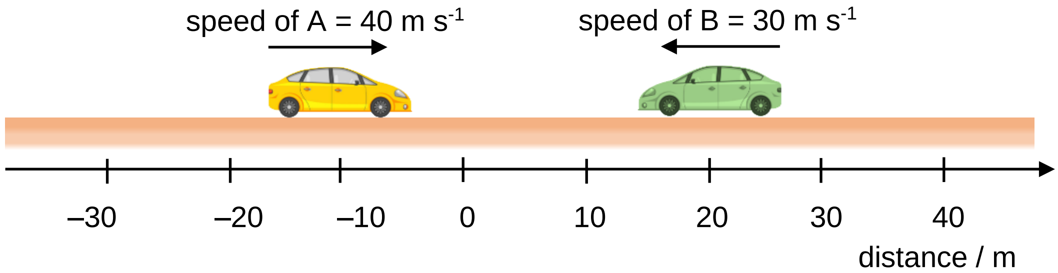


The gradient of the tangent to the graph at t_1 gives the instantaneous velocity.

When we refer simply to velocity, we generally mean the (instantaneous) velocity at that point in time.

Example 3

Cars A and B are travelling in the directions as shown.



The speeds and velocities of the cars are given.

car	speed	velocity
A	40 m s^{-1}	+ 40 m s^{-1}/ 40 m s^{-1} to the right / 40 m s^{-1} in positive direction
B	30 m s^{-1}	-30 m s^{-1}/ 30 m s^{-1} to the left / 30 m s^{-1} in negative direction

Note: The velocity of A relative to B is $v_{AB} = v_A - v_B = v_A + (-v_B) = 70 \text{ m s}^{-1}$ to the right.

2.1.3 Acceleration

Acceleration is

the rate of change of velocity.

$$a = \frac{dv}{dt}$$

v : velocity (m s^{-1})
t : time (s)
a : acceleration (m s^{-2})

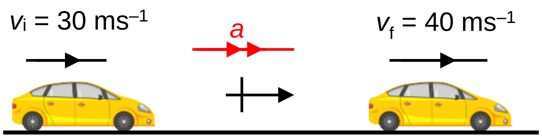
Acceleration is a vector quantity and has direction.

Deceleration refers to decreasing speed, regardless of direction.

Average acceleration can be found by

$$\text{average acceleration } \langle a \rangle = \frac{\text{change in velocity}}{\text{time taken}} = \frac{v_{\text{final}} - v_{\text{initial}}}{\Delta t}$$

<table>
<tr><th>speed up while moving in positive direction</th><th>slow down while moving in positive direction</th></tr>
<tr><td>



$v_{final} > v_{initial}$ so

$$\langle a\rangle = \frac{v_{final} - v_{initial}}{\Delta t}$$

is positive

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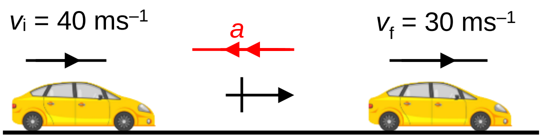

$v_{final} < v_{initial}$ so

$$\langle a\rangle = \frac{v_{final} - v_{initial}}{\Delta t}$$

is negative

(car is decelerating but *a* is negative)

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<tr><th>speed up while moving in negative direction</th><th>slow down while moving in negative direction</th></tr>
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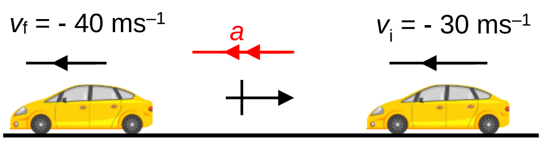

$v_{final} < v_{initial}$ so

$$\langle a\rangle = \frac{v_{final} - v_{initial}}{\Delta t} = \frac{(-40)-(-30)}{\Delta t}$$

is negative

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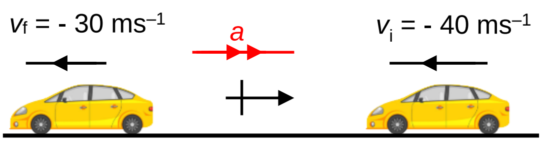

$v_{final} > v_{initial}$ so

$$\langle a\rangle = \frac{v_{final} - v_{initial}}{\Delta t} = \frac{(-30)-(-40)}{\Delta t}$$

is positive

(car is decelerating but *a* is positive)

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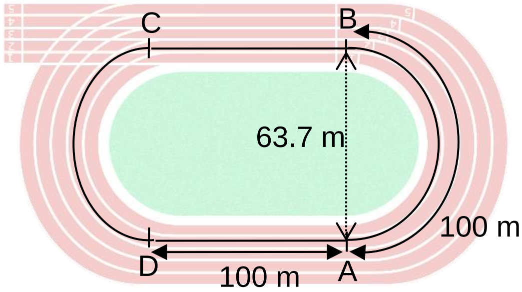
The car speeds up whenever acceleration is in the same direction as the velocity. It slows down when its acceleration is opposite to its velocity.

Example 4

An athlete sprints for 60.0 s at a constant speed round a 400 m track in an anti-clockwise direction.



(a) Determine his
- (i) average speed for the whole journey,
- (ii) average velocity for the whole journey,
- (iii) average velocity from A to C,
- (iv) instantaneous velocity at D,
- (v) acceleration at the midpoint of B and C,
- (vi) average acceleration for the whole journey.

(b) Is the athlete accelerating when he is halfway between C and D?

Solution:

(a)(i) $\langle \text{speed} \rangle = \dfrac{\text{total distance travelled}}{\text{time taken for distance travelled}}$

$= \dfrac{400}{60} = 6.67 \text{ m s}^{-1}$

(a)(ii) $\langle v \rangle = \dfrac{\Delta s}{\Delta t} = \dfrac{s_{\text{final}} - s_{\text{initial}}}{\Delta t}$

$= \dfrac{0}{60} = 0 \text{ m s}^{-1}$

(a)(iii) $s_{\text{AC}} = \sqrt{\text{CD}^2 + \text{DA}^2} = \sqrt{63.7^2 + 100^2}$

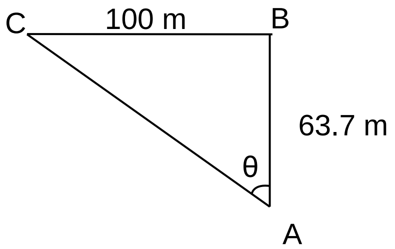
$= 119 \text{ m}$

$\langle v \rangle = \dfrac{\Delta s}{\Delta t} = \dfrac{s_{\text{AC}}}{\Delta t} = \dfrac{119}{60 \div 2}$

$= 3.97 \text{ m s}^{-1}$

$\tan\theta = \dfrac{100}{63.7}$

$\theta = 57.5°$



3.97 m s^{-1} at 57.5° anticlockwise of AB

(a)(iv) $v_{\text{at D}} = 6.67 \text{ m s}^{-1}$ along DA

(a)(v) no change in velocity
so 0 acceleration

(a)(vi) no change in velocity
so 0 acceleration

(b) **[magnitude]** while speed is constant
[direction] direction of motion is constantly changing between C and D

so velocity, being a vector, is changing
so he is accelerating

2.2 Motion Graphs

2.2.1 Calculations in Motion Graphs

Motions graphs track the change in s, v, and a throughout time. Graphs are therefore very rich in the information they can contain and present:

type of graph	gradient represents	area represents
displacement – time s-t	velocity $v = \dfrac{ds}{dt}$	(nil)
velocity – time v-t	acceleration $a = \dfrac{dv}{dt}$	<u>change</u> in displacement $v = \dfrac{ds}{dt}$ so $\Delta s = \int v\ dt$
acceleration-time a-t	(nil)	<u>change</u> in velocity $a = \dfrac{dv}{dt}$ so $\Delta v = \int a\ dt$

Example 5

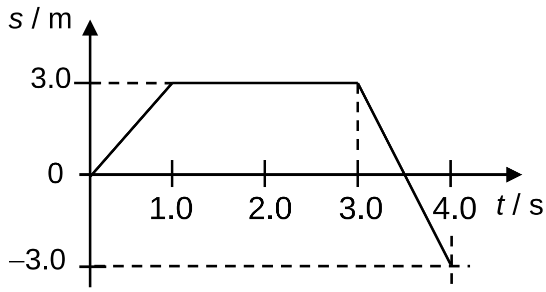
In the displacement-time graph as shown, determine the

(a) average speed from t = 0.0 s to t = 4.0 s,

(b) average velocity from t = 0.0 s to t = 4.0 s,

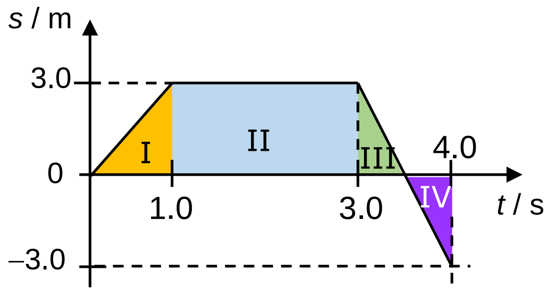
(c) instantaneous velocity at t = 3.5 s and

(d) instantaneous speed at t = 3.5 s.



