RV 2012 Yr 6 H2 MA Prelim Paper 1

Question 1 [7 Marks]

Let $\mathrm{f}(z) = z^4 - 2z^3 + 14z^2 + az + b$

Consider $z^4 - 2z^3 + 14z^2 + az + b = 0$ ---- (1)

Sub $z = 1 + 2\mathrm{i}$ into (1), using GC

$(-7 - 24\mathrm{i}) - 2(-11 - 2\mathrm{i}) + 14(-3 + 4\mathrm{i}) + a(1 + 2\mathrm{i}) + b = 0$

$(-7 + 22 - 42 + a + b) + (-24 + 4 + 56 + 2a)\mathrm{i} = 0$

$(-27 + a + b) + (36 + 2a)\mathrm{i} = 0 + 0\mathrm{i}$

Comparing the real and imaginary coefficients

$$\begin{cases} -27 + a + b = 0 & \text{----(2)} \\ 36 + 2a = 0 & \text{----(3)} \end{cases}$$

Solving (2) and (3),

$a = -18$ and $b = 45$

Therefore $\mathrm{f}(z) = z^4 - 2z^3 + 14z^2 - 18z + 45$

Using GC to solve $\mathrm{f}(z) = z^4 - 2z^3 + 14z^2 - 18z + 45 = 0$,

$z = -\mathrm{i} + 2$, $z = -\mathrm{i} - 2$, $z = 3$, $z = -3$

Replace z with $\mathrm{i}z$ in (1), we obtain

$z^4 + 2iz^3 - 14z^2 - 18iz + 45 = 0$

$\mathrm{i}z = 1 + 2\mathrm{i}$, $\mathrm{i}z = 1 - 2\mathrm{i}$, $\mathrm{i}z = 3\mathrm{i}$, $\mathrm{i}z = -3\mathrm{i}$

$z = -\mathrm{i} + 2$, $z = -\mathrm{i} - 2$, $z = 3$, $z = -3$

Alternatively, since all the coefficients of the polynomial $\mathrm{f}(z)$ are real

$\Rightarrow z = 1 + 2\mathrm{i}$ and $z = 1 - 2\mathrm{i}$ are roots of $\mathrm{f}(z) = 0$

$\Rightarrow [z - (1 + 2\mathrm{i})][z - (1 - 2\mathrm{i})] = z^2 - 2z + 5$ is a quadratic factor of $\mathrm{f}(z)$.

Let $z^2 + qz + r$ be the other quadratic factor of $\mathrm{f}(z)$.

$z^4 - 2z^3 + 14z^2 + az + b = [z^2 - 2z + 5][z^2 + qz + r]$

Comparing coefficient of z^3: $-2 = q - 2 \Rightarrow q = 0$

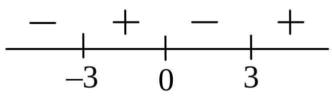
Comparing coefficient of z^2: $14 = r + 5 \Rightarrow r = 9$

Therefore $\mathrm{f}(z) = [z^2 - 2z + 5][z^2 + 9]$

$\mathrm{f}(z) = z^4 - 2z^3 + 14z^2 - 18z + 45$

$a = -18$ and $b = 45$

$\mathrm{f}(z) = 0 \Rightarrow z = 1 + 2\mathrm{i}$, $z = 1 - 2\mathrm{i}$, $z = 3\mathrm{i}$, $z = -3\mathrm{i}$

Question 2 [8 Marks]		
	$x \geq \frac{9}{x}$ $\frac{(x-3)(x+3)}{x} \geq 0$  $\therefore -3 \leq x < 0$ or $x \geq 3$	
	$\int_n^4 \left\lvert x - \frac{9}{x} \right\rvert \mathrm{d}x$ $= \int_n^3 -\left(x - \frac{9}{x} \right)\mathrm{d}x + \int_3^4 \left(x - \frac{9}{x} \right)\mathrm{d}x$ $= \left[9\ln\lvert x\rvert - \frac{x^2}{2} \right]_n^3 + \left[\frac{x^2}{2} - 9\ln\lvert x\rvert \right]_3^4$ $= \left[9\ln 3 - \frac{9}{2} - 9\ln n + \frac{n^2}{2} \right] + \left[8 - 9\ln 4 - \frac{9}{2} + 9\ln 3 \right]$ $= 18\ln 3 - 1 - 9\ln(4n) + \frac{n^2}{2}$ $= I$	
	As $n \to 0$, $\ln(4n) \to -\infty$ $\therefore I \to +\infty$	

