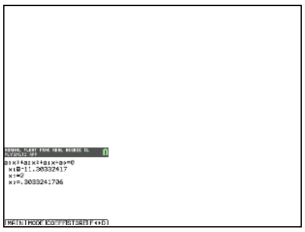
Discrete Random Variable & Binomial Distribution

1(i)	$\frac{k}{40}(1)+\frac{k}{40}(2)+\frac{1}{40}(3)+\frac{1}{40}(4)=1$ $\Rightarrow\ 3k+7=40$ $\Rightarrow\ k=11$ [Shown]
(ii)	$\mathrm{P}\left(X=\ 3\,\vert\, X\geq 2\right)=\frac{\mathrm{P}(X=\ 3\cap X\geq 2)}{\mathrm{P}(X\geq 2)}=\frac{\mathrm{P}(X=3\ )}{\mathrm{P}(X=\ 2,3,4)}=\frac{\frac{3}{40}}{\frac{2k}{40}+\frac{3}{40}+\frac{4}{40}}=\frac{\frac{3}{40}}{1-\frac{k}{40}}=\frac{3}{29}$
(iii)	$\mathrm{E}(X)=1\times\frac{11}{40}+2\times\frac{22}{40}+3\times\frac{3}{40}+4\times\frac{4}{40}\ =2$

2(i)	3 blue counters, 1 red counter and y yellow counters. S = No. of blue counters + 2(No. of red counters) P(S = 3) = P(BBB) + P(RBY in any order) $=\frac{3}{y+4}\times\frac{2}{y+3}\times\frac{1}{y+2}+\frac{1}{y+4}\times\frac{3}{y+3}\times\frac{y}{y+2}\times 3!$ $=\frac{18y+6}{(y+4)(y+3)(y+2)}$ [Shown] or $\frac{{}^{3}C_{3}}{{}^{y+4}C_{3}}+\frac{{}^{1}C_{1}\,{}^{3}C_{1}\,{}^{y}C_{1}}{{}^{y+4}C_{3}}$
(ii)	$\mathrm{P}(S=3)=\frac{7}{20}\Rightarrow\frac{18y+6}{(y+4)(y+3)(y+2)}=\frac{7}{20}$ Expand and simplify: $7y^{3}+63y^{2}-178y+48=0$  a3x³+a2x²+a1x+a0=0 x1≈-11.30332417 x2=2 x3=.3033241706 Since y is a positive integer, y = 2 from GC. Now we have 3 blue, 1 red counter and 2 yellow counters.

Possible values of S are 1(YYB in any order), 2(BBY or RYY in any order), 3 (BBB or RBY in any order) and 4 (BBR in any order)

P(S = 1) = P(YYB in any order)

$$=\frac{2}{6}\times\frac{1}{5}\times\frac{3}{4}\times\frac{3!}{2!}=\frac{3}{20}$$

P(S = 2) = P(BBY in any order) + P(RYY in any order)

$$=\frac{3}{6}\times\frac{2}{5}\times\frac{2}{4}\times\frac{3!}{2!}+\frac{1}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3!}{2!}=\frac{7}{20}$$

P(S = 4) = P(BBR in any order) $=\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3!}{2!}=\frac{3}{20}$

The probability of S is

s	1	2	3	4
$\mathrm{P}(S=s)$	$\frac{3}{20}$	$\frac{7}{20}$	$\frac{7}{20}$	$\frac{3}{20}$

3(i)

x	0	1	2	3	4
P(X = x)	$2k$	k	0	k	$2k$

$$2k+k+k+2k=1$$

$$6k=1$$

$$k=\frac{1}{6}$$

$$\mathrm{E}(X)=0\times\frac{2}{6}+1\times\frac{1}{6}+2\times0+3\times\frac{1}{6}+4\times\frac{2}{6}=2$$

Or By symmetry, E(X) = 2

(ii)

$$\mathrm{E}(|X-2|)$$

$$=\sum_{\text{all } x}|x-2|\,\mathrm{P}(X=x)$$

$$=|0-2|\times\frac{2}{6}+|1-2|\times\frac{1}{6}+0+|3-2|\times\frac{1}{6}+|4-2|\times\frac{2}{6}$$

$$=\frac{5}{3}$$

(iii)

$$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2$$

$$=\left[0+1^2\left(\frac{1}{6}\right)+0+3^2\left(\frac{1}{6}\right)+4^2\left(\frac{2}{6}\right)\right]-2^2=\left(\frac{1}{6}+\frac{9}{6}+\frac{32}{6}\right)-4=3$$

4

Question (i)

$$\mathrm{P}\left(\text{different colour}\right)=\mathrm{P}\left(B,W\right)=\frac{n}{2n+1}\cdot\frac{n+1}{2n}\cdot 2=\frac{n+1}{2n+1}.$$

Question (ii)

$$\mathrm{P}\left(\text{same colour}\right)=\mathrm{P}\left(B,B\right)+\mathrm{P}\left(W,W\right)$$

$$=\frac{n}{2n+1}\cdot\frac{n-1}{2n}+\frac{n+1}{2n+1}\cdot\frac{n}{2n}=\frac{n-1}{2(2n+1)}+\frac{n+1}{2(2n+1)}=\frac{n}{2n+1}$$

Question (iii)

$$\mathrm{P}\left(X=1\right)=\left(\frac{n+1}{2n+1}\right)\left(\frac{1}{2}\right)^2(2)+\left(\frac{n}{2n+1}\right)\left(\frac{1}{2}\right)^4\frac{4!}{3!}=\frac{3n+2}{4\left(2n+1\right)}$$

Alternatively,

$$\mathrm{P}\left(X=1\right)=\mathrm{P}\left(B,W,H,T\right)+\mathrm{P}\left(B,B,H,T,T,T\right)+\mathrm{P}\left(W,W,H,T,T,T\right)$$

$$=\left[\frac{n+1}{2n+1}\right]\left(\frac{1}{2}\right)^2(2)+\left[\frac{n-1}{2(2n+1)}\right]\left(\frac{1}{2}\right)^4\frac{4!}{3!}+\left[\frac{n+1}{2(2n+1)}\right]\left(\frac{1}{2}\right)^4\frac{4!}{3!}=\frac{3n+2}{4\left(2n+1\right)}$$

Question (iv)

If there are 3 blue cards $\Rightarrow n=3$

$$\mathrm{P}\left(B,W\right)=\frac{n+1}{2n+1}=\frac{4}{7};\ \mathrm{P}\left(B,B\right)=\frac{n-1}{2(2n+1)}=\frac{1}{7};\mathrm{P}\left(W,W\right)=\frac{n+1}{2(2n+1)}=\frac{2}{7}$$

$$\mathrm{P}\left(X=1\right)=\frac{3n+2}{4(2n+1)}=\frac{11}{28}$$

$$\mathrm{P}\left(X=4\right)=\mathrm{P}\left(B,B,H,H,H,H\right)+\mathrm{P}\left(W,W,H,H,H,H\right)$$

$$=\frac{1}{7}\left(\frac{1}{2}\right)^4+\frac{2}{7}\left(\frac{1}{2}\right)^4=\frac{3}{112}$$

$$\mathrm{P}\left(X=3\right)=\mathrm{P}\left(B,B,H,H,H,T\right)+\mathrm{P}\left(W,W,H,H,H,T\right)$$

$$=\frac{1}{7}\left(\frac{1}{2}\right)^4\left(\frac{4!}{3!}\right)+\frac{2}{7}\left(\frac{1}{2}\right)^4\left(\frac{4!}{3!}\right)=\frac{3}{28}$$

$$\mathrm{P}\left(X=0\right)=\mathrm{P}\left(B,W,T,T\right)+\mathrm{P}\left(B,B,T,T,T,T\right)+\mathrm{P}\left(W,W,T,T,T,T\right)$$

$$=\frac{4}{7}\left(\frac{1}{2}\right)^2+\frac{1}{7}\left(\frac{1}{2}\right)^4+\frac{2}{7}\left(\frac{1}{2}\right)^4=\frac{19}{112}$$

$$\mathrm{P}\left(X=2\right)=1-\mathrm{P}(X=0)-\mathrm{P}(X=1)-\mathrm{P}(X=3)-\mathrm{P}(X=4)$$

$$=1-\left(\frac{19}{112}+\frac{11}{28}+\frac{3}{28}+\frac{3}{112}\right)$$

$$=1-\frac{39}{56}=\frac{17}{56}$$

Alternatively,

x	0	1	2	3	4
$\mathrm{P}\left(X=x\right)$	$\frac{4}{7}\left(\frac{1}{2}\right)^2+\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{19}{112}$	$\frac{3\left(3\right)+2}{4(2\left(3\right)+1)}$ $=\frac{11}{28}$	$\frac{4}{7}\left(\frac{1}{2}\right)^2+{}^4C_2\,\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{17}{56}$	${}^4C_1\,\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{3}{28}$	$\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{3}{112}$