Solution:

We can divide the graph into 4 regions:



$$(a)\ \langle \text{speed} \rangle = \frac{\text{total distance travelled}}{\text{time taken for distance travelled}} = \frac{(3.0)_{\text{I}} + (0.0)_{\text{II}} + (3.0)_{\text{III}} + (3.0)_{\text{IV}}}{4.0} = 2.25 \text{ m s}^{-1}$$

$$(b)\ \langle v \rangle = \frac{\Delta s}{\Delta t} = \frac{s_{\text{final}} - s_{\text{initial}}}{\Delta t} = \frac{(-3.0) - 0.0}{4.0} = -0.75 \text{ m s}^{-1}$$

(c) gradient represents instantaneous velocity

$$v_{t=3.5\text{ s}} = \frac{s_{t=4\text{ s}} - s_{t=3\text{ s}}}{\Delta t} = \frac{(-3.0) - 3.0}{4.0 - 3.0} = -6.0 \text{ m s}^{-1}$$

(d) instantaneous speed is magnitude of instantaneous velocity, is 6.0 m s^{-1}

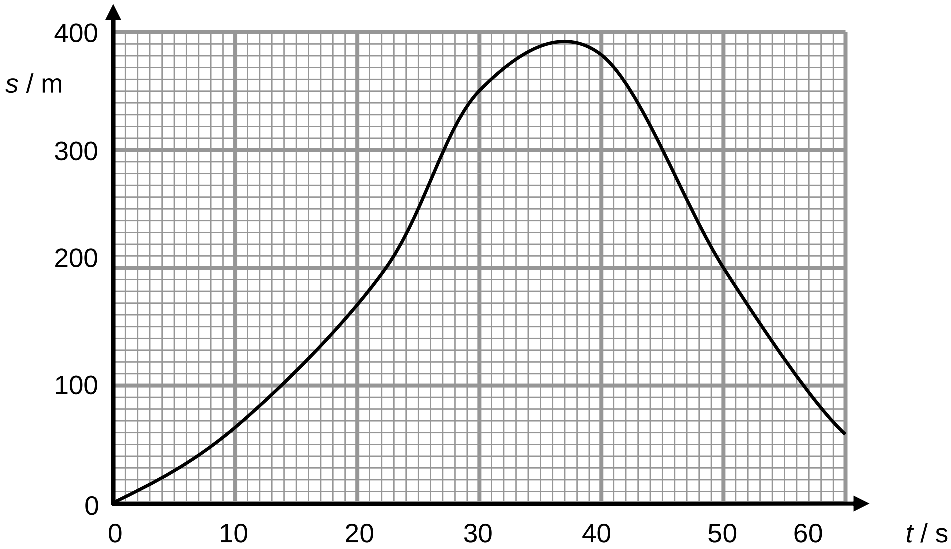
Example 6

In the displacement-time graph as shown,

(a) Explain whether the object is at rest at t = 0.

(b) State the time when the object is at rest.

(c) Determine the velocity at t = 20.0 s.



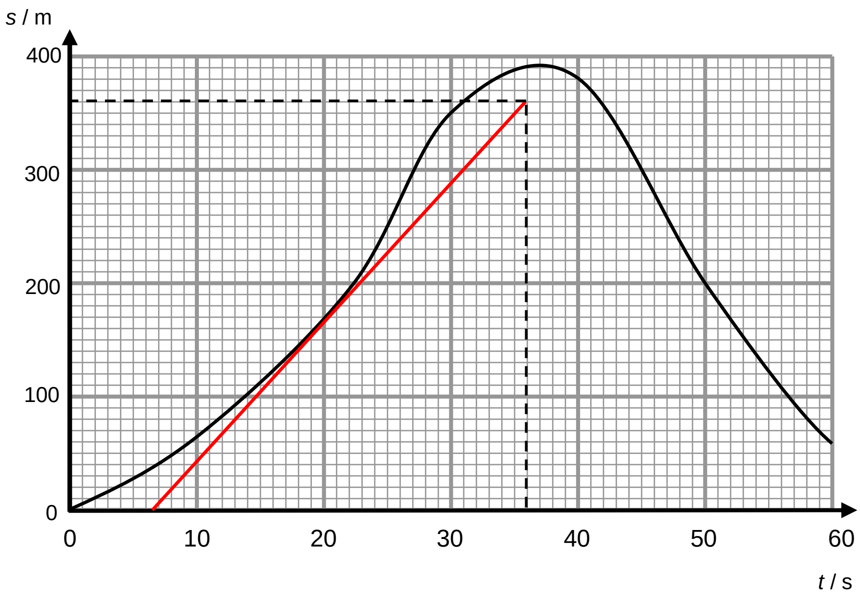
Solution:

(a) No. Gradient represents velocity, gradient is not zero at t = 0 s.

(b) Gradient represents velocity, gradient is zero at t = 37.0 s.

(c) Gradient represents velocity

$$v_{t=20\text{ s}} = \frac{s_1 - s_2}{t_1 - t_2} = \frac{360 - 0}{36.0 - 6.5} = 12.2 \text{ m s}^{-1}$$



Note: We read graphs accurate to half the smallest square.

Example 7

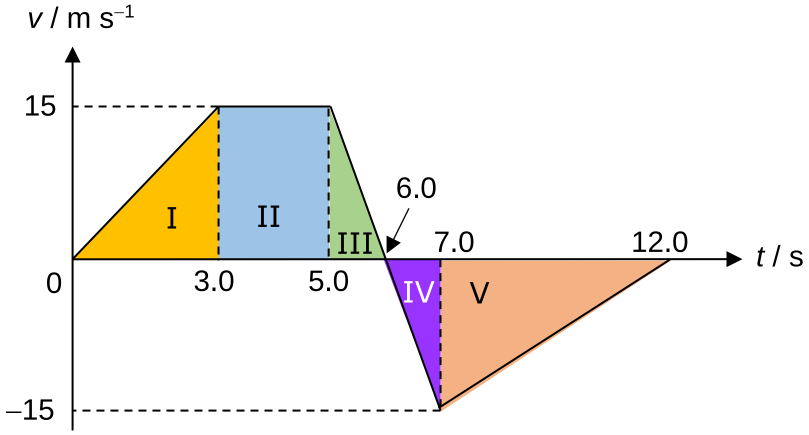
The graph below shows the variation with time of the velocity of a vehicle. Determine

(a) the displacement of the vehicle in the first 7.0 s,

(b) the average speed from $t = 0$ to $t = 12.0$ s,

(c) the average velocity during the time interval from $t = 5.0$ s to $t = 12.0$ s,

(d) whether the vehicle changed its direction of travel? If yes, state the time.



Solution:

(a) area under velocity-time graph represent change in displacement

$$s = \left[\frac{1}{2}(3.0)(15)\right]_{\text{I}} + \left[(5.0-3.0)(15)\right]_{\text{II}} + \left[\frac{1}{2}(6.0-5.0)(15)\right]_{\text{III}} - \left[\frac{1}{2}(7.0-6.0)(15)\right]_{\text{IV}}$$

$$= 52.5 \text{ m}$$

Alternatively (using the formula for trapezium):

$$s = \frac{1}{2}(6.0+2.0)(15) + \frac{1}{2}(1.0)(-15)$$

$$= 52.5 \text{ m}$$

$$(\text{b})\langle \text{speed} \rangle$$

$$= \frac{\text{total distance travelled}}{\text{time taken for distance travelled}}$$

$$= \frac{\left[\frac{1}{2}(3.0)(15)\right]_{\text{I}} + \left[(5.0-3.0)(15)\right]_{\text{II}} + \left[\frac{1}{2}(6.0-5.0)(15)\right]_{\text{III}} + \left[\frac{1}{2}(7.0-6.0)(15)\right]_{\text{IV}} + \left[\frac{1}{2}(12-7)(15)\right]_{\text{V}}}{12.0}$$

$$= 8.8 \text{ m s}^{-1}$$

Alternatively (using the formula for trapezium):

$$\langle v \rangle = \frac{\frac{1}{2}(6.0+2.0)(15) + \frac{1}{2}(6.0)(15)}{12.0}$$

$$= 8.8 \text{ m s}^{-1}$$

$$(\text{c})\langle v_{t=5 \text{ to } t=12} \rangle = \frac{\Delta s_{t=7 \text{ to } t=12}}{\Delta t}$$

$$= \frac{\frac{1}{2}(12-7)(-15)}{12-5} = -5.36 \text{ m s}^{-1}$$

(d) Yes, at $t = 6.0$ s

Note: Notice that the net change in displacement in Region III and IV is zero.

Example 8

Car A moves at a constant velocity of 20 m s^{-1}. At t = 0, A passes a Car B which is starting from rest and accelerating at 1.5 m s^{-2} until it reaches and maintains a constant speed of 30 m s^{-1}.
B overtakes A subsequently. Both cars are moving in the same direction at all time.
Find how far A moves after it passes B and before it is overtaken by B.

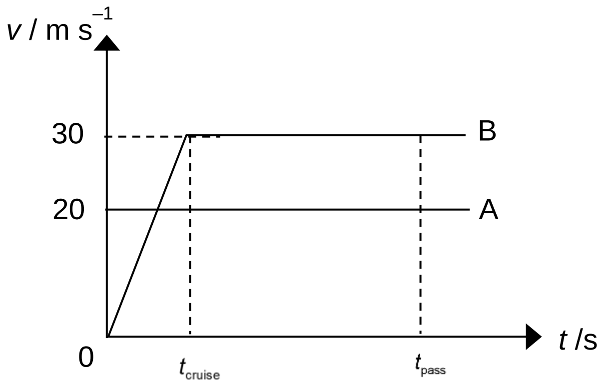
Solution:

Let t_{cruise} be the time B reaches its constant speed of 30 m s^{-1}, and
t_{pass} be the time that B overtakes A.