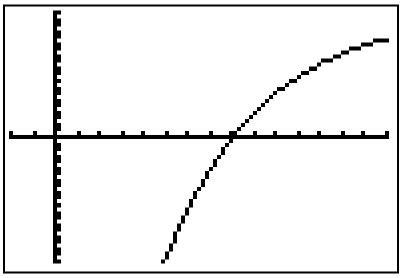
Question 3 [9 Marks]		
i	OAQB is a parallelogram $\Rightarrow \overrightarrow{OA} = \overrightarrow{BQ}$ $\Rightarrow \overrightarrow{OA} = \overrightarrow{OQ} - \overrightarrow{OB}$ $\Rightarrow \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \overrightarrow{OQ} - \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$	
ii	$(\mathbf{a} \cdot \mathbf{c})\mathbf{c}$ is the projection vector of $\mathbf{a}$ onto $\mathbf{b}$.	
iii	$\lvert\mathbf{a}\rvert < \lvert\mathbf{b}\rvert$ $p^2 + (p-1)^2 + 4 < 1 + 4 + 4$ $p^2 - p - 2 < 0$ $(p+1)(p-2) < 0$ $-1 < p < 2$ But $p > 0$, therefore $0 < p < 2$.	

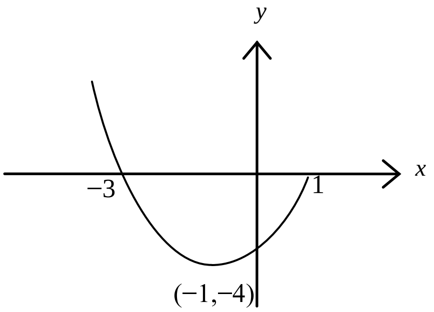
iv	$\left[\begin{pmatrix}2\\1\\2\end{pmatrix}+\begin{pmatrix}1\\-2\\2\end{pmatrix}\right].\left[\begin{pmatrix}2\\1\\2\end{pmatrix}-\begin{pmatrix}1\\-2\\2\end{pmatrix}\right]=\begin{pmatrix}3\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\3\\0\end{pmatrix}=3-3+0=0$ Thus $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ are perpendicular. Note: $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ are the diagonals of the parallelogram with **a** and **b** as the adjacent sides. The parallelogram with **a** and **b** as the adjacent sides must be a rhombus. $\|\mathbf{a}\times\mathbf{b}\|$ is the area of a rhombus formed by the vectors **a** and **b**. OR $\|\mathbf{a}\times\mathbf{b}\|$ is the area of the rhombus OAQB.	

Question 4 [9 Marks]		
	$y=\left(\cos^{-1}x\right)^2$ --- (1) $\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\cos^{-1}x\right)\left(-\frac{1}{\sqrt{1-x^2}}\right)$ --- (2) Squaring both sides, we get $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{4\left(\cos^{-1}x\right)^2}{1-x^2}$ $(1-x^2)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4y$. (shown)	
	$(1-x^2)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4y$ Differentiate with respect to x: $(1-x^2)(2)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(-2x)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4\frac{\mathrm{d}y}{\mathrm{d}x}$ --- (3) Substitute $x = 0$ into (1), (2) and (3): $y=\left(\cos^{-1}0\right)^2=\left(\frac{\pi}{2}\right)^2=\frac{\pi^2}{4}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\cos^{-1}0\right)\left(-\frac{1}{\sqrt{1-0^2}}\right)=2\left(\frac{\pi}{2}\right)(-1)=-\pi$ $(1-0^2)(2)(-\pi)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4\pi \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2$ $\therefore\ y=\frac{\pi^2}{4}+(-\pi)x+\frac{2}{2!}x^2+\ldots$ $y=\frac{\pi^2}{4}-\pi x+x^2+\ldots$	