Question 8(v)

When $n=3$, $\mathrm{P}\left(X>2 \mid X\le 3\right)=\dfrac{\mathrm{P}\left(2<X\le 3\right)}{\mathrm{P}\left(X\le 3\right)}$

$$=\frac{\mathrm{P}\left(X=3\right)}{1-\mathrm{P}\left(X=4\right)}$$

$$=\frac{\frac{3}{28}}{1-\frac{3}{112}}$$

$$=\frac{12}{109}$$

<table>
<tr><td>5(i)</td><td>$\mathrm{P}(X=3)=\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$</td></tr>
<tr><td>(ii)</td><td>X = Number of tries John takes to open the door = { 1, 2, 3, 4}
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X = x)</td><td>$\frac{1}{4}$</td><td>$\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}=\frac{1}{4}$</td></tr>
</table></td></tr>
<tr><td>(iii)</td><td>$\mathrm{E}(X)=1\times\frac{1}{4}+2\times\frac{1}{4}+3\times\frac{1}{4}+4\times\frac{1}{4}=2.5$
$\mathrm{E}(X^2)=1^2\times\frac{1}{4}+2^2\times\frac{1}{4}+3^2\times\frac{1}{4}+4^2\times\frac{1}{4}=7.5$
$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}\left(X\right)\right]^2=7.5-(2.5)^2=1.25$</td></tr>
<tr><td>(iv)</td><td>$\mathrm{P}(X^2<8)=\mathrm{P}(X<\sqrt{8})=\mathrm{P}(X=1)+\mathrm{P}(X=2)=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$</td></tr>
</table>

6

Given $\mathrm{P}\left(X=1\right)=0.04$

For $r=2,3,4,5,6$, $\mathrm{P}\left(X=r\right)=\dfrac{1-0.04}{5}=0.192$.

t	Possible Outcomes (x_1, x_2)	Probability $\mathrm{P}(T=t)$
1	(1, 1)	$0.04(0.04)=0.0016$
2	(2, 2), (1, 2), (2, 1)	$0.192(0.192)+2\left(0.04\right)\left(0.192\right)=0.052224$ [Shown]
3	(3, 3), (1, 3), (3, 1), (2, 3), (3, 2)	$\left(0.192\right)(0.192)+2\left(0.04\right)\left(0.192\right)+2\left(0.192\right)(0.192)$ $=0.125952$
4	(4, 4), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3)	$\left(0.192\right)(0.192)+2\left(0.04\right)\left(0.192\right)+4\left(0.192\right)(0.192)$ $=0.19968$
5	(5, 5), (1, 5), (5, 1), (2, 5), (5, 2), (3, 5), (5, 3), (4, 5), (5, 4)	$\left(0.192\right)(0.192)+2\left(0.04\right)\left(0.192\right)+6\left(0.192\right)(0.192)$ $=0.273408$
6	(6, 6), (1, 6), (6, 1), (2, 6), (6, 2), (3, 6), (6, 3), (4, 6), (6, 4), (6, 5), (5, 6)	$\left(0.192\right)(0.192)+2\left(0.04\right)\left(0.192\right)+8\left(0.192\right)(0.192)$ $=0.347136$

$$\mathrm{E}(T)=\sum_{\text{all } t} t\mathrm{P}(T=t)=4.73248=4.73\ (3\text{ sf})$$

$$\mathrm{E}(T^2)=\sum_{\text{all } t} t^2P(T=t)=23.87104$$

$$\mathrm{Var}(T)=\mathrm{E}(T^2)-\left[\mathrm{E}\left(T\right)\right]^2=23.87104-(4.73248)^2=1.47467305=1.47\ (3\text{ sf})$$

7 i

X = Total score of the two dice = {2, 3, 4, 5, 6, 7, 8}

X	Possible Outcomes	Probability $\mathrm{P}(X=x)$
2	(1, 1)	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$
3	(1, 2), (2, 1)	$\frac{1}{4}\times\frac{1}{4}\times2=\frac{2}{16}=\frac{1}{8}$
4	(2, 2), (1, 3), (3, 1)	$\frac{1}{4}\times\frac{1}{4}\times3=\frac{3}{16}$
5	(2, 3), (3, 2), (1, 4), (4, 1)	$\frac{1}{4}\times\frac{1}{4}\times4=\frac{4}{16}=\frac{1}{4}$

6	(3, 3), (2, 4), (4, 2)	$\frac{1}{4}\times\frac{1}{4}\times3=\frac{3}{16}$
7	(3, 4), (4, 3)	$\frac{1}{4}\times\frac{1}{4}\times2=\frac{2}{16}=\frac{1}{8}$
8	(4, 4)	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

The probability distribution of X is

x	2	3	4	5	6	7	8
P(X = x)	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

$$\mathrm{E}(X)=2\times\frac{1}{16}+3\times\frac{1}{8}+4\times\frac{3}{16}+5\times\frac{1}{4}+6\times\frac{3}{16}+7\times\frac{1}{8}+8\times\frac{1}{16}=5$$

ii Let W = Gain of Harry in the particular game.

w	36	– 3
x	6	≠ 6
P(W = w)	$\frac{3}{16}$	$\frac{13}{16}$

In this game, E(W) = $36\times\frac{3}{16}-3\times\frac{13}{16}$ = $\$\frac{69}{16}$ or \$4.3125

8(i)	The probability that a salesman is successful in closing a deal with each customer is assumed to be constant.
(ii)	<u>Assumption: Deals closed are independent of one another</u> Deals closed may not be independent of one another as customers may collaborate to buy cars as a group for better bargaining power.

(iii)	$\mathrm{P}(C=30)=0.03014$ ${}^{60}C_{30}\times p^{30}\times(1-p)^{30}=0.03014$ $p^2-p+0.2399991029=0$ $\therefore\ p=0.6\ \text{ or } 0.3999955$ Since $p<0.5$, $p=0.4$ (1 d.p)

9(i)	• Whether a call made by an experienced telesales executive is successful or not is independent of any other calls. • The probability that a call made by an experienced telesales executive is successful is a constant at 0.15.
(ii)	Let T be the random variable denoting the number of successful calls, out of 15, made by Tom, an experienced telesales executive. $T\sim\mathrm{B}(15,\ 0.15)$ $\mathrm{P}(T>4)=1-\mathrm{P}(T\le 4)$ $=0.0617053867$ $=0.0617\ (\text{to 3 s.f.})$
(iii)	**<u>Method ①:</u> Define a new binomial random variable** Let D be the random variable denoting the number of days, out of 5, where an experienced telesales executive made more than 4 successful calls. $D\sim\mathrm{B}(5,\ 0.0617053867)$ $\mathrm{P}(D\le 2)=0.998\ (\text{to 3 s.f.})$ **<u>Method ②:</u> Consider cases** $\binom{n}{x}p^x(1-p)^{n-x}$ Required probability $={}^5\mathrm{C}_0\left[\mathrm{P}(T>4)\right]^0\left[1-\mathrm{P}(T>4)\right]^5$ $+{}^5\mathrm{C}_1\left[\mathrm{P}(T>4)\right]^1\left[1-\mathrm{P}(T>4)\right]^4$ $+{}^5\mathrm{C}_2\left[\mathrm{P}(T>4)\right]^2\left[1-\mathrm{P}(T>4)\right]^3$ $=(1-0.0617053867)^5$ $+5(0.0617053867)(1-0.0617053867)^4$ $+10(0.0617053867)^2(1-0.0617053867)^3$ $=0.998$ (3 s.f.)
(iv)	Let J be the random variable denoting the number of successful calls, out of 9, made by Jerry, a novice telesales executive.

