Chapter 1: Permutation and Combination

1. HCI JC2 Prelim 8865/2019/Q6

(i) Janet is going on a holiday trip and is trying to recall the 4-digit numerical combination lock for her luggage. However, she can only remember the numerical combination consists of digits from the set $\{0, 1, 4, 6, 8, 9\}$.

(a) How many different numerical combinations are there if the odd and even digits must alternate and repetitions of digits are not allowed? [1]

(b) If the 4-digit numerical combination must be a number that is more than 6400 and that there are no repetitions of digits, find the maximum number of numerical combinations Janet has to try in order to open her lock? [2]

(ii) Janet has 6 T-shirts, 3 sweaters and 3 pairs of jeans, all of which are of different designs. Find the number of ways she can select her clothes for her trip if she needs 4 T-shirts, a sweater and at least a pair of jeans. [3]

Answer: (ia) 48, (ib) 156, (ii) 315

6. HCI JC2 Prelim 8865/2019/Q6 (Solutions)

(ia) Set of digits to choose from = {0, 1, 4, 6, 8, 9}
Note: Zero is even.

Case 1: <u>odd</u> <u>even</u> <u>odd</u> <u>even</u> $= 2\times 4\times 1\times 3 = 24$
Case 2: <u>even</u> <u>odd</u> <u>even</u> <u>odd</u> $= 4\times 2\times 3\times 1 = 24$

No. of combinations = $2\times\left(4\times 3\times 2\times 1\right) = 48$

(ib) Case 1: 6 _ _ _

4 or 8 or 9

No. of combinations = $1\times 3\times 4\times 3 = 36$

Case 2: _ _ _ _

8 or 9

No. of combinations = $2\times 5\times 4\times 3 = 120$

Total no. of combinations $36 + 120 = 156$

(ii) 6T, 3J, 3W

No. of ways to select T-shirts = $\binom{6}{4} = 15$

	No. of ways to select jeans = $\binom{3}{1}+\binom{3}{2}+\binom{3}{3}=7$ No. of ways to select sweater = 3 Total no. of ways = $15\times7\times3=315$

2.	NJC JC2 Prelim 8865/2019/Q8

Find the number of ways in which the letters of the word QUESTION can be arranged, if

(i) Q and U must be next to each other, [2]
(ii) E and S must be separated, [2]
(iii) consonants (Q, S, T, N) and vowels (U, E, I, O) must alternate. [2]

A code consists of 5 letters chosen from {Q, U, E, S, T, I, O, N}. Repetitions are not allowed.
(iv) Find the probability that a randomly chosen code has only 1 vowel. [2]

Answer: (i) 10080, (ii) 30240, (iii) 1152, (iv) $\frac{1}{14}$

2.	NJC JC2 Prelim 8865/2019/Q8 (Solutions)
(i)	No. of ways to arrange Q and U $=2!$ No. of ways to arrange all letters with Q and U grouped together $=7!=5040$ No. of ways that Q and U must be next to each other $=7!2!=10080$
(ii)	Method 1: Slotting No. of ways to arrange the other 6 letters $=6!=720$ No. of ways to choose slots and arrange E and S $=\binom{7}{2}\times2!=42$ No. of ways E and S must be separated $=6!\ \times\binom{7}{2}\times2!=30240$
(ii)	Method 2: Complementary No. of ways to arrange all the 8 letters $=8!=40320$ No. of ways E and S must be separated $=8!\ -\ \text{(i)}\ =30240$
(iii)	To alternate, the pattern must be CVCVCVCV or VCVCVCVC, where C = consonant, V = vowel. No. of ways to arrange the consonants $=4!=24$ No. of ways to arrange the vowels $=4!=24$ No. of ways to alternate$=2\times4!4!=1152$
(iv)	Method 1: Using Probability

	Probability (consider 5 cases to arrange VCCCC) $=\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times 5=\frac{1}{14}$ Method 2: Using P&C No. of ways to have 1 vowel $=\binom{4}{1}\times 5!=480$ Total number of 5-letter codes $=\binom{8}{5}\times 5!=6720$ Probability $=\frac{480}{6720}=\frac{1}{14}$