Gradient of velocity-time graph represents acceleration. Since acceleration is constant,

$$a = \frac{\Delta v}{t_{\text{cruise}}}$$

$$t_{\text{cruise}} = \frac{\Delta v}{a} = \frac{30-0}{1.5} = 20 \text{ s}$$



When B catches up with A,
displacement of B = displacement of A
area under graph B = area under graph A

$$\frac{1}{2}(t_{\text{cruise}})(30) + (t_{\text{pass}} - t_{\text{cruise}})(30) = (t_{\text{pass}})(20)$$

$$\frac{1}{2}(20)(30) + (t_{\text{pass}} - 20)(30) = (t_{\text{pass}})(20)$$

$$t_{\text{pass}} = 30 \text{ s}$$

$$\text{distance} = (\text{speed})(\text{time}) = (20)(30) = 600 \text{ m}$$

Alternatively (using the formula for trapezium):

$$\frac{1}{2}\left[(t_{pass} - 20) + t_{pass}\right] 30 = 20t_{pass}$$

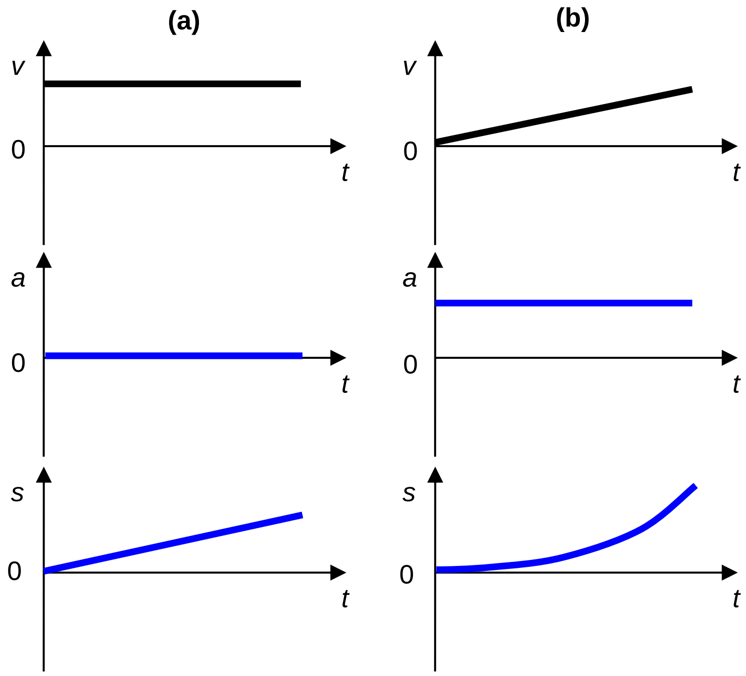
$$30t_{pass} - 300 = 20t_{pass}$$

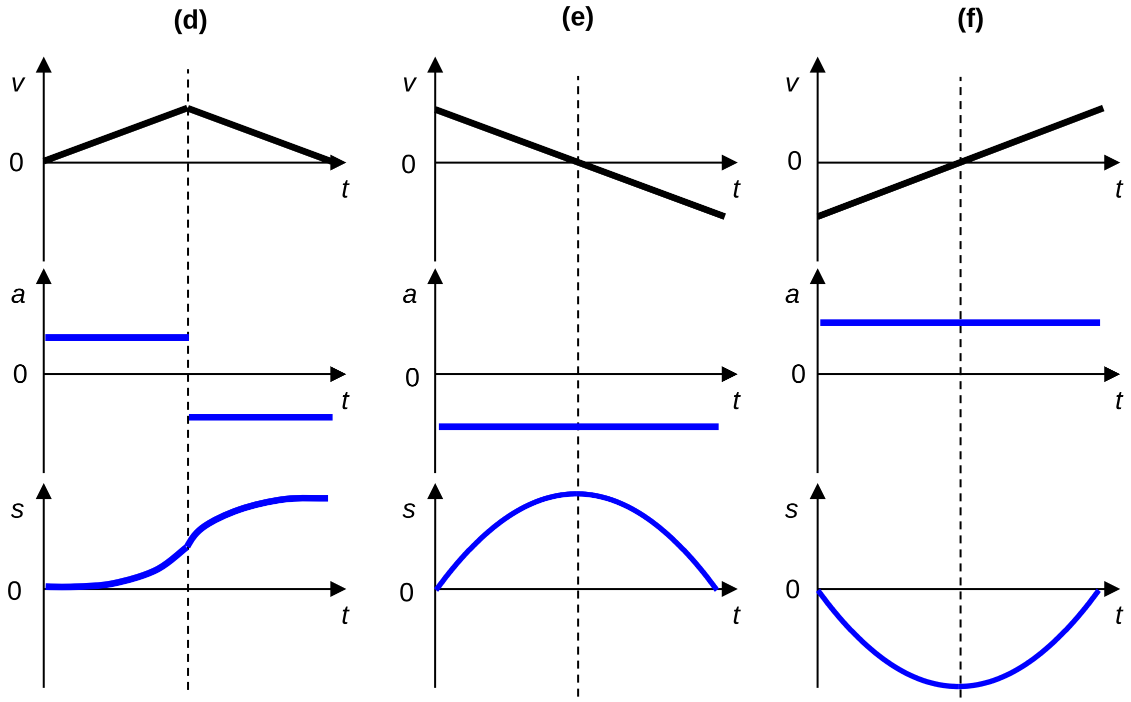
$$t_{\text{pass}} = 30 \text{ s}$$

Distance = 20 × 30 = 600 m

2.2.2 Sketching Motion Graphs

The corresponding displacement-time graphs and acceleration-time graphs are shown together with the velocity-time graphs. Assume that the displacement at time t = 0 is 0.

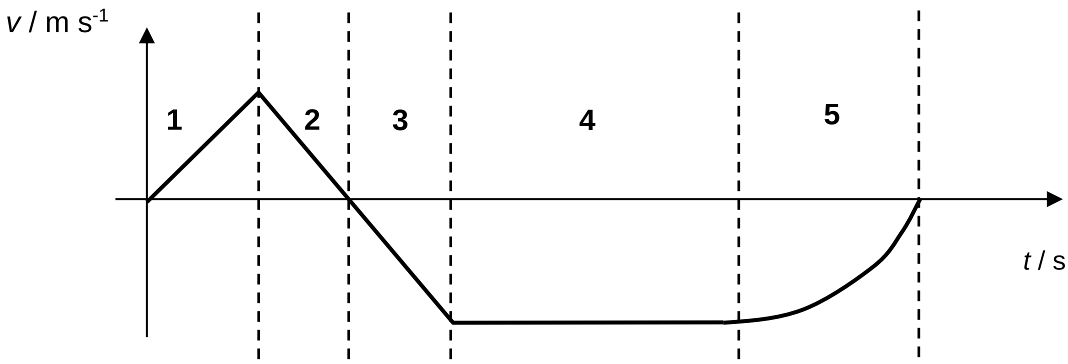




2.2.3 Describing Motion Graphs

In general (within and beyond this topic), when approaching a graph:

First, **segment** the graph into different regions	:	You can consider "cutting up" a graph • where the quantity reaches 0 • where the quantity reaches a turning point • where the gradient of the quantity abruptly changes ("not smooth"
Second, describe change in the ***y*-values**	:	Is the value generally increasing or generally decreasing?
Third, describe the changes in **gradient**	:	Is the graph increasing/decreasing *at an increasing/decreasing rate*? What does the gradient represent? e.g. gradient represents rate of change of velocity: acceleration
Fourth, does **area under graph** represent anything	:	e.g. area under velocity-time graph gives displacement



Time interval 1: The object moves in the positive direction, with speed increasing uniformly.

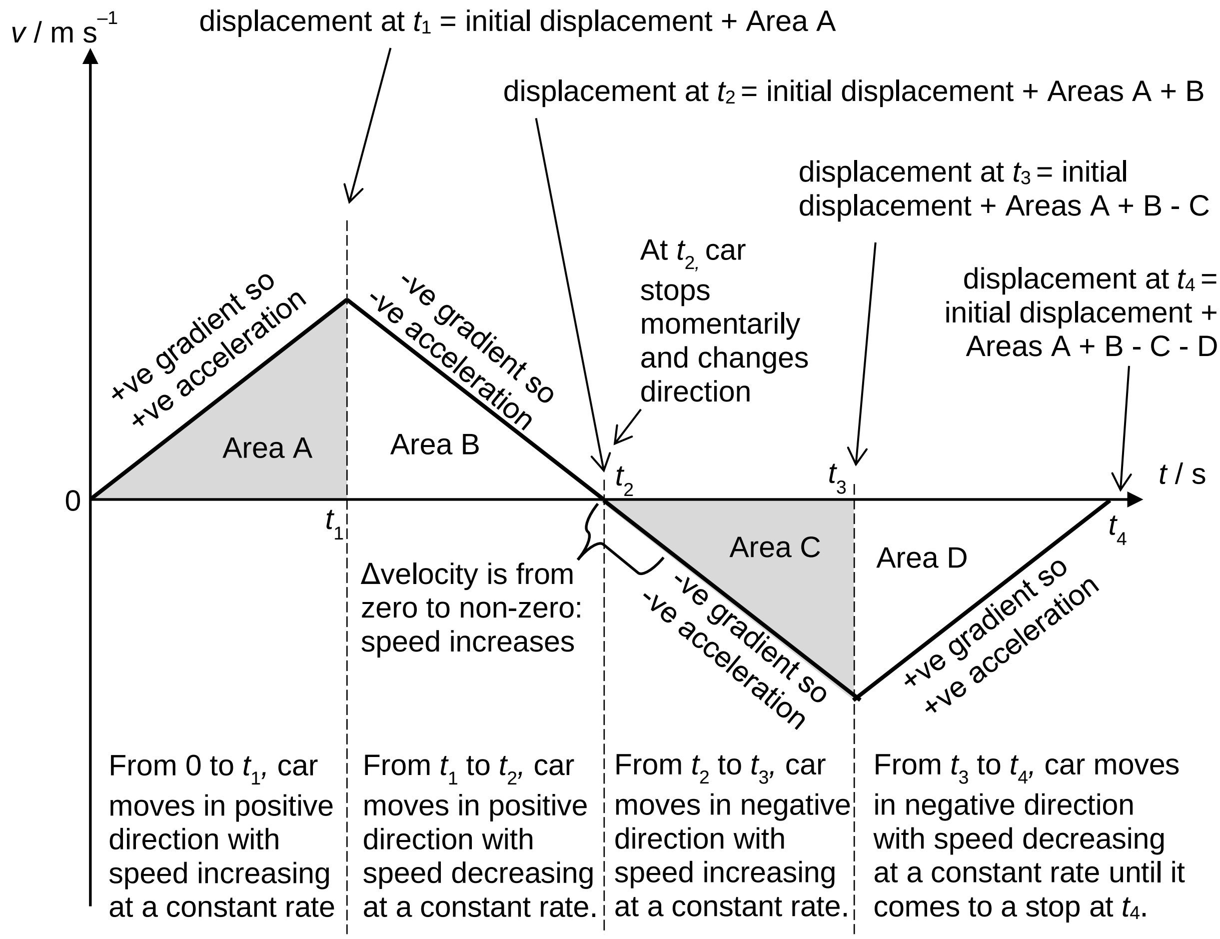
Time interval 2: The object moves in the positive direction, with speed decreasing uniformly, until it comes to a momentary stop.

Time interval 3: The object moves in the negative direction, with speed increasing uniformly.