	$J \sim \mathrm{B}(9,\ 0.09)$ $\mathrm{P}(\text{Tom \& Jerry made 2 sales each}) = \mathrm{P}(T=2)\cdot\mathrm{P}(J=2)$ $=(0.2856392285)(0.1506875132)$ $=0.0430\ (\text{to 3s.f.})$
(v)	$\mathrm{P}(\text{Tom \& Jerry made 3 sales in total} \mid \text{Jerry makes at least two sales})$ $=\dfrac{\mathrm{P}(\text{Tom \& Jerry made 3 sales in total and Jerry makes at least two sales})\cdot}{\mathrm{P}(\text{Jerry makes at least two sales})}$ $=\dfrac{\mathrm{P}(J=2)\cdot\mathrm{P}(T=1)+\mathrm{P}(J=3)\cdot\mathrm{P}(T=0)}{\mathrm{P}(J\geq 2)}$ $=\dfrac{\mathrm{P}(J=2)\cdot\mathrm{P}(T=1)+\mathrm{P}(J=3)\cdot\mathrm{P}(T=0)}{1-\mathrm{P}(J\leq 1)}$ $=\dfrac{(0.1506875132)(0.2312317564)+(0.0347740415)(0.0873542191)}{1-0.8088343476}$ $=0.198\ (\text{to 3s.f.})$
(vi)	Successful calls made by Tom, a novice telesales executive are independent of successful calls made by Jerry, an experienced telesales executive. OR All successful calls made are independent.

10(a)(i)	Let X denote the no of patients out of 20 who will not recover $X \sim B(20, 0.02)$ $P(X>2)=1-P(X\leq 2)=0.00707$
(ii)	Let $\overline{X}$ denote the mean no. of patients (from the 50 samples) who will not recover, i.e. $\overline{X} = \dfrac{X_1+X_2+...+X_{50}}{50}$ $\mathrm{E}(X)=20(0.02)=0.4$ $\mathrm{Var}(X)=20(0.02)(0.98)=0.392$ $\therefore\ \overline{X} \sim N(0.4, \dfrac{0.392}{50})$ by CLT, since n is large Hence, $P(\overline{X}>0.5)=0.129$
(b)(i)	Let Y denote the no of patients out of 50 who will recover $Y \sim B(50, p)$

	$P(Y > 48) = 0.64$ $P(Y \leq 48) = 0.36$ From GC, p = 0.97481 = 0.975 (to 3 sig fig)

11(i)	Let X and Y be the number of rectangular tables and round tables that are occupied. $X \sim \text{B}(6, 0.8)$ $\quad Y \sim \text{B}(9, 0.65)$ Required probability $= \text{P}(X = 4)\,\text{P}(Y = 7) = 0.24576 \times 0.21619 = 0.0531$ (3 s.f.)
(ii)	• Customers may arrive as a big group that requires them to be split into two separate tables next to each other. OR • The restaurant may choose to seat the customers at tables in a particular section first.

12(i)	The event of Ben losing a game is independent of other games played. OR Whether Ben loses a game is independent of other games played.
(ii)	Let X be the r.v. denoting the number of games that Ben loses out of 10 games. $X \sim \text{B}(10, 0.7)$ $\text{P}(X > 5) = 1 - \text{P}(X \leq 5) = 0.84973 \approx 0.850$ (3 sig fig)
(iii)	Let Y be the r.v. denoting the number of games that Ben loses out of n games. $Y \sim \text{B}(n, 0.3)$ $\text{P}(Y > 8) \leq 0.01$ $1 - \text{P}(Y \leq 8) \leq 0.01$ Using GC, When $n = 14$, $\text{P}(Y > 8) = 0.00829$ (< 0.01) When $n = 15$, $\text{P}(Y > 8) = 0.01524$ (> 0.01) $\therefore$ the greatest value of n is 14.

13(i)	P(first person that uses Voyager is the third person selected) $= 0.92 \times 0.92 \times 0.08 = 0.067712$
(ii)	1. Whether a person uses Voyager is independent of another person. 2.The probability that a person uses Voyager is constant for every person in the sample.
(iii)	Let V be the number of people who use Voyager out of n people.

