Section A: Pure Mathematics [40 marks]

1 A company sells three types of cars: gas-powered, electric and hybrid. The total selling price of 4 electric cars is \$172 800 more than the total selling price of 3 gas-powered cars. The selling price of a hybrid car is 1.2 times that of a gas-powered car. The company sold 3 hybrid cars, 2 gas-powered cars and 2 electric cars for \$716 880.

Find the selling price of an electric car. [3]

2 Find the set of values of k for which

$$(k^2+36)x^2+(k-6)x+0.5\geq 0$$

for all real values of x. [4]

Hence find the set of values of k for which the expression

$$\ln[(k^2+36)x^2+(k-6)x+0.5)]$$

is defined for all real values of x. [1]

3 **(a)** Find $\int \frac{\left(25x^4+4\right)}{\sqrt{x}}\,\mathrm{d}x$. [2]

(b) Express $\frac{4x-3}{4-x}$ in the form $A+\frac{B}{4-x}$, where A and B are integers.

Hence find $\int \frac{4x-3}{4-x}\,\mathrm{d}x$, where $x<4$. [3]

4 The curve C has equation $y=-\ln(x+1)+\frac{5}{x-2}+2$.

(i) Show that curve C has no stationary points. [3]

(ii) Sketch the graph of C, stating the x-coordinates of any points of intersection with the x-axes and the equation(s) of any asymptotes. [3]

(iii) Find the numerical value of the area bounded by C, the x-axis and the lines $x = 3$ and $x = 5$. [1]

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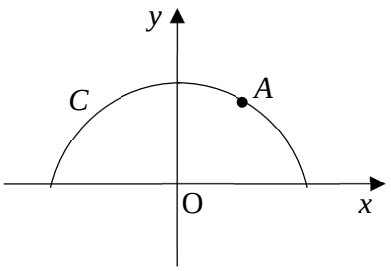

The diagram shows the curve C with equation $y = k^2 - x^2$, where k is a positive constant.

The x- coordinate of A is positive and the y-coordinate is $\frac{3}{4}k^2$.

(i) Find the equation of the tangent to C at the point A. Give your answer in the form $y = mx + c$, where m and c are in terms of k. [4]

(ii) Find the area bounded by the curve C, the tangent to the curve at point A and the x-axis, giving your answer in the form $\frac{p}{q}k^3$, where p and q are integers. [4]

6 Mr Lim has just set up a company selling handphones. Over a period of 9 months, his total profit P thousand dollars per month can be modelled by

$$P = -(\mathrm{e}^{0.2t} - 4)^2 + 9,$$

where t is the time in months after he set up his company.

(i) Use differentiation to find the value of t which gives a stationary point on the graph of P against t. Justify whether this point is a maximum or minimum point. [4]

(ii) Sketch the graph of P against t for $0 \le t \le 9$. [1]

(iii) Using integration, find $\int_0^9 -(\mathrm{e}^{0.2t} - 4)^2 + 9 \, \mathrm{d}t$, correct to the nearest integer, showing your workings clearly.

Explain, in the context of the question, what this value represent. [4]

(iv) Mr Lim has a budget for advertising his handphones for sale. At any time, t months, measured from the start of his advertising campaign, the remaining budget, A dollars, is modelled by

$$A = \frac{200}{3}(-20t + 75 + \frac{375}{2t+5}).$$

Using differentiation, find $\frac{\mathrm{d}A}{\mathrm{d}t}$ when $t = 3,$ giving your answer to the nearest integer.

Explain, in the context of this question, what this value represent. [3]

Section B: Probability and Statistics [60 marks]

7 It is known that on average p% of the elderly patients in ABC hospital suffer from hypertension. A research study on 30 randomly selected elderly patients in the hospital was conducted. The number of elderly patients in the hospital with hypertension is denoted by the random variable X. Assume that X has a binomial distribution.

(a) If X has mean 18, calculate $\mathrm{P}(X \geq 20)$. [3]

It is also known that on average k% of the elderly patients in ABC hospital suffer from diabetes. The hospital conducted a research study on another 40 randomly selected elderly patients. The number of elderly patients in the hospital with diabetes also follows a binomial distribution.