i	Equation of tangent: $y=-\pi x+\dfrac{\pi^2}{4}$.	
ii	$\dfrac{2\cos^{-1}x}{x^2-1}$ $=-\dfrac{2\cos^{-1}x}{1-x^2}$ $=-\dfrac{2\cos^{-1}x}{\sqrt{1-x^2}\sqrt{1-x^2}}$ $=\dfrac{dy}{dx}(1-x^2)^{-\frac{1}{2}}$ $=(-\pi+2x+\ldots)\left[1+\dfrac{1}{2}x^2+\ldots\right]$ $\approx -\pi+2x$	

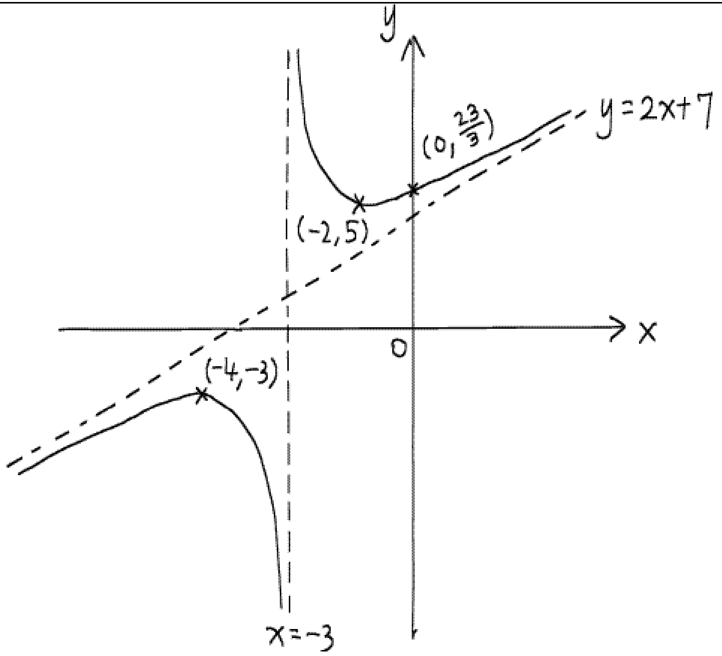
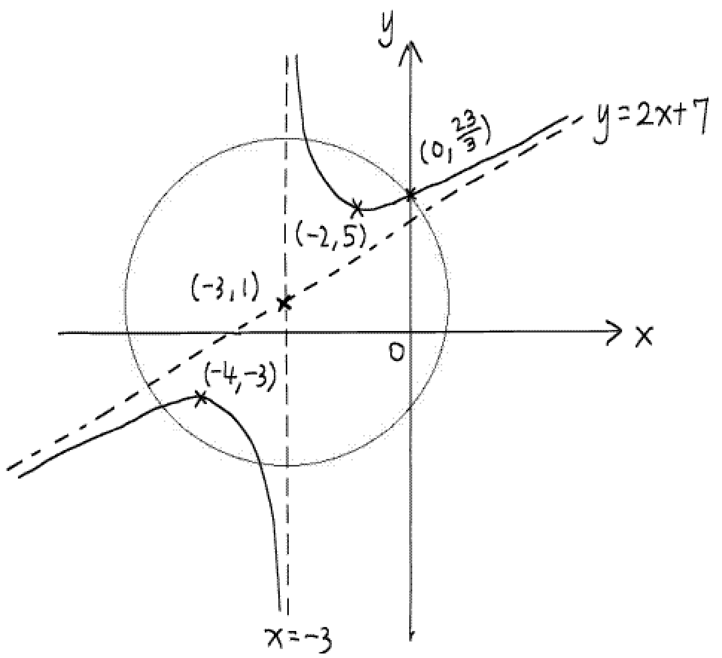
Question 5 [10 Marks]		
i	Amount of water at the: End of 1st day = 80(0.8) End of 2nd day = (80(0.8)+40)(0.8) $= 80(0.8)^2 + 40(0.8)$ $= 83.2\text{ cm}^3$	
ii	End of 2nd day $= 80(0.8)^2 + 40(0.8)$ End of 3rd day $= (80(0.8)^2 + 40(0.8)+40)(0.8)$ $= 80(0.8)^3 + 40(0.8)^2 + 40(0.8)$ ⋮ End of *n*th day $= 80(0.8)^n + 40[0.8 + 0.8^2 + 0.8^3 + \ldots + 0.8^{n-1}]$ $= 80(0.8)^n + 40\left[\dfrac{0.8(1-0.8^{n-1})}{1-0.8}\right]$ $= 80(0.8)^n + 160\left(1-(0.8)^{n-1}\right)$ $= 80(0.8)^n + 160 - 160(0.8)^{n-1}$ $= 80(0.8)^n + 160 - 200(0.8)^n$ $= 160 - 120(0.8)^n$ So, the amount of water at the end of the *n*th day is $\left(160-120(0.8)^n\right)\text{cm}^3$ (shown)	
iii	Since the maximum capacity of the glass is 180 cm^3, we will find the least *n* value such that the amount of water at the end of *n*th day is more than 140 cm^3. $160 - 120(0.8)^n + 40 > 180$ $160 - 120(0.8)^n > 140$	