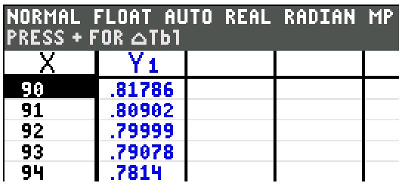
$V \sim \mathrm{B}(n, 0.08)$

$\mathrm{P}(V \geq 10) > 0.2$

$1 - \mathrm{P}(V \leq 9) > 0.2$

$\quad \mathrm{P}(V \leq 9) < 0.8$

Using GC,



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NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl
  X     Y1
  90    .81786
  91    .80902
  92    .79999
  93    .79078
  94    .7814
```

Least value of n = 92

14(i) The mode of payment made by a customer is independent of other customers.

(ii) $X \sim \mathrm{B}(10, p)$

$\mathrm{P}(X = 10) = 0.268$

$$\binom{10}{10} p^{10} (1-p)^{0} = 0.268$$

$p = 0.877$ (3sf) shown

(iii) $X \sim \mathrm{B}(10, 0.877)$

$\mathrm{P}(X = a) =$ Highest Probability

a	$\mathrm{P}(X = a)$
8	0.238
9	0.377
10	0.269

Hence most likely number is 9.

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<td>15</td>
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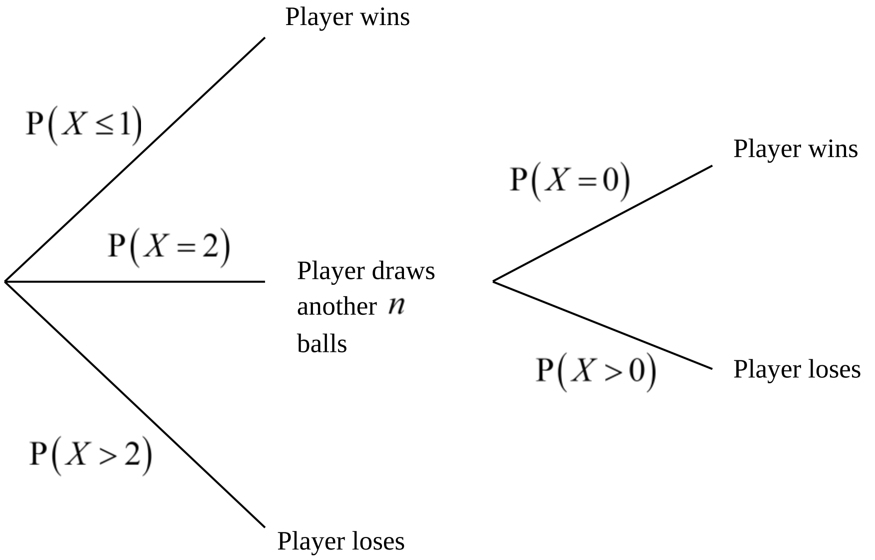
Let X be the number of red balls drawn, out of n balls.

$X \sim \mathrm{B}(n, p)$



$$P = \mathrm{P}(\text{player wins})$$
$$= \mathrm{P}(X \le 1) + \mathrm{P}(X = 2)\mathrm{P}(X = 0)$$
$$= \mathrm{P}(X = 0) + \mathrm{P}(X = 1) + \mathrm{P}(X = 2)\mathrm{P}(X = 0)$$
$$= \binom{n}{0}(0.2)^0(0.8)^n + \binom{n}{1}(0.2)^1(0.8)^{n-1} + \left[\binom{n}{2}(0.2)^2(0.8)^{n-2}\right]\left[\binom{n}{0}(0.2)^0(0.8)^n\right]$$
$$= (0.8)^n + n(0.2)(0.8)^{n-1} + \left[\binom{n}{2}(0.2)^2(0.8)^{n-2}\right](0.8)^n$$
$$= (0.8 + 0.2n)(0.8)^{n-1} + \binom{n}{2}(0.2)^2(0.8)^{2n-2}$$

(Shown)

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<tr>
<td>(i)</td>
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$$P = (0.8 + 0.2n)(0.8)^{n-1} + \binom{n}{2}(0.2)^2(0.8)^{2n-2} < 0.1$$

Using G.C.,

When $n = 18,\ P = 0.10218(> 0.1)$

When $n = 19,\ P = 0.08509(< 0.1)$

$\therefore$ least $n = 19$

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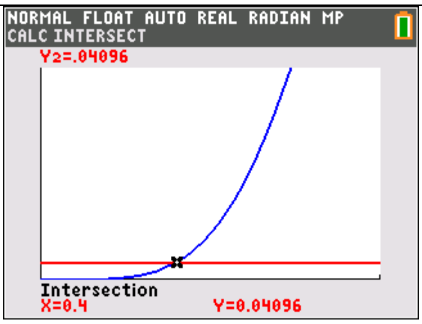
(ii)	Let Y be the number of games won, out of 100 games played. $Y \sim \mathrm{B}(100, 0.3)$. Required probability $= \mathrm{P}(Y \geq 40) = 1 - \mathrm{P}(\mathrm{Y} \leq 39) = 0.020988 \approx 0.0210$ (3 s.f.)

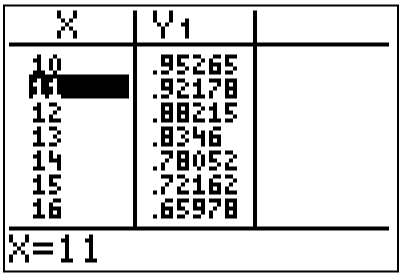
16(a)	Let $Y \sim \mathrm{B}(n, p)$ Given $np = 1.6$ -----(1) $(1-p)^n = 0.1296$ -----(2) From (1), $p = \frac{1.6}{n}$, sub into (2) $\left(1 - \frac{1.6}{n}\right)^n = 0.1296$ Using GC, $n = 4$ and $p = \frac{1.6}{4} = 0.4$ $\mathrm{P}(Y > 2) = 1 - \mathrm{P}(Y \leq 2) = 0.1792$
(b)	The probability of a parking lot being occupied is constant throughout the 80 parking lots. Whether a parking lot is occupied is independent of whether any other parking lot is occupied

17(i)	The probability of picking a brown egg from a box is constant. The colour of an egg is independent of other eggs.
(ii)	Let X be the r.v. "number of brown eggs in a box of 6 eggs" $X \sim \mathrm{B}(6, p)$ $P(X \geq 5) = 0.04096$ $P(X = 5) + P(X = 6) = 0.04096$ $\binom{6}{5} p^5 (1-p) + \binom{6}{6} p^6 (1-p)^0 = 0.04096$

	 Using GC, $p = 0.4$
(iii)	$Y \sim \mathrm{B}(100,\ 0.04096)$ $E(Y) = 100(0.04096) = 4.096$ $Var(Y) = 100(0.04096)(0.95904) = 3.9282$ In 8 weeks, there are 56 days altogether. Mean number of boxes with at least 5 brown eggs is $\overline{Y} = \dfrac{Y_1 + Y_2 + Y_3 + \cdots + Y_{56}}{56}$ Since sample size = 56 is large, by Central Limit Theorem, $\overline{Y} \sim \mathrm{N}\left(4.096, \dfrac{3.9282}{56}\right)$ approximately i.e. $\overline{Y} \sim \mathrm{N}(4.096, 0.0701469)$ $\mathrm{P}(4 < \overline{Y} < 7) = 0.641$ (correct to 3 sig fig)

18(i)	The number of SMSes that John receives in each 30 minute period is independent of the number of SMSes John received in other 30 minute periods. (i.e. independent trials) The probability that John receives at least one SMS in a 30 minute period remains constant for all such periods. (i.e. constant probability of success)
(ii)	The people who contact John will do so at their convenient time, which should not be constant throughout the day.
(iii)	$X \sim \mathrm{B}(14,\ 0.95)$ $P(X \leq 10) = 0.00417$

19(i)	Each of the students is equally likely to answer the Differential Equation question correctly (i.e. constant p throughout all trials) The event of a student answering the Differential Equation question correctly is independent of the other students.

(ii)	Let X be the random variable "no of students out of 30 students who could do the Differential Equation question" $X \sim B(30, 0.3)$ $\text{P}(X \geq 6) \ = 1 - \text{P}(X \leq 5) \ = 0.92341 \approx 0.923$
(iii)	Let S be the r.v. "no of students out of 8 who could do that question" Let T be the r.v "no of students out of 22 who could do that question" $S \sim B(8, 0.3)$ $T \sim B(22, 0.3)$ P(only 2 among first 8 could do that question| X $\geq$ 6) $= \dfrac{\text{P}(S = 2)\text{P}(T \geq 4)}{\text{P}(X \geq 6)} = 0.299$
(iv)	Let Y be the r.v. "no of students out of *n* who could do that question" $Y \sim B(n, 0.3)$ P(Y $\leq$ 5) >0.9 From G.C,  X Y1 10 .95265 11 .92178 12 .88215 13 .8346 14 .78052 15 .72162 16 .65978 X=11 Therefore, the largest possible value of *n* is 11.
	Since sample size = 50 is large, $\overline{X} \sim N(9, \dfrac{6.3}{50})$ approximately by Central Limit Theorem $\text{P}(\overline{X} \geq 10) = 0.00242$

20(i)	The event that a call is successfully connected is independent from the event of other calls being successfully connected.
(ii)	Let *X* be the random variable denoting the number of successful calls, out of a sample of 60 calls. $X \sim \text{B}(60, 0.92)$ $\text{P}(X \geq 50 \mid X \leq 55) = \dfrac{\text{P}(50 \leq X \leq 55)}{\text{P}(X \leq 55)} = \dfrac{\text{P}(X \leq 55) - \text{P}(X \leq 49)}{\text{P}(X \leq 55)} = 0.986$
(iii)	$X \sim \text{B}(60, 0.92)$ Since *n* = 70 is large, by Central Limit Theorem,

$\bar{X} \sim \mathrm{N}\left(55.2, \dfrac{4.416}{70}\right)$ approx

$\mathrm{P}\left(\bar{X} \le 55\right) = 0.213\left(\text{to 3 s.f}\right)$

21(i)

Let U and S be the number of subscribers who purchased a uPhone and a Samseng phone respectively in a sample of size n

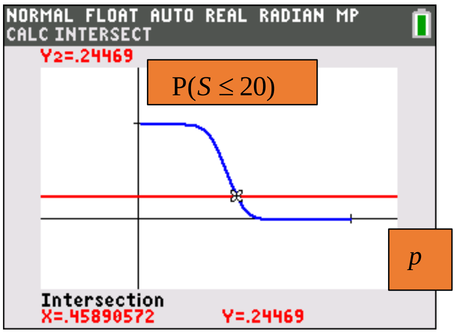
$U \sim \mathrm{B}(n, 0.3)$ $S \sim \mathrm{B}(n, p)$

When $n = 50$,

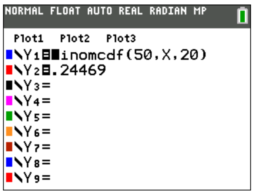
$\mathrm{P}(S \le 20) = 2\mathrm{P}(U = 15)$

$\mathrm{P}(S \le 20) = 0.24469$

From GC,

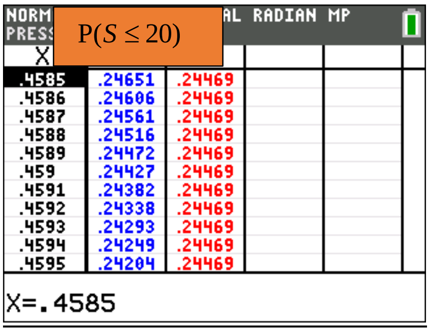


```
NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTERSECT
Y2=.24469
Intersection
X=.45890572        Y=.24469
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
  Plot1  Plot2  Plot3
\Y1=█inomcdf(50,X,20)
\Y2=.24469
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=
\Y9=
```

or