3.	ACJC JC2 Prelim 8865/2019/Q7
	A certain restaurant employs 4 cashiers, 5 ushers and 6 food servers as full-time service staff. A shift consists of a team of 2 cashiers, 4 ushers and 4 food servers on duty. (i) How many different teams can be formed to work in a shift? [1] (ii) Out of the 6 food servers, there is a pair of sisters working together as food servers in the restaurant. How many different selections of food servers are there in order to include exactly one of the two sisters in a shift? [1] (iii) The restaurant also employs a part-timer who can work either as a cashier or as an usher. How many different ways can a shift be formed that must include this particular part-timer?[3] (iv) All 15 full-time service staff gather in a straight line for a briefing. Find the number of ways the line can be formed such that all 5 ushers stand alternately with all 6 food servers. [2] Answer: (i) 450, (ii) 8, (iii) 1200, (iv) 10368000
3.	ACJC JC2 Prelim 8865/2019/Q7 (Solutions)
(i)	No. of teams $= {}^4C_2\times{}^5C_4\times{}^6C_4=6\times5\times15=450$
(ii)	No. of selections of food servers to include exactly 1 sister $={}^2C_1\times{}^4C_3=2\times4=8$
(iii)	Case 1: Part-timer as cashier No. of cashier-usher teams $= {}^4C_1\times{}^5C_4\times{}^6C_4=4\times5\times15=300$ Case 2: Part-timer as usher No. of cashier-usher teams $= {}^4C_2\times{}^5C_3\times{}^6C_4=6\times10\times15=900$ $\therefore$ No. of teams $=300+900=1200$
(iv)	No. of ways $=5!\times6!\times5!=120\times720\times120$ $=10,368,000$

4.	TMJC JC2 Prelim 8865/2019/Q6

	A company has 10 software engineers, 2 civil engineers and 5 electrical engineers. The company would like to send a delegation of 8 engineers for a conference. (i) How many different delegations can be formed if at least 2 of each type of engineers must be included in the delegation for the conference? [3] It is given that the delegation of 8 engineers for the conference must include 5 software engineers, 1 civil engineer and 2 electrical engineers. (ii) How many different delegations can be formed by the company? [1] (iii) One of the software engineers in the company is the brother of one of the electrical engineers in the company. How many different delegations can be formed which include exactly one of the two brothers? [3] Answer: (i) 3525, (ii) 5040, (iii) 2520
4.	TMJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	Number of delegations formed = (3 software, 2 civil, 3 electrical) + (4 software, 2 civil, 2 electrical) + (2 software, 2 civil, 4 electrical) = $\left({}^{10}C_3 \times {}^{2}C_2 \times {}^{5}C_3\right) + \left({}^{10}C_4 \times {}^{2}C_2 \times {}^{5}C_2\right) + \left({}^{10}C_2 \times {}^{2}C_2 \times {}^{5}C_4\right) = 3525$
(ii)	Number of delegations = ${}^{10}C_5 \times {}^{2}C_1 \times {}^{5}C_2 = 5040$
(iii)	Case 1: only software engineer brother is in Number of delegations = $1 \times {}^{9}C_4 \times {}^{2}C_1 \times {}^{4}C_2 = 1512$ Case 2: only electrical engineer brother is in Number of delegations = ${}^{9}C_5 \times {}^{2}C_1 \times 1 \times {}^{4}C_1 = 1008$ Total number of delegations formed = 1512 + 1008 = 2520

5.	ASRJC JC2 Prelim 8865/2019/Q6
	A game is played with 12 cards, consisting of 4 colours, Red, Green, Blue, and Yellow. Each set of colour consists of 3 cards labelled X, Y, and Z. The 12 cards are arranged in a row.
(a)	(i) How many different ways can the 12 cards be arranged so that the 3 cards in each coloured set are next to each other? [2]
	(ii) How many different ways can the cards be arranged so that not all Red cards are together? [2]

(iii) How many different ways can the cards be arranged so that all four cards labelled X are next to each other and all three Green cards are next to each other? [3]

(b) The cards are shuffled and arranged in a row.