	Method 1 (Algebraic approach): $160 - 120(0.8)^n > 140$ $(0.8)^n < \frac{1}{6}$ $n > \frac{\ln(\frac{1}{6})}{\ln(0.8)}$ $n > 8.0296\ldots$ $\therefore$ least value of n is 9. At the end of the 9th day, amount of water is more than 140 cm^3. So, the day when overflowing happens is the 10th day.	
	Method 2 (Graphical approach): $160 - 120(0.8)^n > 140$ $20 - 120(0.8)^n > 0$ Plot the graph $y = 20 - 120(0.8)^n$  From the graph, $n > 8.0296\ldots$ $\therefore$ least value of n is 9. So, the day when overflowing happens is the 10th day.	
iv	As $n \to \infty$, $(0.8)^n \to 0$ $160 - 120(0.8)^n \to 160$ Amount of water at the end of any day will not exceed $160cm^3$, so the minimum capacity of the glass to be used is $(160+40)cm^3 = 200cm^3$.	

Question 6 [10 marks]		
(i)	$\text{f}: x \mapsto x^2 + 2x - 3, \quad x \le 1.$  A horizontal line $y = k$ where $-4 < k \le 0$ cuts the graph of $y = \text{f}(x)$ twice, thus f is not one-to-one. Therefore f^{-1} does not exist. Quoting a specific line eg $y = -3$ is acceptable.	
(ii)	For f^{-1} to exist, largest domain is $(-\infty, -1]$. Largest value of $a = -1$. Let $y = x^2 + 2x - 3$. $x^2 + 2x - 3 - y = 0$ $\therefore x = \dfrac{-2 \pm \sqrt{4 - 4(-3 - y)}}{2}$ $= \dfrac{-2 \pm \sqrt{4y + 16}}{2}$ $= -1 \pm \sqrt{y + 4}$ Since $x \le -1, x = -1 - \sqrt{y + 4}$ $\therefore \text{f}^{-1}(x) = -1 - \sqrt{x + 4}, \ \ x \ge -4$	
(iii)	$y = \text{f}(x)$, $y = \text{f}^{-1}(x)$ and $y = x$ intersect at the same point. $\text{f}(x) = x$ $x^2 + 2x - 3 = x$ $x^2 + x - 3 = 0$ $x = \dfrac{-1 \pm \sqrt{13}}{2}$ Since $x \le -1, x = \dfrac{-1 - \sqrt{13}}{2}$.	