```
NORM               AL RADIAN MP
PRESS
  X
.4585  .24651  .24469
.4586  .24606  .24469
.4587  .24561  .24469
.4588  .24516  .24469
.4589  .24472  .24469
.459   .24427  .24469
.4591  .24382  .24469
.4592  .24338  .24469
.4593  .24293  .24469
.4594  .24249  .24469
.4595  .24204  .24469
X=.4585
```

$p = 0.459$ (3 sig fig)

(ii)

Given $p = 0.4$, $n = 50$, $S \sim \mathrm{B}(50, 0.4)$

$\mathrm{E}(S) = 50\ (0.4) = 20$

$\mathrm{P}(U > 20) = 1 - \mathrm{P}(U \le 20) = 0.0478$ (3 sig fig)

22(i) $p > 0.6$

	$\Rightarrow \frac{2}{65}\left(40-d\right)>0.6$ $\Rightarrow 40-d>19.5$ $\Rightarrow d<20.5$ Maximum distance of d = 20 metres.
(ii)	Let W denote the number of kicks that hit the net $W \sim B(15, p)$ $P\left(W \geq 2\right)=0.9$ $1-P\left(W \leq 1\right)=0.9$ $P\left(W \leq 1\right)=0.1$ $\left(1-p\right)^{15}+15p\left(1-p\right)^{14}=0.1$ $\left(1-p\right)^{14}\left(1+14p\right)-0.1=0$ Using GC, p = 0.23557 $0.23557=\frac{2}{65}\left(40-x\right)$ $\Rightarrow x=32.343975$ $\Rightarrow x=32\ metres\left(nearest\ metres\right)$
	Let S be the number of kicks that hit into the net out of 100 kicks at a distance of 24 metres from the goalpost. $S \sim B\left(100, 0.4923077\right)$ $E(S)=100\times 0.4923077=49.231$ $Var(S)=100\times 0.4923077\times 0.5076923=24.9941$ Since sample size is large (60 days), by CLT, $\bar{S} \sim N(49.231, \frac{24.9941}{60})$ Required prob = $P\left(\left\|\bar{S}-50\right\|<1\right)=P\left(49<\bar{S}<51\right)=0.637(3sf)$

23(i)	$X \sim \mathrm{B}(20,\ 0.15)$ $\mathrm{P}(X \geq 20\ \times\ 0.15)=\mathrm{P}(X \geq 3)=1-\mathrm{P}(X \leq 2)=0.595$ (3 s.f)

(ii)

$$\mathrm{P}(X > n) < 0.1$$

$$1 - \mathrm{P}(X \le n) < 0.1$$

$$\mathrm{P}(X \le n) > 0.9$$

From GC,

n	$\mathrm{P}(X \le n)$
4	0.82985 < 0.9
5	0.93269 > 0.9

$\therefore$ Least integer n = 5.

24(i) Assume that the event of obtaining a rotten cherry is independent of another. Assume that the probability of obtaining a rotten cherry is constant.

(ii) Let X be the random variable denoting the number of rotten cherries out of 26.

$X \sim \mathrm{B}(26, 0.08)$

X	$\mathrm{P}(X = x)$
1	0.25868
2	0.28117
3	0.19560

The most likely number of rotten cherries is 2.

(iii) Let Y be the random variable denoting the number of rotten cherries out of n.

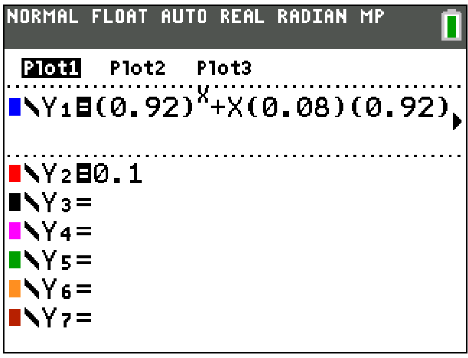
$Y \sim \mathrm{B}(n, 0.08)$

$$\mathrm{P}(Y=0)+\mathrm{P}(Y=1)<0.1$$

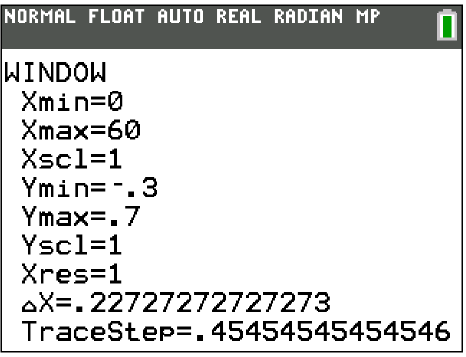
$$^{n}\mathrm{C}_0\,(0.08)^0\,(1-0.08)^n + {}^{n}\mathrm{C}_1\,(0.08)^1\,(1-0.08)^{n-1} < 0.1$$

$$(0.92)^n + n(0.08)(0.92)^{n-1} < 0.1$$

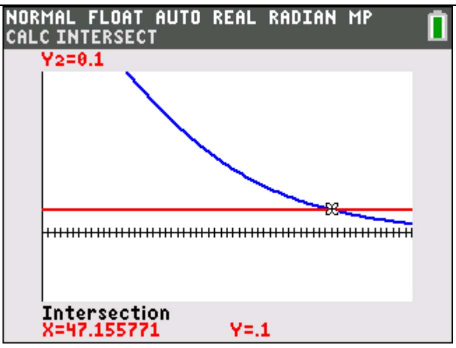
Method ①:



```
NORMAL FLOAT AUTO REAL RADIAN MP
Plot1  Plot2  Plot3
\Y1=(0.92)^X+X(0.08)(0.92)▸
\Y2=0.1
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW
 Xmin=0
 Xmax=60
 Xscl=1
 Ymin=-.3
 Ymax=.7
 Yscl=1
 Xres=1
 ΔX=.22727272727273
 TraceStep=.45454545454546
```

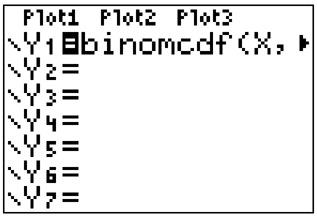


```
NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTERSECT
Y2=0.1
Intersection
X=47.155771     Y=.1
```

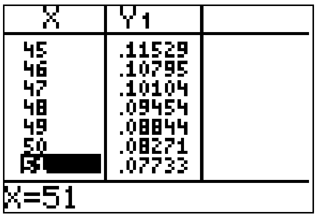
The least number of n is 48.

Method ②:

```
binomcdf
 trials:X
 p:.08
 x value:1
 Paste
```



```
Plot1 Plot2 Plot3
\Y1=binomcdf(X,
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



X	Y1
45	.11529
46	.10795
47	.10104
48	.09454
49	.08844
50	.08271
51	.07733

X=51

n	$\mathrm{P}(Y \le 1)$
47	0.10104 (> 0.1)
48	0.09454 (< 0.1)
49	0.08844 (< 0.1)

The least number of n is 48.

25(i)

Mean score = $\frac{1}{5}+\frac{2}{6}+\frac{3}{7}+\frac{4}{8}+5p+6q=3.5 \Rightarrow 5p+6q=\frac{214}{105}$ … (1)

$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+p+q=1 \Rightarrow p+q=\frac{307}{840}$ … (2)

From graphing calculator, $p = \frac{13}{84}$ and $q = \frac{59}{280}$.

(ii)

Let X be the score of the die and Y be the outcome of coin.

For w = 0, 1,

$\mathrm{P}(W = w) = \mathrm{P}(X = w+1, Y = \text{tail}) = \frac{1}{2}\mathrm{P}(X = w+1)$.

<table>
<tr>
<td></td>
<td>
For $w = 2, 3, 4, 5$,
$\mathrm{P}(W = w) = \mathrm{P}(X = w+1, Y = \text{tail}) + \mathrm{P}(X = w-1, Y = \text{head})$
$= \frac{1}{2}\left[\mathrm{P}(X = w-1) + \mathrm{P}(X = w+1)\right]$
For $w = 6, 7$,
$\mathrm{P}(W = w) = \mathrm{P}(X = w-1, Y = \text{head}) = \frac{1}{2}\mathrm{P}(X = w-1)$.
The probability distribution of W is given by:
<table>
<tr><td>w</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$\mathrm{P}(W = w)$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{12}$</td><td>$\frac{6}{35}$</td><td>$\frac{7}{48}$</td><td>$\frac{25}{168}$</td><td>$\frac{47}{280}$</td><td>$\frac{13}{168}$</td><td>$\frac{59}{560}$</td></tr>
</table>
</td>
</tr>
<tr>
<td>(iii)</td>
<td>Since the mean score of the die is 3.5 and the coin is fair, the expected winnings is \$3.5, hence I would not play this game since amount pay = \$4 > \$3.5 = expected winnings.</td>
</tr>
</table>

26(i)

Let X be the amount of winnings Sally gets after one game in dollars.