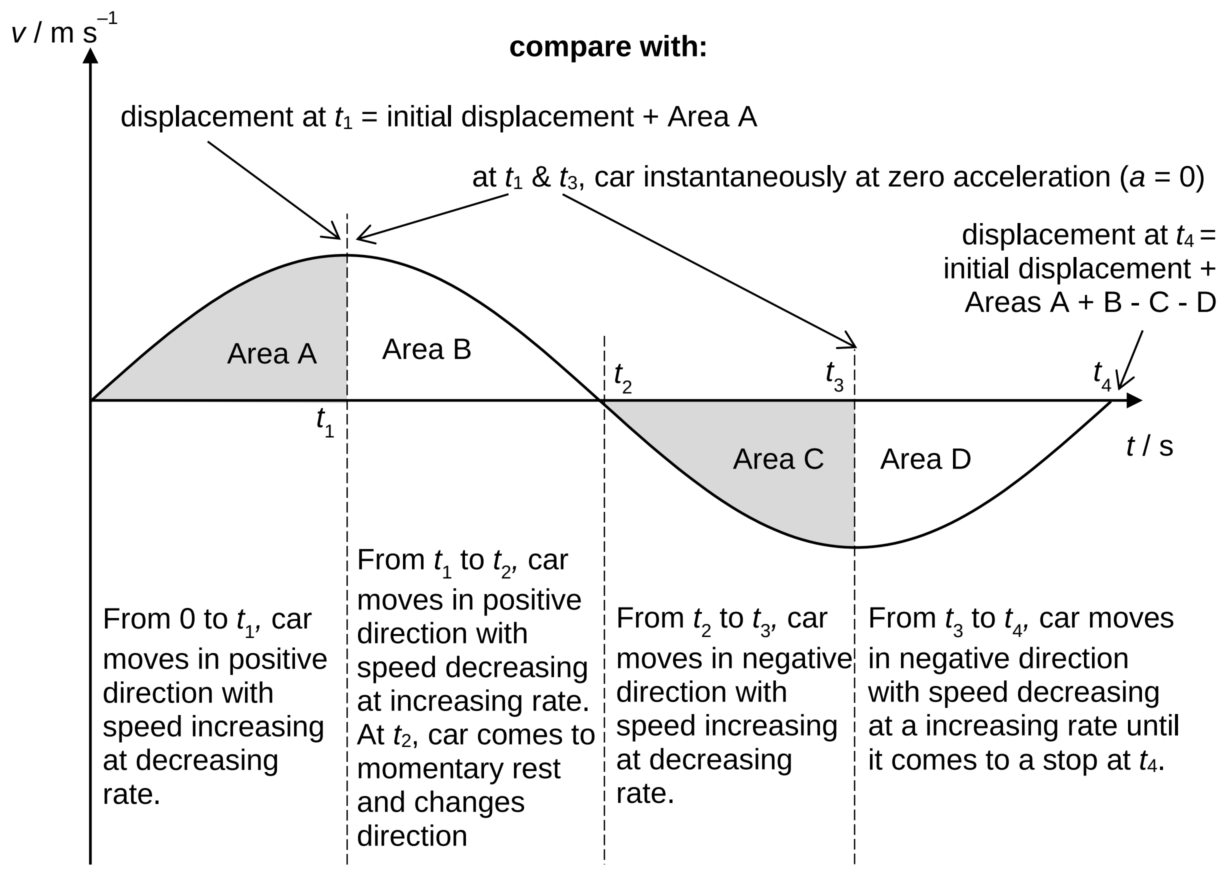
Time interval 4: The object moves in the negative direction, with constant speed.

Time interval 5: The object moves in the negative direction, with speed decreasing at an increasing rate.

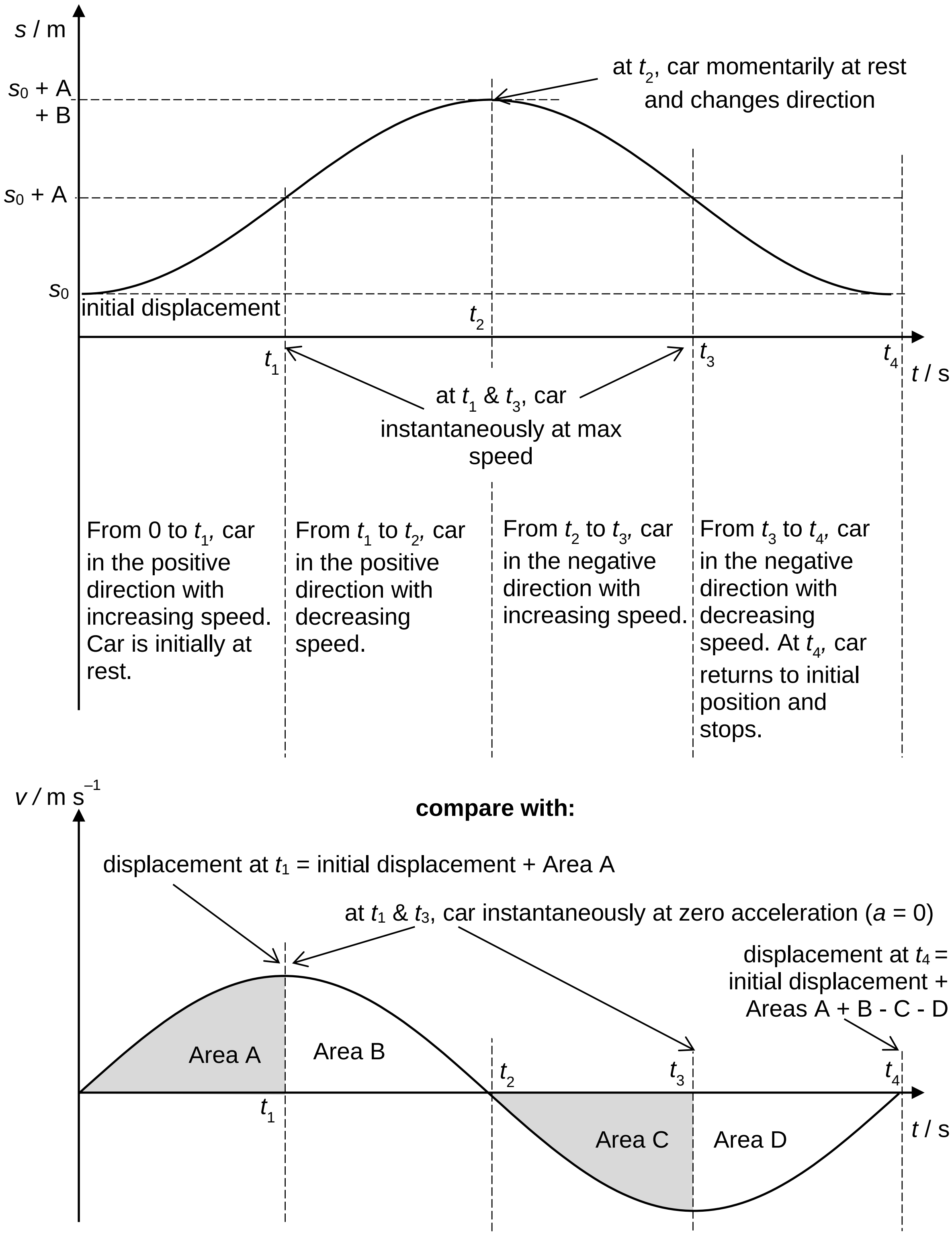
Compare and contrast the **velocity-time profiles** of two separate cars below.



compare with:



Study the **displacement-time profile** of a car below.

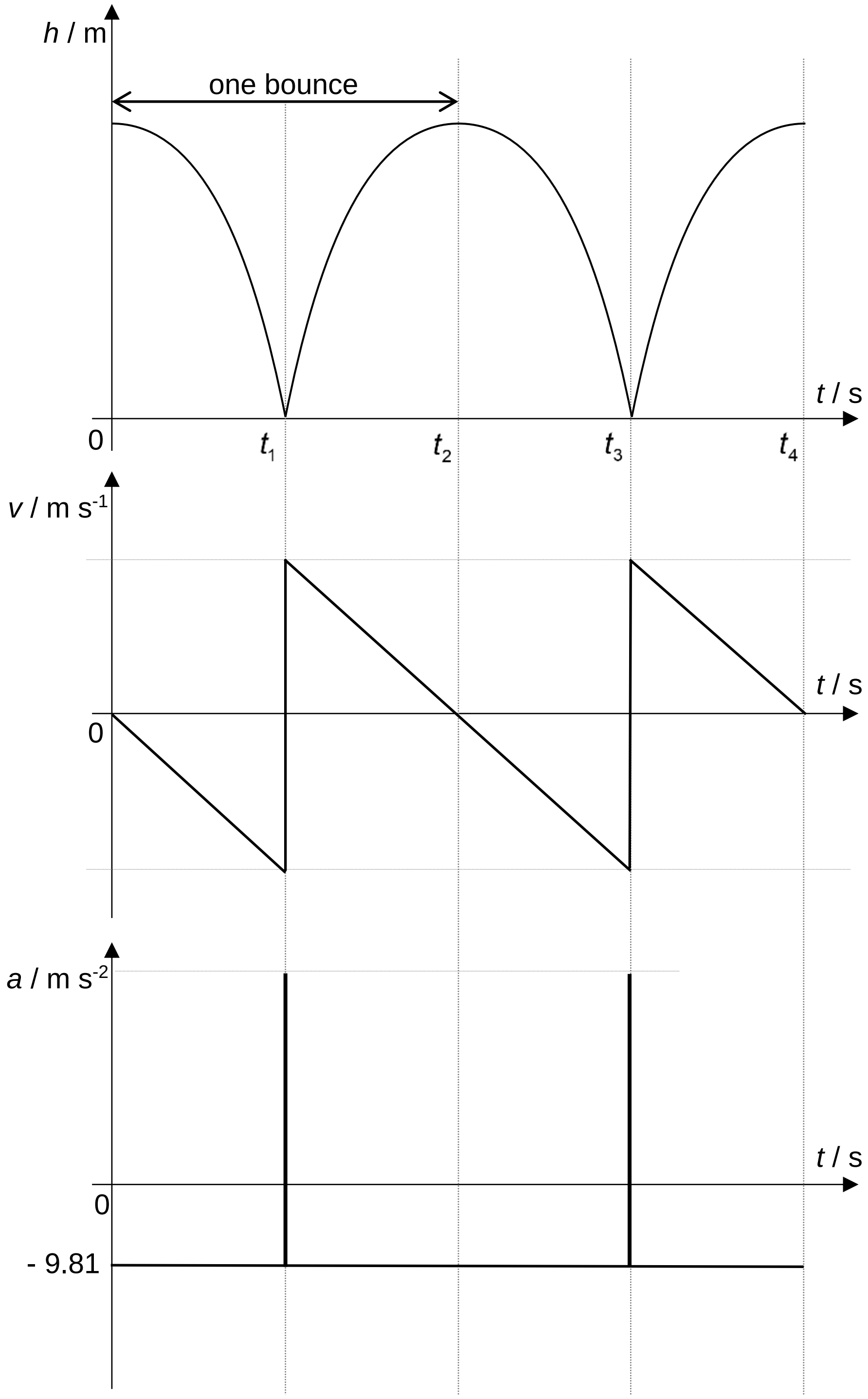


2.2.3.1 Motion Graphs of a Bouncing Ball

Consider a ball released from a height *h* above the floor that encounters no air resistance.