Given that the first card is labelled Z, find the exact probability that no two cards labelled Z are next to each other. [4]

Answer: (ai) 31104, (aii) 457 228 800, (aiii) 120960, (b) $\frac{56}{165}$

5. ASRJC JC2 Prelim 8865/2019/Q6 (Solutions)

(a) (i) No. of Ways $=4!(3!)^4=31104$

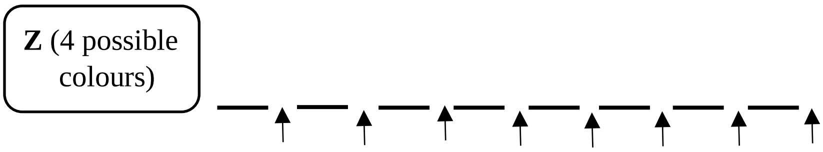
(ii) No. of Ways $=12!-(10!\times 3!)$

$=457\,228\,800$

(iii) No. of Ways $=7!(3!)(2)(2)=120960$

Green X must be beside the group of Xs (Left/right, 2 ways) so that it can be next to its colour set, while there are 3! ways to permute the other Xs.

(b)



Required Probability =

$$\frac{P(\text{no two Z next to each other and first card Z})}{P(\text{first card Z})}$$

$$=\frac{\frac{{}^4C_1\times{}^8C_3\times 3!\times 8!}{12!}}{\frac{4}{12}}$$

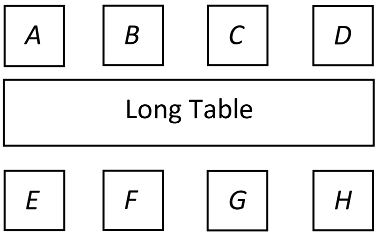
$$=\frac{\frac{56}{495}}{\frac{1}{3}}=\frac{56}{165}$$

6. CJC JC2 Prelim 8865/2019/Q10

Marie is planning to hold a birthday celebration dinner and she wants to prepare a cake for the occasion. She purchases six different types of toppings to enhance the flavour of the cake.

(i) Find the total number of different cakes she can make if she uses at least two toppings. [2]

Marie invites seven guests for the dinner, which includes a couple and a family of three. Marie and the guests are seated in eight chairs marked *A*, *B*, *C*, …, *H*, with four on each side of the long table, as shown in the following diagram.



Find the total number of table arrangements if

(ii) there are no restrictions, [1]

(iii) the couple must be facing each other across the table, and the others do not mind where they are seated, [3]

(iv) the family of three must be seated together on the same side of the table, and the others do not mind where they are seated. [3]

Answer: (i) 57, (ii) 40320, (iii) 5760, (iv) 2880

6.	CJC JC2 Prelim 8865/2019/Q10 (Solutions)
(i)	Method 1: Complement Number of ways $= 2^6 - {}^6C_1 - {}^6C_0$ $= 57$ Method 2: Number of ways $= {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$ $= 57$
(ii)	Number of ways $= 8!$ $= 40320$
(iii)	Arrange within the couple, then arrange the remaining 6 people. There are 4 possible places that the couple can sit. Hence, Number of ways $= 2! \times 6! \times 4$ $= 5760$
(iv)	Method 1:

	There are four possible places that the family of three can sit. Within the family, they can arrange themselves. Number of ways to arrange the family. $= 4 \times 3!$ $= 24$ Number of ways to arrange the remaining people $= 5!$ $= 120$ Total number of ways $= 24 \times 120$ $= 2880$
	Method 2: The family must sit together on the same side, so choose 1 person to sit on the same side with the family. Number of ways to arrange the people on the family side $= {}^{5}C_{1} \times 2!(\text{family, 1 person}) \times 3!(\text{within family})$ = 60 For the remaining people, number of ways to arrange them = 4! = 24 Total number of ways $= 60 \times 24 \times 2(\text{switch of two rows})$ $= 2880$

7. DHS JC2 Prelim 8865/2019/Q6

A group of neighbours consisting of 9 adults and 3 children attends a concert together. They are members from 3 households, Ahmad, Gupta and Lee. They stand in line to enter the concert hall together. The number of members from the respective households are shown in the following table.