(b) The probability of at least nine but no more than twenty elderly patients suffering from diabetes is 0.25. Given that $k < 50,$ find the value of k. [3]

8 Events A and B are such that $\mathrm{P}(A \mid B) = \frac{7}{10},$ $\mathrm{P}(B) = \frac{1}{3}$ and $\mathrm{P}(A \cup B) = \frac{3}{5}.$

(a) Find $\mathrm{P}(A \cap B)$. Hence show that $\mathrm{P}(A) = \frac{1}{2}.$ [4]

(b) Find the probability that exactly one of A and B occurs. [2]

(c) Determine whether the events A and B are independent. [1]

9 There are two bags of balls. Bag *A* contains 2 red balls and 1 black ball. Bag *B* contains 1 red ball and 1 black ball.

- A bag is selected at random, and the first ball is drawn at random from it.
- If the first ball is black, it is returned to the selected bag.
- If the first ball is red, it is placed in the other bag.
- Regardless of whether the first ball is red or black, the second ball is drawn at random from the other bag.

(a) Draw a tree diagram to represent this information. [2]

(b) Find the probability that both balls are of the same colour. [2]

(c) Given that both balls drawn are red, find the probability that the first bag chosen was *A*. [3]

10 A horticulturist carries out research on the effects of varying amounts of fertilizer on the average crop yield of potatoes. Eight potato plants of the same variety were selected at random. The results are shown in the table below.

Plant	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Amount of fertiliser (x grams per square metre)	3.2	1.5	0.8	0	2.5	8.0	4.0	4.1
Average crop yield (y kilograms)	2.6	2.2	2.3	2.0	2.3	3.6	2.7	2.8

(i) Give a sketch of the scatter diagram of the data. [2]

(ii) Find the product moment correlation coefficient, r, and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x in the form $y = a + bx$, giving the values of a and b correct to 3 significant figures. [1]

(iv) Use the equation of your regression line to estimate the amount of fertiliser used on a potato plant of the same variety when the average crop yield of potatoes is 2.5 kilograms. Comment on the reliability of the estimate. [2]

(v) Without any further calculations, explain briefly what will happen to the value of r found in **(ii)** if y was recorded in grams instead. [1]

11 A family of seven, consisting of a father, mother, two sisters, and three brothers is queuing in a line.

(i) Find the number of different possible arrangements

(a) if the sisters stand next to each other and the brothers are all next to each other. [2]

(b) if either the father is first in the queue or mother is last in the queue (or both). [3]

(ii) The family is queuing in a line in a random order. Find the probability that no two brothers stand next to each other. [3]

12 **In this question you should state the parameters of any normal distributions you use**.

In a certain college, the heights of boys and girls have independent normal distributions. The heights of boys are normally distributed with mean 172 cm and standard deviation σ cm. The probability that the boys are taller than 156 cm is 0.948.

(a) Show that $\sigma = 9.84$, correct to 3 significant figures. [2]

Use $\sigma = 9.84$ for the rest of this question.

(b) A sample of 100 randomly chosen boys are taken. $\bar{X}$ denote the mean height of the boys in the sample. 80 such samples are taken.

(i) Find the expected number of samples where $\bar{X} > 172$. [2]

(ii) Find the probability that at most 35 samples have $\bar{X} > 172$. [2]

The heights of girls are normally distributed with mean 165 cm and standard deviation 8 cm.

(c) A boy and a girl are chosen at random. Find the probability that their heights differ by less than 5 cm. [3]

(d) Find the probability that the total height of 3 randomly chosen girls exceeds twice the height of a randomly chosen boy by at most 149 cm. [4]

13 The manager of a firm that produces tyres claims that the mean lifetime of the tyres (thousand kilometres) is 18.5. He selects a random sample of 200 tyres and the lifetimes of these tyres , x (thousand kilometres) are summarised as follows:

$$\sum(x-18.5)=75 \qquad \Sigma(x-18.5)^2=1968.04$$

(a) Test at the 5% significance level whether the manager's claim is supported by the data. [6]

(b) Explain what is meant by 'at the 5% significance level' in the context of the question. [1]

A new manufacturing process is developed to increase the lifetime of tyres (in thousands of kilometres). Over a period of time, it was found that the mean lifetime of tyres produced by the new process is μ_0. The lifetime of tyres is known to follow a normal distribution with variance 9.5. A random sample of 400 tyres made by the new process is examined and found to have a mean lifetime of 20.

(c) Find the set of values of μ_0 in order that the firm may claim that the new process is effective at 1% level of significance. [4]