Bouncing Ball undergoing <u>Elastic</u> Collisions with Ground

Elastic Collision implies that the kinetic energy after collision with the ground is the same as the kinetic energy before collision.



***h-t* graph**

The maximum height reached after each rebound is constant. Note the constant height on h-t graph.

***v-t* graph**

When the ball is not touching the ground, it is in free fall. As the only force acting on the ball is its weight, it experiences a constant downwards acceleration of *g*.

Since gradient of velocity-time graph is acceleration, the gradient is constant throughout free fall and acceleration-time graph has a constant magnitude.

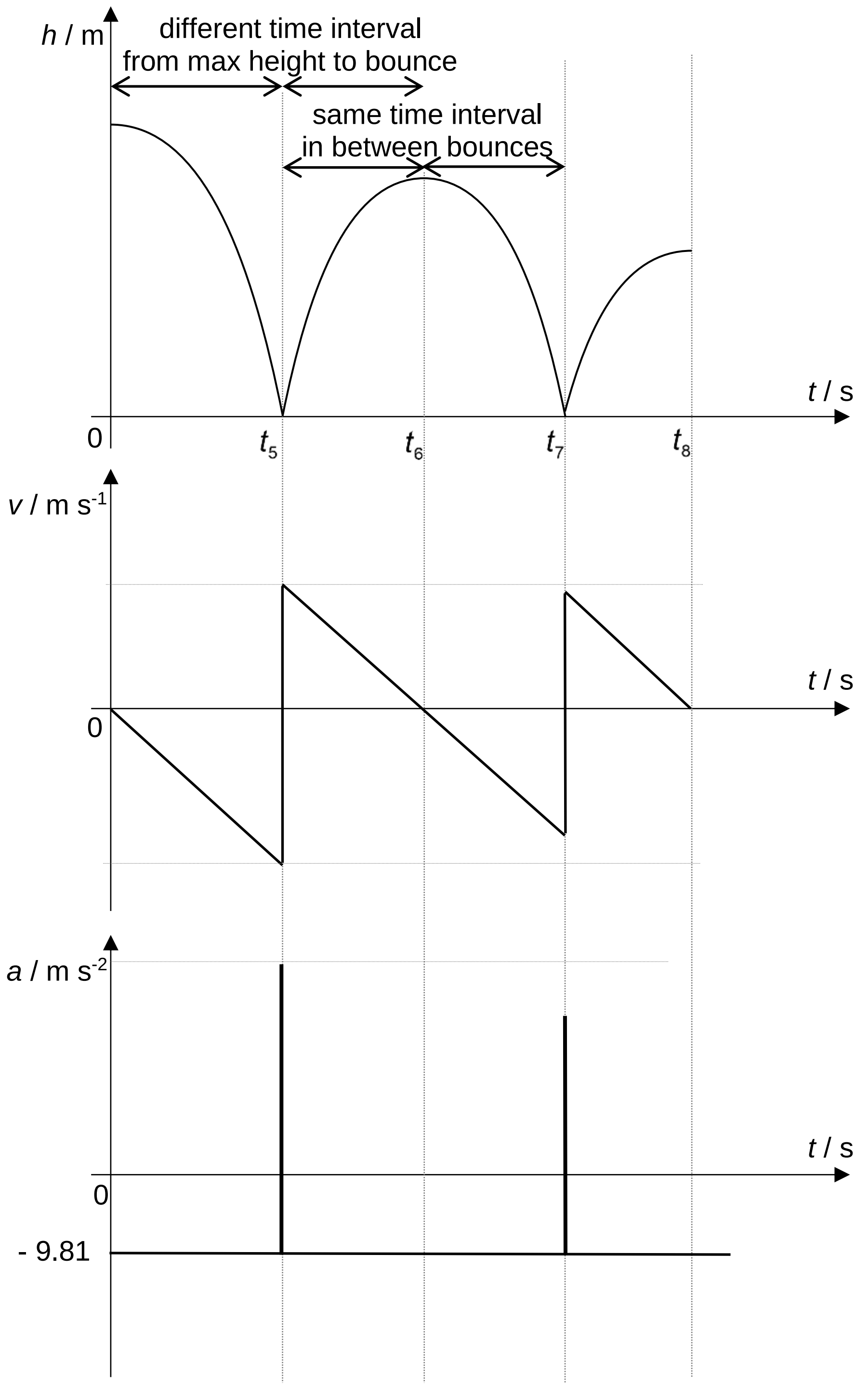
The area of the v-t graph for the part when the ball travels upwards is the same as the part when it travels downwards since it is the same distance covered by the ball.

***a-t* graph**

During the short time interval of contact with the ground, net force on ball becomes upwards, hence acceleration becomes positive.

Bouncing Ball undergoing <u>Inelastic</u> Collisions with Ground

During inelastic collisions, some of the ball's energy is dissipated during a bounce.



The time interval between impacts decreases.

***h-t* graph**

The maximum height reached after each bounce decreases.

***v-t* graph**

The area of the v-t graph for the part when the ball travels upwards is still the same as the part when it travels downwards.

Since the maximum height of the ball decreases after each bounce, the area after each bounce is smaller than the area before the bounce.

***a-t* graph**

Successive maximum accelerations during impacts decrease as the ball hits the surface with decreasing speeds on impacts.

2.3 Equations of Motion

When a body undergoes constant acceleration and motion in a straight line, we can apply kinematics equations to work with its displacement, velocity, acceleration and time interval during its motion.

$v = u + at$	$v^2 = u^2 + 2as$
$s = \frac{1}{2}(u+v)t$	$s = ut + \frac{1}{2}at^2$

s : displacement
u : initial velocity
v : final velocity
a : acceleration
t : time

Note:

(a) These 4 equations apply <u>only</u> to **motion in a straight line** with **constant acceleration**.

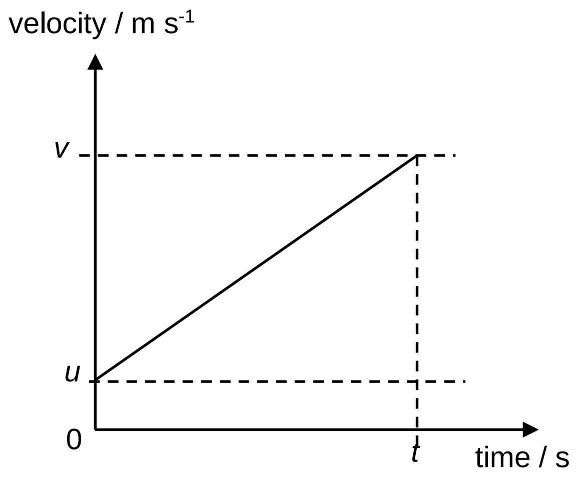
(b) **The terms in the equations are all vectors** (expect for time, t). **It is a good habit to define the positive direction before starting, and then ensure that the correct value is assigned to each term.** For example, we define upwards as positive. Then a ball falling due to the effects of gravity will experience acceleration a = – 9.81 m s^{-2}.

(c) If the motion has zero acceleration, $s = ut$, i.e. distance = speed × time.

(d) The use of such equations set s = 0 when t = 0.

Derivation

We consider the velocity-time graph of a body moving along 1 direction with constant acceleration a, initial velocity u, and final velocity v.



Gradient of velocity-time graph represents acceleration so

$$a = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$$

$$v = u + at \quad \text{---(1)}$$

Area under velocity-time graph represents displacement so

$$s = \frac{1}{2}(u+v)t \quad \text{--- (2)}$$

substitute (1) into v of (2):

$$s = \frac{1}{2}(u+v)t = \frac{1}{2}(u+[u+at])t = \frac{1}{2}(2u+at)t = ut + \frac{1}{2}at^2$$

substitute (1) into t of (2):

$$s = \frac{1}{2}(u+v)t = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{1}{2a}(v^2 - u^2)$$

$$v^2 = u^2 + 2as$$

General Steps to Solving Kinematics Questions

1. Sketch diagram of motion path
2. Identify the portion of motion to work with (select the start and end points)
3. Indicate which direction(s) is/are positive
4. Identify known quantities (includes drawing vectors and resolving)
5. Apply kinematic equation(s)
6. Check if answer is logical (i.e. not too big/small, positive/negative)

1.3.1 Solving 1D Kinematics (Linear Motion)

Example 9

A car is moving at 17 m s^{-1} when it begins to slow down with a deceleration of 1.5 m s^{-2}.
(a) Find the time taken to travel a distance of 70 m from when the car starts to slow down.
(b) Sketch its velocity-time graph.

Solution:

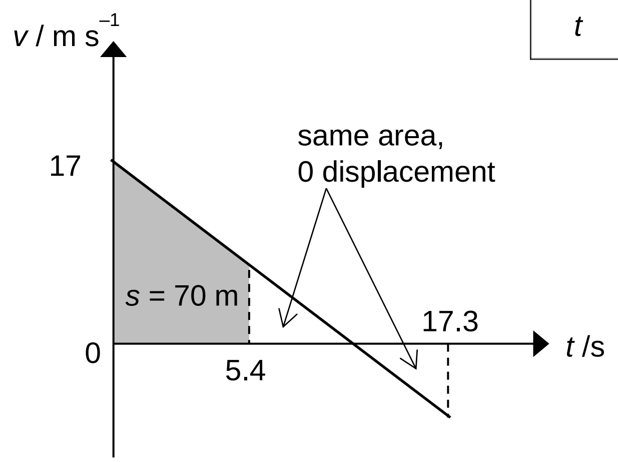
(a) $s = ut + \frac{1}{2}at^2$

$70 = 17t + \frac{1}{2}(-1.5)t^2$

$t = 5.41 \text{ s or } 17.3 \text{ s (reject)}$

u	17
v	
s	70
a	- 1.5
t	?