Let the random variables F be the scores on the fair dice.

$$\begin{aligned}&\mathrm{P}(X=-5)\\&=\mathrm{P}\left(\left|F-Y\right|>3\right)\\&=\mathrm{P}\left[F=5,Y=1\right]+\mathrm{P}\left[F=6,Y=1\right]+\mathrm{P}\left[F=6,Y=2\right]+\mathrm{P}\left[F=1,Y=5\right]\\&+\mathrm{P}\left[F=1,Y=6\right]+\mathrm{P}\left[F=2,Y=6\right]\\&=\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{18}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{5}{18}\right)+\left(\frac{1}{6}\times\frac{5}{18}\right)\\&=\frac{5}{27}\end{aligned}$$

$$\begin{aligned}&\mathrm{P}(X=0)\\&=\mathrm{P}\left[F=1,Y=1\right]+\mathrm{P}\left[F=2,Y=2\right]+\mathrm{P}\left[F=3,Y=3\right]+\mathrm{P}\left[F=4,Y=4\right]\\&+\mathrm{P}\left[F=5,Y=5\right]+\mathrm{P}\left[F=6,Y=6\right]\\&=\frac{1}{6}\left(\frac{1}{6}+\frac{1}{18}+\frac{1}{6}+\frac{3}{18}+\frac{1}{6}+\frac{5}{18}\right)\\&=\frac{1}{6}\end{aligned}$$

$$\begin{aligned}\mathrm{P}(X=3)&=1-\mathrm{P}(X=0)-\mathrm{P}(X=-5)\\&=1-\frac{5}{27}-\frac{1}{6}=\frac{35}{54}\end{aligned}$$

x	-5	0	3
$\mathrm{P}(X=x)$	$\frac{5}{27}$	$\frac{1}{6}$	$\frac{35}{54}$

$$\mathrm{E}(X)=\sum_{\text{all }x}x\mathrm{P}(X=x)=-5\left(\frac{5}{27}\right)+0\left(\frac{1}{6}\right)+(3)\left(\frac{35}{54}\right)=\frac{55}{54}$$

(ii)

$$\mathrm{E}(X^2)=\sum_{all\ x}x^2\mathrm{P}(X=x)=(-5)^2\left(\frac{5}{27}\right)+(0)^2\left(\frac{1}{6}\right)+(3)^2\left(\frac{35}{54}\right)=\frac{565}{54}$$

$$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left[\mathrm{E}(X)\right]^2=\frac{565}{54}-\left(\frac{55}{54}\right)^2=\frac{27485}{2916}$$

Since the number of games of 50 is sufficiently large, by Central Limit Theorem,

	$X_1+X_2+\cdots+X_{50} \sim \mathrm{N}\left(50\left(\frac{55}{54}\right),50\left(\frac{27485}{2916}\right)\right)$ approximately $\mathrm{P}\left(X_1+X_2+\cdots+X_{50} \ge 65\right)$ $=0.25839=0.258$ (to 3 sf)

27 (i)	Let *W* denote the number of shots that hits the bullseye 40m away from the target out of 18 shots. $W \sim B(18,\frac{2}{195}(95-40))$ $W \sim B(18,\frac{22}{39})$ $\mathrm{P}\left(W>6 \mid W \le 10\right)=\frac{\mathrm{P}\left([W>6]\cap[W\le 10]\right)}{\mathrm{P}(W\le 10)}$ $=\frac{\mathrm{P}\left(7\le W\le 10\right)}{\mathrm{P}(W\le 10)}$ $=\frac{\mathrm{P}\left(W\le 10\right)-\mathrm{P}\left(W\le 6\right)}{\mathrm{P}(W\le 10)}$ $=0.926$ (to 3 s.f.)
(ii)	Let *Y* denote the number of shots that hits the bullseye *x* m away from the target out of 18 shots. $Y \sim B(18,\frac{2}{195}(95-x))$ $Y \sim B(18,\left(\frac{190}{195}-\frac{2}{195}x\right))$ $\mathrm{P}\left(Y\ge 2\right)=0.98$ $1-\mathrm{P}\left(Y=0\right)-\mathrm{P}\left(Y=1\right)=0.98$ $\mathrm{P}\left(Y=0\right)+\mathrm{P}\left(Y=1\right)=0.02$ $\left[1-\left(\frac{190}{195}-\frac{2}{195}x\right)\right]^{18}+\left\{{}^{18}C_1\times\left(\frac{190}{195}-\frac{2}{195}x\right)^1\times\left[1-\left(\frac{190}{195}-\frac{2}{195}x\right)\right]^{17}\right\}=0.02$ $\left(\frac{1}{39}+\frac{2}{195}x\right)^{18}+\left[{}^{18}C_1\times\left(\frac{190}{195}-\frac{2}{195}x\right)^1\times\left(\frac{1}{39}+\frac{2}{195}x\right)^{17}\right]=0.02$ Using the GC, *x* = 67.3 m (to 3 s.f.)

28(i)	$\bar{x}=2$
(ii)	The probability that a frog carries the genetic trait is constant for all frogs **in a box.** The presence of the genetic trait in a frog is independent among the frogs **in a box**.
(iii)	$X \sim \mathrm{B}\left(6,\frac{1}{3}\right)$ $\mathrm{P}\left(X=2\right)=0.32922$ Let Y be the number of boxes that contain exactly 2 frogs with the genetic trait. $Y \sim \mathrm{B}\left(10,0.32922\right)$ $\mathrm{P}\left(Y>3\right)=1-\mathrm{P}\left(Y\le 2\right)=0.691$

29 (a)(i)	Let X be the number of packets with a toy, in a carton of 12. $X \sim \mathrm{B}\left(12,\frac{2}{7}\right)$ $\mathrm{P}(X<2)=\mathrm{P}(X\le 1)$ =0.10230 =0.102 (3s.f)
(ii)	$\mathrm{P}(1<X\le 6)=\mathrm{P}(X\le 6)-\mathrm{P}(X\le 1)$ =0.868
(b)	Required probability = $\left(\mathrm{P}(X=4)\right)^2\times(\mathrm{P}(X=2))^3\times\frac{5!}{2!3!}$ = 0.00323 (3s.f.)
(c)	Let Y be the number of cartons out of 40 cartons that contains less than 2 toys. $Y \sim \mathrm{B}\left(40,0.1023\right)$ P(Y=0) = 0.0133 **Alternative method** Required probability = $\left(1-0.10230\right)^{40}=0.0133$
	P(A = 2 or 3) = 0.25 $\binom{25}{2}p^2(1-p)^{23}+\binom{25}{3}p^3(1-p)^{22}$=0.25 300 $p^2(1-p)^{23}$+2300 $p^3(1-p)^{22}$=0.25 From GC, p = 0.186 or p = 0.0403 (reject since p >0.1)

30. Suggested solution

(i)

Let *X* be the number randomly generated by the computer chip.

$$\mathrm{E}(X) = p + \frac{2(1-p)}{3} + \frac{3(1-p)}{3} + \frac{4(1-p)}{3} = 3 - 2p$$

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$$

$$= (1^2)p + (2^2 + 3^2 + 4^2)\frac{(1-p)}{3} - (3-2p)^2$$

$$= \frac{2}{3} + \frac{10}{3}p - 4p^2 = \frac{2}{3}(1-p)(1+6p)$$

(ii)

