Xinmin Secondary School
Additional Mathematics Practice
Topic: Logarithm

Name: _____ ( ) Date: _____

Question 1

Solve $2\log_2(1-x)-\log_2 x=3+\log_2 2x$.

Question 2

Solve $\log_5 50+4\log_{25} x=\log_5(2x+4)+2$.

Question 3

(a) Given that $\log_2 x=p$ and $\log_4 y=q$, express the following in terms of p and/or q.

(i) $\log_2 \sqrt{x}$,

(ii) $\log_2 xy^2$,

(iii) $\log_4 \dfrac{4x}{y}$.

(b) Solve the equation $\lg x+\lg[5(x+1)]-2=0$.

Question 4

Solve the equation

(a) $\log_2 x+\dfrac{2}{\log_2 x}=3$.

(b) $\log_3(x+1)-\log_{\sqrt{3}}(x-1)=1$.

Question 5

Solve the equation $\log_3(x+2)=3-\log_3(x-4)$.

Question 6

Given that $\log_4 x+\log_2 y=\log_2(x+y)$, express y in terms of x.

Question 7

Solve the equation $\log_3(2x-1)-\dfrac{1}{2}\log_3(x^2+2)=\log_{25}5$.

Solutions

Question 1

$$2\log_2(1-x)-\log_2 x=3+\log_2 2x$$
$$\log_2(1-x)^2-\log_2 x=\log_2 2^3+\log_2 2x$$
$$\log_2\frac{(1-x)^2}{x}=\log_2 8(2x)$$
$$\frac{(1-x)^2}{x}=16x$$
$$x^2-2x+1=16x^2$$
$$15x^2+2x-1=0$$
$$(5x-1)(3x+1)=0$$
$$x=\frac{1}{5}\ \text{or}\ -\frac{1}{3}\ \ \text{(reject)}$$

Question 2

$$\log_5 50+4\log_{25} x=\log_5(2x+4)+2$$
$$\log_5 50+4\frac{\log_5 x}{\log_5 25}=\log_5(2x+4)+\log_5 5^2$$
$$\log_5 50+2\log_5 x=\log_5 25(2x+4)$$
$$\log_5 50x^2=\log_5 25(2x+4)$$
$$50x^2=50x+100$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2\ \text{or}\ -1\ \ \text{(reject)}$$

Question 3

(a) (i)

$$\log_2\sqrt{x}=\log_2 x^{\frac{1}{2}}$$
$$=\frac{1}{2}\log_2 x$$
$$=\frac{1}{2}p\ \text{or}\ 0.5p$$

Alternative Method,

$$\log_2 \sqrt{x} = \log_2 x^{\frac{1}{2}}$$
$$= \frac{1}{2}\log_2 2^p$$
$$= \frac{1}{2}p$$

(ii)

$$\log_4 y = \frac{\log_2 y}{\log_2 4}$$
$$= \frac{\log_2 y}{2}$$

$$\frac{1}{2}\log_2 y = q$$
$$\log_2 y = 2q$$

$$\log_2 xy^2 = \log_2 x + 2\log_2 y$$
$$= p + 2(2q)$$
$$= p + 4q$$

Alternative method,

$$y = 4^q$$
$$= 2^{2q}$$

$$\log_2 xy^2 = \log_2 x + \log_2 y^2$$
$$= p + 2(\log_2 y)$$
$$= p + 2(\log_2 2^{2q})$$
$$= p + 2(2q)$$
$$= p + 4q$$