(b)



The car will eventually come to a rest (v = 0) and begin to accelerate with increasingly speed in the opposite (negative) direction.

At t = 17.3 s, the car is back to the same displacement as it were at t = 5.41 s.

Example 10

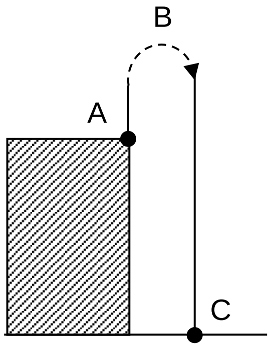
A stone is projected vertically upwards with a velocity of 10.0 m s^{-1} near the edge of a cliff. Find

(a) time taken for stone to reach maximum height

(b) maximum height of stone

(c) time taken for stone to reach the ground, 20.0 m below cliff edge

(d) speed of stone just before hitting ground



Solution:

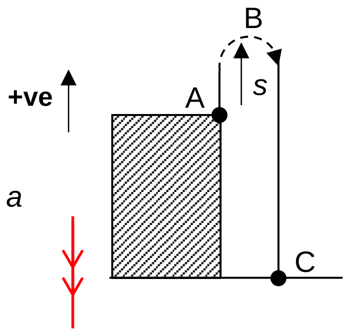
Take upwards as positive,

(a) velocity is zero at maximum height

From A to B:

$$v = u + at$$
$$0 = 10.0 + (-9.81)t$$
$$t = 1.02 \text{ s}$$

u	10.0
v	0
s	?
a	- 9.81
t	?



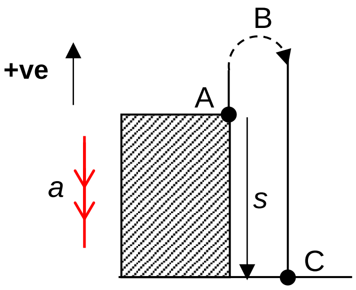
(b) From A to B:

$$v^2 = u^2 + 2as$$
$$0 = 10.0^2 + 2(-9.81)s_{max}$$
$$s_{max} = 5.10 \text{ m}$$

(c) From A to C:

$$s = ut + \frac{1}{2}at^2$$
$$-20.0 = 10t + \frac{1}{2}(-9.81)t^2$$
$$t = 3.28 \text{ s or } -1.24 \text{ s (reject)}$$

u	10.0
v	?
s	- 20.0
a	- 9.81
t	?



(d) From A to C:

$$v^2 = u^2 + 2as$$
$$v = \sqrt{10.0^2 + 2(-9.81)(-20.0)}$$
$$= 22.2 \text{ m s}^{-1} \text{ or } -22.2 \text{ m s}^{-1}$$

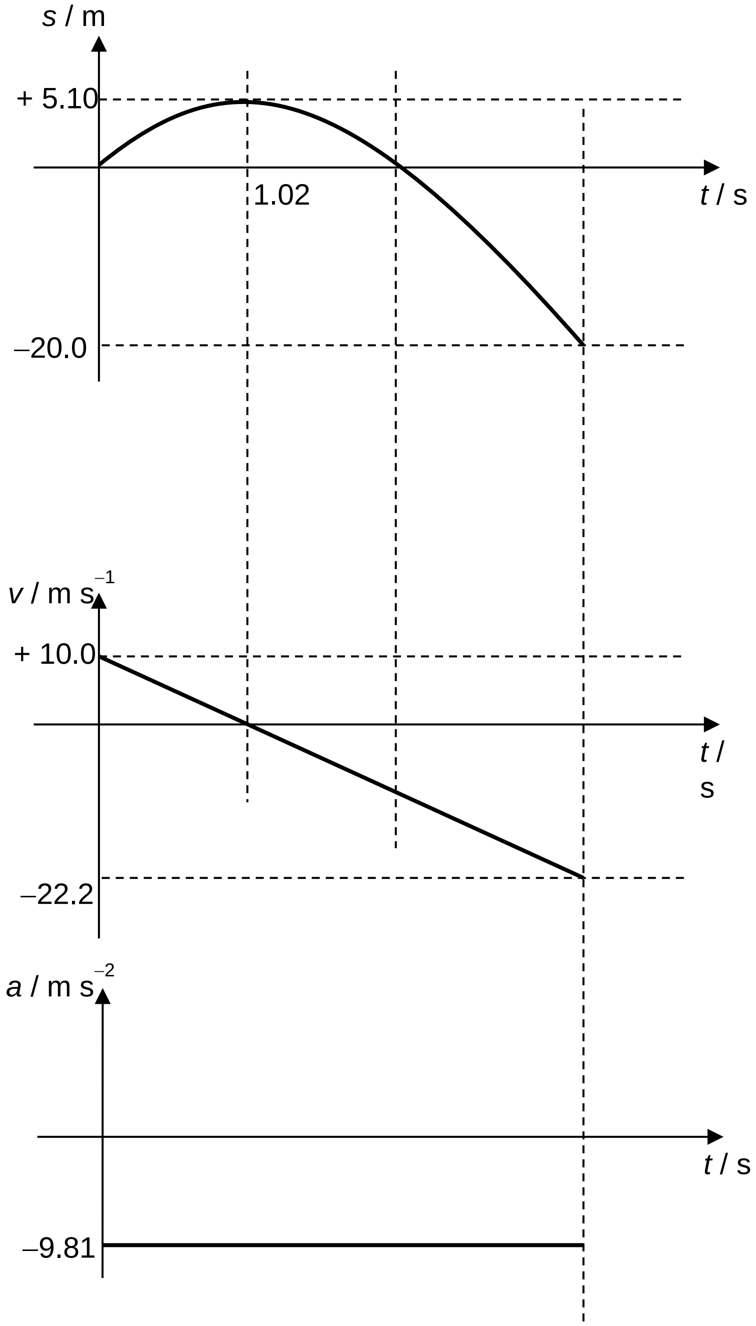
speed = 22.2 m s^{-1}

Note: For (d) we can quote the *speed* as the magnitude found. However, if asked for velocity we should reject the positive value because the stone is falling towards the ground moving along the negative direction.

Example 10 (cont'd)

Hence, sketch the variation of displacement, velocity and acceleration with time of the stone.

Solution:



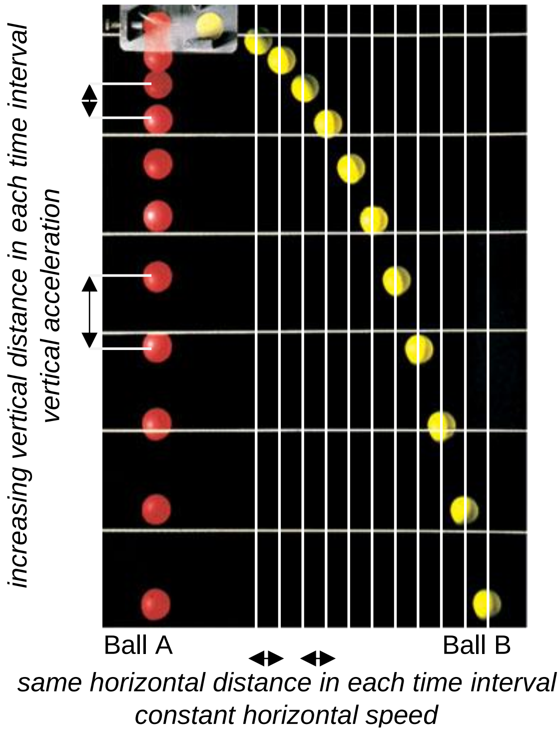
Note: the displacement graph is symmetric about the highest height to s = 0.

2.3.2 Solving 2D Kinematics (Projectile Motion)

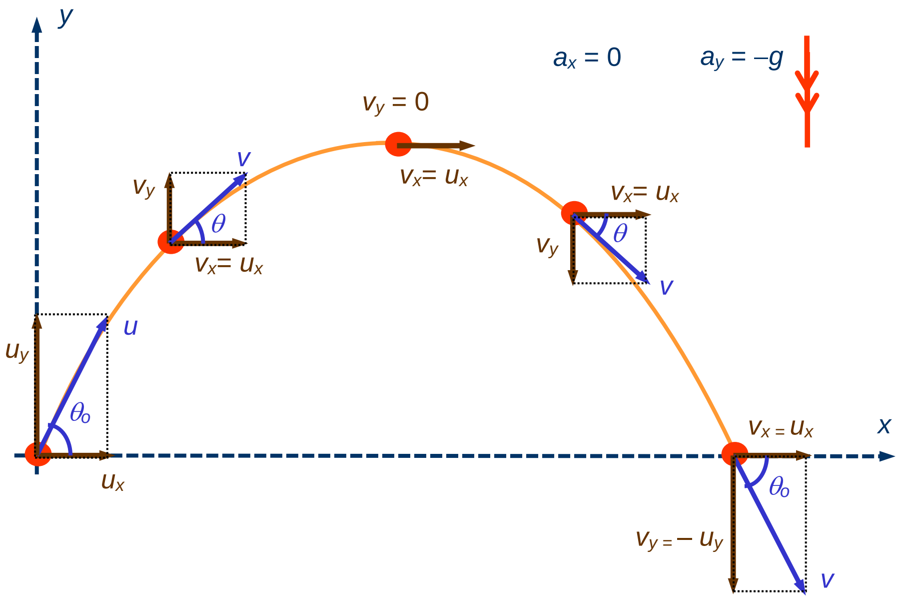
Near the Earth's surface, gravitational force acts on all masses and at all times. In this uniform gravitational field, masses accelerate towards the centre of the earth with constant acceleration g = 9.81 m s^{-2}.

When the only force acting on a mass is gravitational force (ignoring air resistance), the mass is said to be in free fall.