Consider (X_1, X_2) **as the values for the r.v.** X_1 and X_2

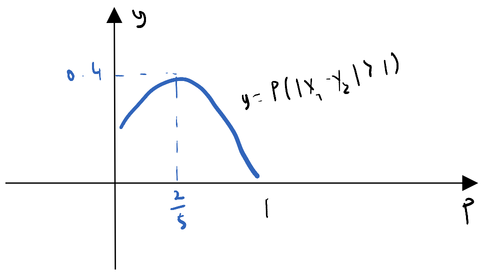
$$P(|X_1 - X_2| > 1) = P((1,3),(3,1),(1,4),(4,1),(2,4),(4,2))$$

$$= p \times \frac{(1-p)}{3} \times 2 + p \times \frac{(1-p)}{3} \times 2 + \left[\frac{(1-p)}{3}\right]^2 \times 2$$

$$= \frac{4}{3}p(1-p) + \frac{2}{9}(1-p)^2$$

$$= \frac{2}{9}(1-p)[6p + (1-p)]$$

$$= \frac{2}{9}(1-p)(5p+1)$$



Max value of $P(|X_1 - X_2| > 1)$ **occurs when** $p = \frac{-\frac{1}{5}+1}{2} = \frac{2}{5}$

Max value of $P(|X_1 - X_2| > 1) = \frac{2}{9}\left(1 - \frac{2}{5}\right)(2+1) = 0.4$

(iii)

Required probability = $P(X_1 + X_2 + \cdots + X_{50} > 110)$

By Central Limit Theorem, since *n*=50 is large

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_{50}}{50} \sim N\left(2, \frac{4/3}{50}\right) \text{approximately}$$

Hence, $P\left(X_1+X_2+\cdots+X_{50}>110\right)=P\left(\overline{X}>2.2\right)=0.11034\approx 0.110$ **Alternative solution:** **By Central Limit Theorem, since *n*=50 is large** $\Rightarrow 50\overline{X}=X_1+X_2+\cdots+X_{50}\sim N\left(100,\frac{200}{3}\right)$approximately **Hence,** $P\left(X_1+X_2+\cdots+X_{50}>110\right)$ $=0.11034\approx 0.110$	

31. Solution

(i)

The number of customers who do not turn up for the 9 am ferry, on a randomly chosen day, is denoted by *X*.

$X\sim \mathrm{B}\left(113,\frac{p}{100}\right)$

$\mathrm{P}\left(X\le 1\right)=0.012$

$\mathrm{P}\left(X=0\right)+\mathrm{P}\left(X=1\right)=0.012$

$$\left(1-\frac{p}{100}\right)^{113}+113\left(\frac{p}{100}\right)\left(1-\frac{p}{100}\right)^{112}=0.012$$

Using GC, $p=5.554$ (3 decimal place)

(ii) Let *Y* be the random variable that denotes the number of customers who purchased a ticket and turn up for the 9 am ferry.

$Y\sim \mathrm{B}\left(113,0.94\right)$

$\mathrm{P}(Y\ge 108\,|\,Y\le 108)$

$=\frac{\mathrm{P}(Y=108)}{\mathrm{P}(Y\le 108)}$

$=\frac{0.136717}{0.814523}$

= 0.168

(iii) Let *W* be the random variable that denotes the number of days out of 7 days where every customer who turns up get a seat on the 9 am ferry.

$$W\sim \mathrm{B}\left(7,0.814523\right)$$

$$\mathrm{P}\left(W\ge 5\right)=1-\mathrm{P}\left(W\le 4\right)=0.876$$

(iv) Let *T* denote the number of customers who purchased a ticket and turn up for the 9am ferry out of *n* passengers.

$T \sim \text{B}(n\ , 0.94)$

$\text{P}(T > 108) \le 0.01$

$\text{P}(T \le 108) \ge 0.99$

Using GC,

When $n = 109$, $\text{P}(T \le 108) = 0.99882 > 0.99$

When $n = 110$, $\text{P}(T \le 108) = 0.99112 > 0.99$

When $n = 111$, $\text{P}(T \le 108) = 0.96571 < 0.99$

maximum $n = 110$ *awarded only if method is clearly shown*

(v) Since $n = 40$ is large, by Central Limit Theorem,

$\bar{Y} \sim \text{N}\left(106.22, 0.15933\right)$ approximately.

$\text{P}\left(\bar{Y} \le 106\right) = 0.291$

32. Solution

$$\text{P}\left(X = 0\right) = \frac{1}{2} + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)^3\left(\frac{1}{2}\right) + \ldots$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{3}}$$

$$= \frac{3}{4} \quad \text{(shown)}$$

$$\text{P}\left(X = 1 \,|\, \text{did not lose}\right) = \frac{\text{P}\left(\text{first roll is '6'}\right)}{\text{P}\left(\text{did not lose}\right)}$$

$$= \frac{\frac{1}{6}}{1 - \frac{3}{4}}$$

$$= \frac{2}{3}$$

X, is the number of tosses that it takes for the player to toss the first ‘6’, and is 0 if the player loses the game

x	0	1	2	3	…
P(X= x)	$\frac{3}{4}$	$\frac{1}{6}$	$\left(\frac{2}{6}\right)\frac{1}{6}$	$\left(\frac{2}{6}\right)^2\frac{1}{6}$	

P(X = 1) = P(6 on first toss)

P(X = 2) = P(2 or 4 on first toss, 6 on second toss)

P(X = 3) = P(2 or 4 on first and second toss, 6 on third toss)

$$\mathrm{E}(X)=0+1\left(\frac{1}{6}\right)+2\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)+3\left(\frac{1}{3}\right)^2\left(\frac{1}{6}\right)+\ldots$$

$$=\sum_{x=1}^{\infty}\left[x\left(\frac{1}{3}\right)^{x-1}\frac{1}{6}\right]$$

$$=\frac{1}{6}\sum_{x=1}^{\infty}x\left(\frac{1}{3}\right)^{x-1}$$

$$\mathrm{E}(X)=\frac{1}{6}\times\frac{1}{\left(1-\frac{1}{3}\right)^2}=\frac{3}{8}$$

$$\mathrm{E}\left(X^2\right)=0+1^2\left(\frac{1}{6}\right)+2^2\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)+3^2\left(\frac{1}{3}\right)^2\left(\frac{1}{6}\right)+..$$

$$=\sum_{x=1}^{\infty}\left[x^2\left(\frac{1}{3}\right)^{x-1}\frac{1}{6}\right]$$

$$=\frac{1}{6}\times\frac{1+\frac{1}{3}}{\left(1-\frac{1}{3}\right)^3}=\frac{3}{4}$$

$$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left(\mathrm{E}(X)\right)^2=\frac{3}{4}-\left(\frac{3}{8}\right)^2=\frac{39}{64}$$

33.	Suggested solution	
(i)	Each mask has the same probability of being faulty of 0.06. The event that a mask is faulty is independent of any other mask.	
(ii)	Let X be the number of faulty masks in a box of 25. $X\sim \mathrm{B}(25,0.06)$ Probability $=P(X\geq 3)$ $=1-P(X\leq 2)=0.18711\quad(5\text{ s.f.})=0.187\quad(3\text{ s.f.})$	
(iii)	Probability $=\mathrm{P}(X\leq 5 \mid X\geq 3)$ $=\dfrac{\mathrm{P}(X\leq 5\ \cap\ X\geq 3)}{\mathrm{P}(X\geq 3)}=\dfrac{\mathrm{P}(3\leq X\leq 5)}{\mathrm{P}(X\geq 3)}$ $=\dfrac{\mathrm{P}(X\leq 5)-\mathrm{P}(X\leq 2)}{\mathrm{P}(X\geq 3)}=\dfrac{0.99694-0.81289}{0.18711}$ $=0.98359\quad(5\text{ s.f.})=0.984\quad(3\text{ s.f.})$	
(iv)	Let Y be the number of boxes out of 100 which have at least 3 faulty masks. $Y\sim \mathrm{B}(100,0.18711)$ Probability $=\mathrm{P}(Y\leq 20)$ $=0.68423\quad(5\text{ s.f.})=0.684\quad(3\text{ s.f.})$	