(iv)	$R_g = (0,2)$ $D_f = (-\infty,1]$ $R_g \not\subset D_f$ Thus, fg does not exist.	
(v)	For fg to exist, $R_g \subseteq D_f$. Let $R_g = (0,1]$ Thus $D_g = [1.2,3)$ $[1.2,3) \xrightarrow{g} (0,1] \xrightarrow{f} (-3,0]$	

Question 7 [11 Marks]		
i	$y = \dfrac{2x^2 + 13x + 23}{x+3}$ $2x^2 + 13x + 23 = xy + 3y$ $2x^2 + (13 - y)x + (23 - 3y) = 0$ The equation above has no real roots when $b^2 - 4ac < 0$ $(13 - y)^2 - 4(2)(23 - 3y) < 0$ $y^2 - 2y - 15 < 0$ $(y+3)(y-5) < 0$ $\therefore\ -3 < y < 5$ So, C cannot lie between –3 and 5.	
ii	$y = \dfrac{2x^2 + 13x + 23}{x+3} = 2x + 7 + \dfrac{2}{x+3}$ The asymptotes are $y = 2x + 7$ and $x = -3$.	

iii		
iv	$(x+3)^2+\left(\frac{2x^2+12x+20}{x+3}\right)^2=k^2$ $(x+3)^2+\left(\frac{2x^2+13x+23-x-3}{x+3}\right)^2=k^2$ $(x+3)^2+(y-1)^2=k^2$ Add a circle with centre (–3, 1) and radius k  To have a positive root, we first find the distance between (–3, 1) and $\left(0, \frac{23}{3}\right)$.	

	$\sqrt{3^2+\left(\frac{20}{3}\right)^2} = \frac{\sqrt{481}}{3}$ So, range of values of k: $k^2 > \left(\frac{\sqrt{481}}{3}\right)^2$ $k < -\frac{\sqrt{481}}{3}$ or $k > \frac{\sqrt{481}}{3}$.	

Question 8 [11 Marks]		
(i)	$x = \sec t \Rightarrow \frac{dx}{dt} = \sec t \tan t$ $y = \tan t \Rightarrow \frac{dy}{dt} = \sec^2 t$ $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ $= \frac{\sec^2 t}{\sec t \tan t}$ $= \frac{\sec t}{\tan t}$ $= \frac{1}{\sin t} = \operatorname{cosec} t$	
(ii)	At point P $(\sec\theta, \tan\theta)$, $t=\theta$ $\frac{dy}{dx} = \operatorname{cosec}\theta$ Equation of tangent at P: $y - \tan\theta = (\operatorname{cosec}\theta)(x - \sec\theta)$ $y = (\operatorname{cosec}\theta)x + \tan\theta - (\operatorname{cosec}\theta)(\sec\theta)$ $y = (\operatorname{cosec}\theta)x + \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta\cos\theta}$ $y = (\operatorname{cosec}\theta)x + \frac{\sin^2\theta - 1}{\sin\theta\cos\theta}$ $y = (\operatorname{cosec}\theta)x - \frac{\cos^2\theta}{\sin\theta\cos\theta}$ $y = (\operatorname{cosec}\theta)x - \frac{\cos\theta}{\sin\theta}$ $y = (\operatorname{cosec}\theta)x - \cot\theta$	

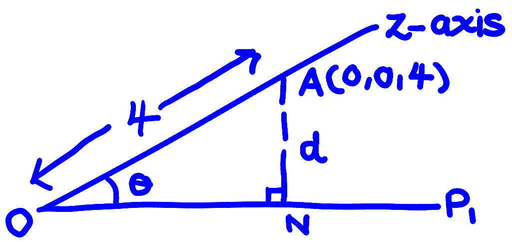
(iii)	When $y=0$, $x=\dfrac{\cot\theta}{\text{cosec}\ \theta}=\dfrac{\cos\theta}{\sin\theta}\cdot\ \sin\theta=\cos\theta$ So, coordinates of $A=(\cos\theta,\ \ 0\ )$ When $x=0,\ y=-\cot\theta$ So, coordinates of $B=(0,\ \ \ -\cot\theta\ )$ Area of triangle *AOB* $=\dfrac{1}{2}\ \times\ \cos\theta\ \times\ \cot\theta$ When $\theta=\dfrac{\pi}{6}$, Area = $\dfrac{1}{2}\times\cos\dfrac{\pi}{6}\times\ \cot\dfrac{\pi}{6}$ $=\dfrac{1}{2}\times\dfrac{\sqrt{3}}{2}\times\sqrt{3}$ $=\dfrac{3}{4}\text{units}^2$	
(iv)	$A=(\cos\theta,\ \ 0\ )$ $B=(0,\ \ \ -\cot\theta\ )$ So, mid-point of *AB* = $\left(\dfrac{\cos\theta}{2},\ \ \ -\dfrac{\cot\theta}{2}\right)$ $\Rightarrow\ x=\dfrac{\cos\theta}{2},\ \ y=-\dfrac{\cot\theta}{2}$ $\Rightarrow\ \sec\theta=\dfrac{1}{2x},\ \ \tan\theta=-\dfrac{1}{2y}$ Since $\tan^2\theta+1=\sec^2\theta$ Then $\dfrac{1}{4y^2}+1=\dfrac{1}{4x^2}$ $\therefore\dfrac{1}{4x^2}-\dfrac{1}{4y^2}=1$ [Note: $0<\theta<\dfrac{\pi}{2}\Rightarrow\ 0<x=\dfrac{\cos\theta}{2}<\dfrac{1}{2}$ The corresponding domain for the cartesian curve is $0<x<\dfrac{1}{2}$.] ――――――――――――――― Alternatively $A=(\cos\theta,\ \ 0\ )$ $B=(0,\ -\cot\theta\ )$	