Also, if air resistance is negligible, then any horizontal velocity will remain constant, resulting in a parabola / parabolic path like that of Ball B.



The ***x*-direction** and ***y*-direction** motions of a projectile are **independent** of each other.



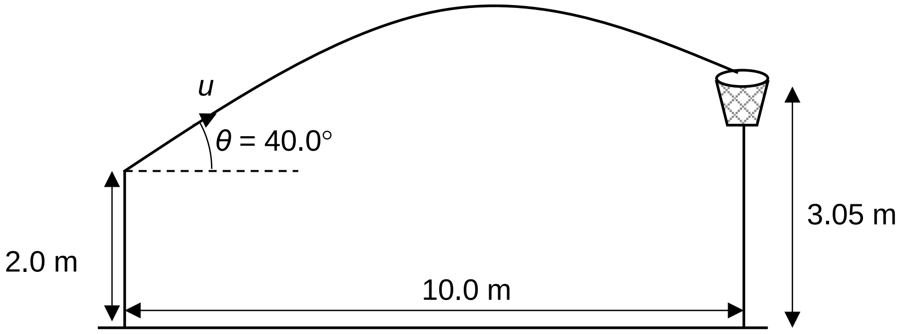
Projectile motion can be analysed by:

- Resolving displacement, velocity and acceleration vectors into their horizontal and vertical components
- Applying kinematics equations in each direction independently and separately
  - In the horizontal direction, since there is no horizontal acceleration, the kinematics equation is reduced to only one equation: $\boxed{s_x = u_x t}$
- Exploiting symmetry in the parabolic path i.e. the angle θ and speed will be the same for any two points which are at the same height of path
- Recalling that the time taken t is the same for both motion descriptions because they are describing the same motion path

Example 11

A basketball player 2.0 m tall is standing on the floor 10.0 m from the basket. The top of the basket is 3.05 m from the floor. He throws the ball at an angle of 40.0° above the horizontal.
Find the initial speed of the ball so that it scores without striking the backboard.

Solution:



Take upwards and rightwards as positive,
Take $s_y = 0$ at 2 m above ground

	horizontal	vertical
u	$u\cos(40.0°)$	$u\sin(40.0°)$
v	$u\cos(40.0°)$	
s	+ 10	+1.05
a	0	- 9.81
t		

consider horizontal motion:

$$s_x = u_x t$$

$$t = \frac{s_x}{u_x} = \frac{10.0}{u\cos(40.0°)}$$

consider vertical motion

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

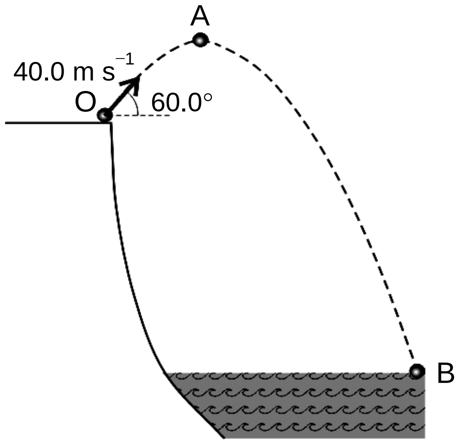
$$1.05 = [u\sin(40.0°)]\left(\frac{10.0}{u\cos(40.0°)}\right) + \frac{1}{2}(-9.81)\left(\frac{10.0}{u\cos(40.0°)}\right)^2$$

$$u = 10.7 \text{ m s}^{-1}$$

Note: here we used the fact that time taken for both the horizontal and vertical motion is same.

Example 12

A ball is projected from the edge of a cliff with a velocity of 40.0 m s^{-1} at an angle of 60.0° above horizontal as shown. It hits the water after 9.00 s.



(a) Determine the height of the cliff above the level of the water.
(b) Determine the velocity of the ball when it hits the water.
(c) Determine the horizontal distance travelled by the ball before it hits the water.
(d) State the velocity of the ball at A.
(e) Determine the greatest height reached by the ball.

Solution

Take upwards and rightwards as positive,

	horizontal	vertical
u	$40.0\cos(60.0°)$	$40.0\sin(60.0°)$
v	$40.0\cos(60.0°)$	
s		
a	0	- 9.81
t	9.00	

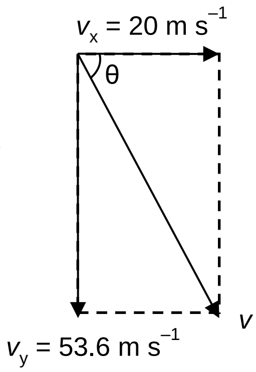
(a)
$$\begin{aligned} s_y &= u_y t + \frac{1}{2}a_y t^2 \\ &= (40.0\ \sin(60.0°))(9.00) + \frac{1}{2}(-9.81)(9.00)^2 \\ &= -85.5 \text{ m} \end{aligned}$$

Cliff is 85.5 m above sea level.

(b)
$$\begin{aligned} u_x &= v_x \\ &= 40\cos(60°) = 20.0 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} v_y &= u_y + a_y t \\ &= 40.0\sin(60.0°) + (-9.81)(9.0) \\ &= -53.6 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} v &= \sqrt{v_x^{\ 2} + v_y^{\ 2}} \\ &= \sqrt{20^2 + (-53.6)^2} \\ &= 57.3 \text{ m s}^{-1} \end{aligned}$$



$$\begin{aligned} \tan\theta &= \frac{53.6}{20} \\ \theta &= 69.5° \end{aligned}$$

final velocity is 57.3 m s^{-1}, 69.5° below horizontal.

(c)
$$\begin{aligned} s_x &= u_x t \\ &= 20(9.0) \\ &= 180 \text{ m} \end{aligned}$$

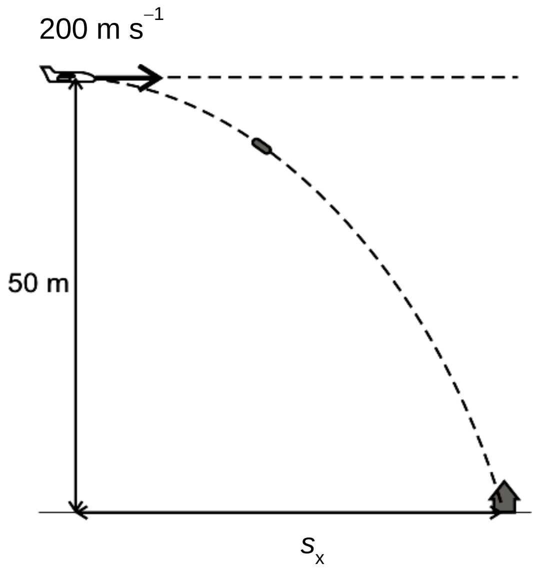
(d) velocity at A is 20.0 m s^{-1} to the right.

(e) at maximum height s_y, vertical velocity is zero

$$\begin{aligned} v_y^{\ 2} &= u_y^{\ 2} + 2a_y s_y \\ 0 &= (40\ \sin(60°))^2 + 2(-9.81)s_y \\ s_y &= 61.1 \text{ m} \end{aligned}$$

Example 13

A bomber flying horizontally at a speed of 200 m s^{-1} and a height of 50 m above the ground is approaching its target. Calculate how far horizontally before its target should the bomber release the bomb.



Solution

	horizontal	vertical
u	200	0
v	200	
s		+50
a	0	+ 9.81
T		

Take **down**wards and rightwards as positive, considering time of flight of bomb:

$$s_y = u_y t + \frac{1}{2} a_y t^2$$
$$50 = 0 + \frac{1}{2}(9.81)t^2$$
$$t = 3.2 \text{ s}$$

$$s_x = u_x t$$
$$= 200(3.2)$$
$$= 640 \text{ m}$$

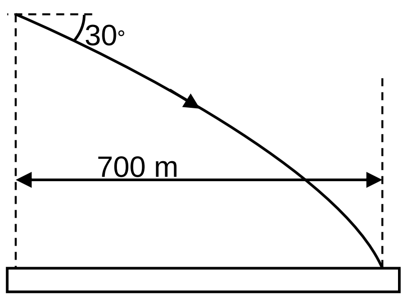
Note: it is not a must to define upwards as positive. In this case, it is more convenient otherwise.

Example 14

An airplane has a speed of 80.0 m s^{-1} and is diving at an angle of 30.0^o below the horizontal when the pilot releases a bomb. The horizontal distance between the release and the point where the decoy strikes the ground is 700 m.

(a) Find the time of flight of the bomb.

(b) Find the height at which the bomb was released.



Solution

Take downwards and rightwards as positive,

	horizontal	vertical
u	$80.0\cos(30.0°)$	$80.0\sin(30.0°)$
v	$80.0\cos(30.0°)$	
s	700	
a	0	+ 9.81
t		

(a) $$s_x = u_x t$$
$$700 = (80.0\cos 30.0^o)t$$
$$t = 10.1 \text{ s}$$

(b) $$s_y = u_y t + \frac{1}{2} a_y t^2$$
$$s_y = (80.0\sin 30.0°)(10.1) + \frac{1}{2}(9.81)(10.1)^2$$
$$s_y = 904 \text{ m}$$

2.4 Motion in Air Resistance

2.4.1 Motion of Falling Bodies With Air Resistance

w/o air resistance | **w/ air resistance**

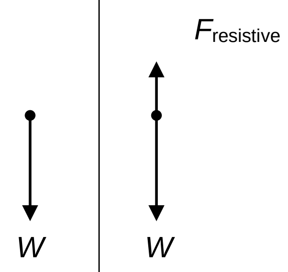
Free Body Diagram

w/o air resistance:

object is in free-fall, downward gravitational force is the only force acting on the mass.

$$F_{net} = W$$
$$ma = mg$$
$$a = g$$

vertically downwards



w/ air resistance:

magnitude of resistive force increases with relative speed between object and air:

$$F_{net} = W - F_r$$

vertically downwards

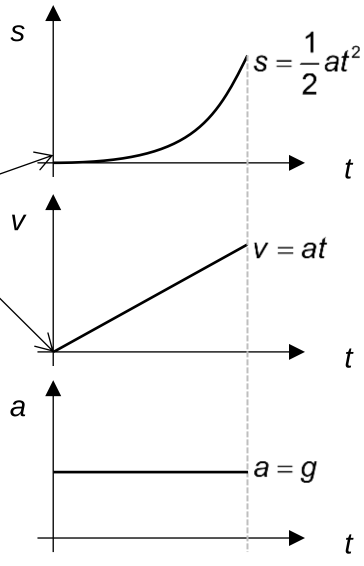
magnitude of F_r will increase until matches weight. With zero net force, object will reach a final constant speed called terminal velocity.