Household	Number of members
Ahmad	3 children, 2 adults
Gupta	4 adults
Lee	3 adults

(i) Find the number of possible arrangements if members of each household stand together.[2]

(ii) Find the number of possible arrangements if no two children stand next to each other. [2]

	(iii) The organiser randomly picks 6 out of the 12 members for a survey such that at least one member from each household will be included. Find the number of possible selections. [3] Answer: (i) 103680, (ii) 261273600, (iii) 805
7.	DHS JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	No. of ways $= 5! \times 4! \times 3! \times 3! = 103680$
(ii)	No. of ways $= 9! \times {}^{10}P_3 = 261273600$
(iii)	Using Complement Method – Case 1 [Household Gupta & Ahmad (No Lee)] – Case 2 [Household Ahmad & Lee (No Gupta)] – Case 3 [Household Lee & Gupta & (No Ahmad)] Case 1: No. of ways $= {}^{9}C_6$ Case 2: No. of ways $= {}^{8}C_6$ Case 3: No. of ways $= {}^{7}C_6$ Total number of ways $= {}^{12}C_6 - {}^{9}C_6 - {}^{8}C_6 - {}^{7}C_6 = 805$

8.	SAJC JC2 Prelim 8865/2019/Q6
	A group of 12 people consisting of 3 married couples, 4 single men and 2 single women sit in a row to watch a movie. (i) Find the number of possible seating arrangements such that each married man sits beside his wife. [3] A committee of 6 members is to be formed from the above group to rate the movie. (ii) Find the number of ways to form the committee such that there are more men than women chosen. [3] Answer: (i) 2903040, (ii) 462
8.	SAJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	No. of possible arrangements $= 9! \times 2! \times 2! \times 2!$ $= 2903040$
(ii)	Case 1: 6 men and 0 woman chosen No. of ways = ${}^{7}C_6 \times {}^{5}C_0 = 7$ Case 2: 5 men and 1 woman chosen No. of ways = ${}^{7}C_5 \times {}^{5}C_1 = 105$ Case 3: 4 men and 2 women chosen No. of ways = ${}^{7}C_4 \times {}^{5}C_2 = 350$ $\therefore$ Total number of ways = 7 + 105 + 350 = 462

9.	YIJC JC2 Prelim 8865/2019/Q6
	A softball coach is forming a new female softball team comprising of 1 pitcher, 5 infielders and 3 outfielders. A certain club has 4 pitchers, 8 infielders and 6 outfielders. (i) How many different teams can be formed by the coach? [1] One of the infielders in the club is the sister of one of the outfielders in the club. (ii) How many different teams can be formed which include exactly one of the two sisters? [3] (iii) The coach selects the members of the team at random. Find the probability that none of the sisters is in the team. [2]
	Answer: (i) 4480, (ii) 2240, (iii) $\frac{3}{16}$

9.	YIJC JC2 Prelim 8865/2019/Q6 (Solutions)
(i)	Number of ways $= {}^4C_1 \times {}^8C_5 \times {}^6C_3$ $= 4480$
(ii)	Case 1: Infielder sister in, outfielder sister out No. of ways $= {}^4C_1 \times {}^7C_4 \times {}^5C_3$ $= 1400$ Case 1: Infielder sister out, outfielder sister in No. of ways $= {}^4C_1 \times {}^7C_5 \times {}^5C_2$ $= 840$ Total number of ways = 2240 Alternative Number of ways = total number of ways $-$ number of ways where both sisters are in $-$ number of ways where both sisters are out $= 4480 - \left({}^4C_1 \times {}^7C_4 \times {}^5C_2\right) - \left({}^4C_1 \times {}^7C_5 \times {}^5C_3\right)$ $= 4480 - 1400 - 840$ $= 2240$
(iii)	Required probability

	$= \dfrac{{}^{4}C_{1} \times {}^{7}C_{5} \times {}^{5}C_{3}}{4480}$ $= \dfrac{3}{16}$