(iii)

$$\log_4 \frac{4x}{y} = \log_4 4 + \log_4 x - \log_4 y$$

$$= 1 + \frac{\log_2 x}{\log_2 4} - q$$

$$= 1 + \frac{1}{2} p - q$$

$$\text{or } \frac{2+p}{2} - q$$

$$\text{or } \frac{2+p-2q}{2}$$

Alternative method,

$$\log_4 \frac{4x}{y} = \log_4 \left(\frac{4(2^p)}{4^q} \right)$$

$$= \log_4 \left(\frac{2^{p+2}}{2^{2q}} \right)$$

$$= \log_4 (2^{p+2-2q})$$

$$= (p+2-2q) \log_4 2$$

$$= \frac{1}{2}(p+2-2q)$$

$$= \frac{1}{2} p + 1 - q$$

(b)

$$\lg x + \lg[5(x+1)] - 2 = 0$$

$$\lg x[5(x+1)] = 2$$

$$x(5x+5) = 10^2$$

$$5x^2 + 5x - 100 = 0$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0$$

$$x = -5 \text{ (rej) or } 4$$

Question 4

(a)

$$\log_2 x + \frac{2}{\log_2 x} = 3$$

$$(\log_2 x)^2 - 3\log_2 x + 2 = 0$$

$$(\log_2 x - 2)(\log_2 x - 1) = 0$$

$$\log_2 x = 2 \text{ or } \log_2 x = 1$$

$$x = 4 \qquad x = 2$$

Or

Let $y = \log_2 x$

$$y^2 - 3y + 2 = 0$$

$$(y-2)(y-1) = 0$$

$$y = 2 \text{ or } y = 1$$

$$\log_2 x = 2 \qquad \log_2 x = 1$$

$$x = 4 \qquad x = 2$$

(b)

$$\log_3(x+1) - \log_{\sqrt{3}}(x-1) = 1$$

$$\log_3(x+1) - \frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = 1$$

$$\log_3(x+1) - \frac{\log_3(x-1)}{\frac{1}{2}\log_3 3} = 1$$

$$\log_3(x+1) - 2\log_3(x-1) = 1$$

$$\log_3(x+1) - \log_3(x-1)^2 = 1$$

$$\log_3 \frac{x+1}{(x-1)^2} = 1$$

$$\frac{x+1}{(x-1)^2} = 3^1$$

$$3(x-1)^2 - (x+1) = 0$$

$$3(x^2 - 2x + 1) - x - 1 = 0$$

$$3x^2 - 7x + 2 = 0$$

$$(3x-1)(x-2) = 0$$

$$x = \frac{1}{3} \text{ (reject) or } 2$$

Question 5

$$\log_3(x+2) = 3 - \log_3(x-4)$$
$$\log_3(x+2)(x-4) = 3$$
$$x^2 - 2x - 8 = 3^3$$
$$x^2 - 2x - 35 = 0$$
$$(x+5)(x-7) = 0$$
$$x = -5 \text{ (NA) or } 7$$

Question 6

$$\log_4 x + \log_2 y = \log_2(x+y)$$
$$\frac{\log_2 x}{\log_2 2^2} + \log_2 y = \log_2(x+y)$$
$$\log_2 x^{\frac{1}{2}} + \log_2 y = \log_2(x+y)$$
$$\log_2\left(x^{\frac{1}{2}} y\right) = \log_2(x+y)$$
$$x^{\frac{1}{2}} y = x + y$$
$$x^{\frac{1}{2}} y - y = x$$
$$y\left(\sqrt{x} - 1\right) = x$$
$$y = \frac{x}{\sqrt{x}-1} \text{ or } \frac{x\sqrt{x}+x}{x-1}$$

Question 7

$$\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$$

$$\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\frac{1}{2}$$

$$2\log_3(2x-1)-\log_3(x^2+2)=1$$

$$\log_3(2x-1)^2-\log_3(x^2+2)=1$$

$$\log_3\frac{(2x-1)^2}{(x^2+2)}=1$$

$$\frac{(2x-1)^2}{(x^2+2)}=3$$

$$4x^2-4x+1=3x^2+6$$

$$x^2-4x-5=0$$

$$(x+1)(x-5)=0$$

$$x=5 \text{ or } -1 \text{ (rejected)}$$