Motion Graphs

w/o air resistance:

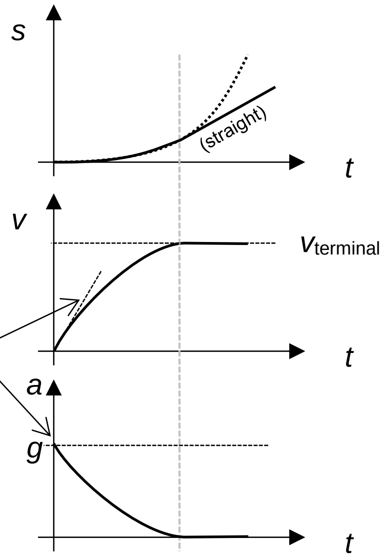
displacement is a quadratic curve (x^2) as $s = ut + \frac{1}{2}at^2$. If initially at rest:

gradient initially zero as $v = 0$

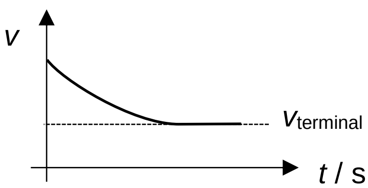


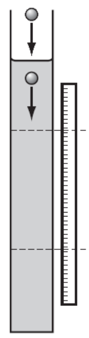
w/ air resistance:

acceleration initially is g as $v = 0$ so no resistance



Terminal velocity is not always maximum velocity e.g. small sphere falling from air into oil:



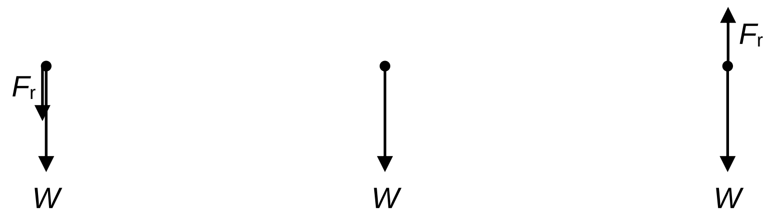


2.4.2 Motion of An Object Thrown Vertically Upwards With Air Resistance

on the way up | **on the way down**

Free Body Diagram

with air resistance | w/o air resistance | with air resistance



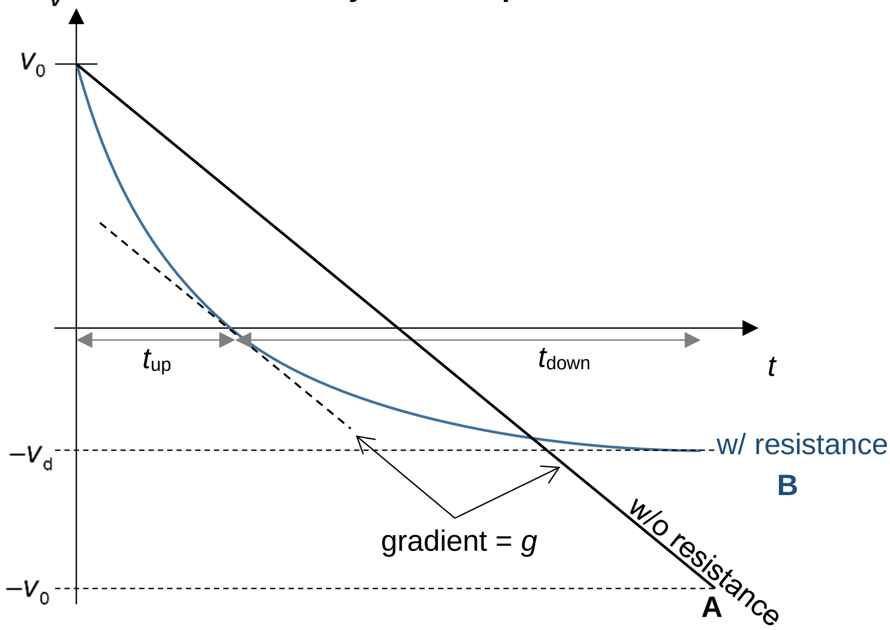
as air resistance opposes relative motion, it acts down vertically when object moves up. The magnitude of acceleration downwards is greater than g:

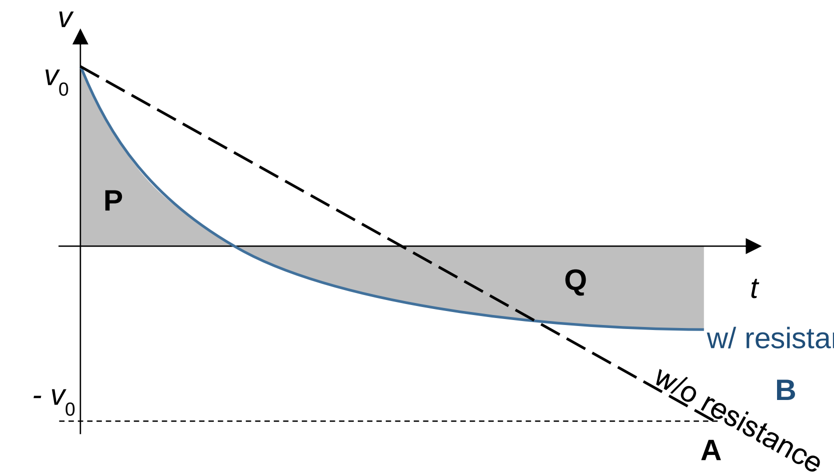
$$a = -g - \frac{F_r}{m}$$

as air resistance opposes relative motion, it acts up vertically when object moves down. The magnitude of acceleration downwards is less than g:

$$a = -g + \frac{F_r}{m}$$

Velocity-Time Graphs





Some features about the graphs:

1. During the upwards motion, gradient of B is steeper than A
   - gravitational force and air resistance both acts downwards
   - downwards net force is larger than gravitational force so larger acceleration
2. During the downwards motion, gradient of B is less steep than A
   - gravitational force acts downwards while air resistance acts upwards
   - downwards net force is smaller than gravitational force so smaller acceleration

Consequence of 1 & 2:

- for same vertical distance, upwards motion takes less time than downwards motion

3. At maximum height ($v_y = 0$), gradient of both graphs same at $g = 9.81\text{m s}^{-2}$
   - magnitude of air resistance increases with relative speed of object.
   - at maximum height, the ball is momentarily at rest so no air resistance
   - net force equals gravitational force

4. Area under graph (max height reached) for upward motion is smaller for B
   - with no air resistance all kinetic energy is converted to gravitational potential energy
   - with air resistance, some kinetic energy is lost as work done against air resistance
     - only some of initial kinetic energy is converted to gravitational potential energy
     - lower maximum height
   - area under *v*-*t* graph represents change in displacement
     - area P = area Q
     - smaller area under B represents lower maximum height reached
5. Speed when ball returns to original height is smaller for graph B
   - Let body have an initial upwards speed
   - Without air resistance, the ball will have same speed on return to ground
   - With air resistance, some energy is lost throughout the journey as thermal energy
     - kinetic energy is less than initial so less speed on return

2.4.3 Projectile Motion of An Object With Air Resistance

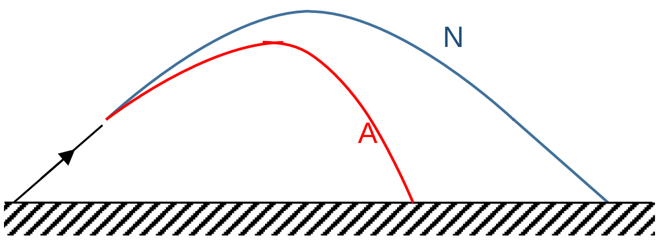
Example 15

A ball is thrown from the ground at an angle to the horizontal as shown. Sketch the path

(a) assuming no air resistance and label this path N,

(b) under the influence of air resistance and label this path A.

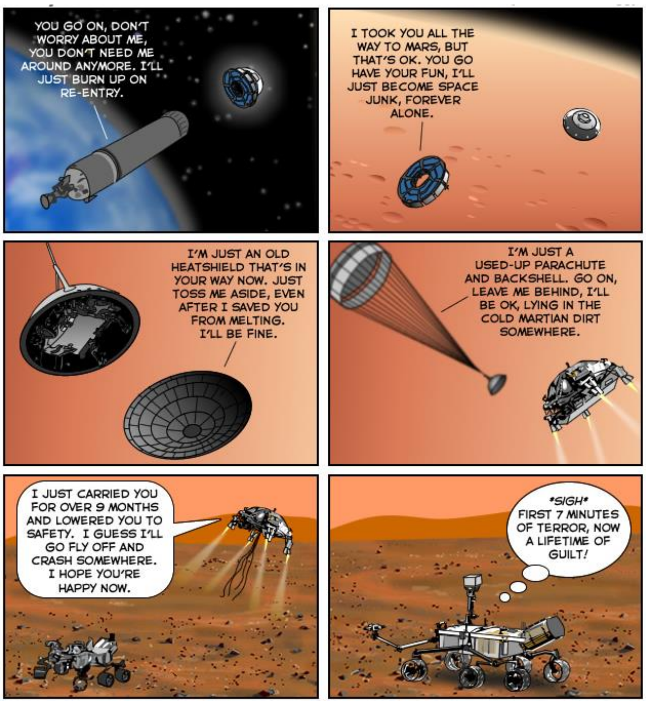
Explain the differences.



Lower vertical height and shorter horizontal range: air resistance opposes relative motion of the ball so average horizontal and vertical velocities of ball decreases,

Path is asymmetric: ball's horizontal velocity decreases with time so horizontal displacement per unit time decreases with motion.

2.5 Ending Notes



Kinematics can be very mathematical and therefore lends itself well to practice.

Do not lose yourself in doing so though, keep in mind how the principles of conservation of energy and the application of force considerations is intertwined in the motion.

You can use the space below to do up your own mind-map to summarise this topic.