<table>
<tr><td rowspan="1">(v)</td><td colspan="2">$F \sim \mathrm{B}(n, 0.1)$
Given that mode is 4 (i.e. $P(F=4)$ has the highest probability)
$\mathrm{P}(F=3) < \mathrm{P}(F=4)$ and $\mathrm{P}(F=4) > \mathrm{P}(F=5)$</td></tr>
<tr><td></td><td>$\binom{n}{3}(0.1)^3(0.9)^{n-3} < \binom{n}{4}(0.1)^4(0.9)^{n-4}$
$\frac{n!}{3!(n-3)!}(0.1)^3(0.9)^{n-3} < \frac{n!}{4!(n-4)!}(0.1)^4(0.9)^{n-4}$
$\frac{4!}{3!}\frac{(0.9)^{n-3}}{(0.9)^{n-4}} < \frac{(n-3)!}{(n-4)!}\frac{(0.1)^4}{(0.1)^3}$
$4(0.9) < (n-3)(0.1)$
$n > 39$</td><td>$\binom{n}{4}(0.1)^4(0.9)^{n-4} > \binom{n}{5}(0.1)^5(0.9)^{n-5}$
$\frac{n!}{4!(n-4)!}(0.1)^4(0.9)^{n-4} > \frac{n!}{5!(n-5)!}(0.1)^5(0.9)^{n-5}$
$\frac{5!}{4!}\frac{(0.9)^{n-4}}{(0.9)^{n-5}} < \frac{(n-4)!}{(n-5)!}\frac{(0.1)^5}{(0.1)^4}$
$5(0.9) > (n-4)(0.1)$
$n < 49$</td></tr>
<tr><td></td><td colspan="2">$\therefore 39 < n < 49$
Since n is an integer, $\Rightarrow n = 40, 41, 42, 43, 44, 45, 46, 47$ or 48</td></tr>
</table>

Qn	**Suggested Solution**
34(a)	$\sum_{r=1}^{\infty} \mathrm{P}(X=r) = 1$ $\sum_{r=1}^{\infty} \frac{a}{r^3} = 1$ $a = \frac{1}{1.2021} = 0.83188 = 0.832$ (3 sf)
(b)	$\mathrm{E}(X) = \sum_{r=1}^{\infty} r\ \mathrm{P}(X=r) = a\sum_{r=1}^{\infty}\frac{1}{r^2} = 1.37$ (3 s.f) $\mathrm{E}(X^2) = \sum_{r=1}^{\infty} r^2\ \mathrm{P}(X=r) = a\sum_{r=1}^{\infty}\frac{1}{r}$ does not exist. Therefore Var(*X*) cannot be calculated.
(c)	Method 1 $\mathrm{P}(X \geq 2 \mid X \leq 15)$ $= 1 - \mathrm{P}(X = 1 \mid X \leq 15)$ $= 1 - \frac{\mathrm{P}(X=1)}{\mathrm{P}(X \leq 15)}$ $= 1 - \frac{a}{\sum_{r=1}^{15}\frac{a}{r^3}}$ $= 0.16665 = 0.167$ (3 s.f) Method 2 $\mathrm{P}(X \geq 2 \mid X \leq 15)$ $= \frac{\mathrm{P}(2 \leq X \leq 15)}{\mathrm{P}(X \leq 15)}$ $= \frac{\sum_{r=2}^{15}\frac{a}{r^3}}{\sum_{r=1}^{15}\frac{a}{r^3}}$ $= 0.16665 = 0.167$ (3 s.f)
(d)	$Y \sim \mathrm{B}(10, \mathrm{P}(X=3))$ where $\mathrm{P}(X=3) = \frac{a}{27} = \frac{0.83188}{27} = 0.030810$ $\mathrm{P}(Y > 2) = 1 - \mathrm{P}(Y \leq 2) = 0.00298$

(e)

Note: X_1 and Y are dependent variables.

Case 1: $X_1 = 1$ and $Y = 2$

The first number must be a '1' and the rest of 9 numbers must have two '3's.

$$\mathrm{P}(X=1)\left[\binom{9}{2}(\mathrm{P}(X=3))^2(1-\mathrm{P}(X=3))^7\right]=0.022835$$

Case 2: $X_1 = 2$ and $Y = 1$

The first number must be a '2' and the rest of 9 numbers must have one '3'.

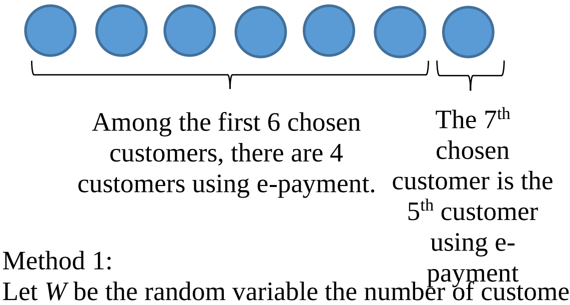
$$\mathrm{P}(X=2)\left[\binom{9}{1}(\mathrm{P}(X=3))(1-\mathrm{P}(X=3))^8\right]=0.022448$$

Note: $X_1 = 3$ and $Y = 0$

This case is impossible as Y is counting the number of '3' generated, probability is 0 for this case.

$$\therefore \mathrm{P}(X_1+Y=3)=0.022835+0.022448+0=0.0453 \text{ (3 sf)}$$

35
(a)
(i)



Method 1:

Let W be the random variable the number of customers out of first 6 chosen customers who uses e-payment at a hawker stall.

$$W \sim \mathrm{B}(6, 0.25)$$

$$\begin{aligned}\text{Required Probability} &= \mathrm{P}(W=4)\times 0.25\\ &= 0.00824 \text{ (3.s.f)}\end{aligned}$$

Method 2:

$$\begin{aligned}\text{Required Probability} &= (0.25)^4(0.75)^2\left(\frac{6!}{4!2!}\right)\times 0.25\\ &= 0.00824 \text{ (3.s.f)}\end{aligned}$$

(a)
(ii)

Let C be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall.

$$C \sim \mathrm{B}(40, 0.25)$$

$$\mathrm{P}(C \le 10) = 0.583904$$

Let A be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall.

$$A \sim \mathrm{B}(30, 0.583904)$$

	$\mathrm{P}(A\geq 15)=1-\mathrm{P}(A\leq 14)$ $=1-0.1322406$ $=0.867759$ $=0.868$ (3s.f)
(b) **(i)**	A random sample in this context means that each hawker has an equal chance of being selected for the sample and the selection of one hawker is independent of any other hawkers.
(b) **(ii)**	$X\sim \mathrm{B}(n,p)$ Since $\mathrm{E}(X)=3.96$, $\therefore np=3.96$ --------------(1) Since $\mathrm{P}(X\leq 1)=0.05303$, $\mathrm{P}(X=0)+\mathrm{P}(X=1)=0.05303$ ${}^{n}C_{0}p^{0}(1-p)^{n}+{}^{n}C_{1}p^{1}(1-p)^{n-1}=0.05303$ $(1-p)^{n}+np(1-p)^{n-1}=0.05303$ --------------(2) Sub. (1) into (2): $(1-p)^{3.96/p}+(3.96)(1-p)^{3.96/p-1}=0.05303$ Using GC, $p=0.3600375=0.360$ (3s.f) $\therefore n=\dfrac{3.96}{0.3600375}=11$ (nearest integer)
(b) **(iii)**	$X\sim \mathrm{B}(15,p)$ Since mode $=5\Rightarrow$ $\mathrm{P}(X=4)<\mathrm{P}(X=5)$ and $\mathrm{P}(X=5)>\mathrm{P}(X=6)$ Considering $\mathrm{P}(X=4)<\mathrm{P}(X=5)$, ${}^{15}C_{4}p^{4}(1-p)^{11}<{}^{15}C_{5}p^{5}(1-p)^{10}$ $(1365)(p^{4})(1-p)^{11}<(3003)(p^{5})(1-p)^{10}$ $(p^{4})(1-p)^{10}\left[1365(1-p)-3003p\right]<0$ Since $(1-p)>0$ and $p>0$, $1365(1-p)-3003p<0$ $1-p-2.2p<0$ $p>\dfrac{5}{16}$ Considering $\mathrm{P}(X=5)>\mathrm{P}(X=6)$, ${}^{15}C_{5}p^{5}(1-p)^{10}>{}^{15}C_{6}p^{6}(1-p)^{9}$ $(3003)(p^{5})(1-p)^{10}>(5005)(p^{6})(1-p)^{9}$ $(p^{5})(1-p)^{9}\left[3003(1-p)-5005p\right]>0$

Since $(1-p)>0$ and $p>0$,

$3003(1-p)-5005p>0$

$1-p-\frac{5}{3}p>0$

$p<\frac{3}{8}$

Combining both results,

$\therefore \frac{5}{16}<p<\frac{3}{8}$