	So, mid-point of $AB = \left(\dfrac{\cos\theta}{2}, \quad -\dfrac{\cot\theta}{2}\right)$ $\Rightarrow \quad x = \dfrac{\cos\theta}{2}, \quad y = -\dfrac{\cot\theta}{2}$ $\Rightarrow \quad \cos\theta = 2x --- (1) \quad \tan\theta = -\dfrac{1}{2y} --- (2)$ Using (1): $\cos\theta = 2x \quad \Rightarrow \quad \tan\theta = \dfrac{\sqrt{1-4x^2}}{2x}$ ----(3) Equating (2) and (3), $\dfrac{\sqrt{1-4x^2}}{2x} = -\dfrac{1}{2y}$ $1-4x^2 = \dfrac{x^2}{y^2}$ $\therefore \dfrac{1}{4x^2} - \dfrac{1}{4y^2} = 1$	

Question 9 [12 Marks]		
(a)	$\dfrac{r+1}{r+2} - \dfrac{r}{r+1} = \dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$ $= \dfrac{1}{(r+2)(r+1)}$	
	$\sum_{r=1}^{10}\left[\dfrac{1}{r^2+3r+2} - \ln r^2\right]$ $= \sum_{r=1}^{10}\left[\dfrac{r+1}{r+2} - \dfrac{r}{r+1} - 2\ln r\right]$ $= \dfrac{2}{3} - \dfrac{1}{2} - 2\ln 1$ $+ \dfrac{3}{4} - \dfrac{2}{3} - 2\ln 2$ $+ \dfrac{4}{5} - \dfrac{3}{4} - 2\ln 3$ $\vdots$ $+ \dfrac{10}{11} - \dfrac{9}{10} - 2\ln 9$ $+ \dfrac{11}{12} - \dfrac{10}{11} - 2\ln 10$ $= \dfrac{11}{12} - \dfrac{1}{2} - 2[\ln 1 + \ln 2 + \ln 3 + ... + \ln 10]$	

	$= \frac{5}{12} - 2[\ln(1)(2)(3)...(10)]$ $= \frac{5}{12} - 2[\ln(10)!] = \frac{5}{12} - 2\ln(3628800)$	
(b)	Let P(n) be the statement: $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2 ,\ n \in Z^+$$ When n = 1, LHS = $\sum_{r=1}^{1} r^3 = 1^3 = 1$ RHS = $\frac{1}{4}(1)^2(1+1)^2$ = 1 = LHS $\therefore$ P(1) is true. Assume P(k) is true for some $k \in Z^+$, i.e. $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$ We need to show that P(k+1) is true, i.e. $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}(k+1)^2(k+2)^2$ LHS = $\sum_{r=1}^{k+1} r^3$ $= \sum_{r=1}^{k} r^3 + (k+1)^3$ $= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ $= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)]$ $= \frac{1}{4}(k+1)^2(k^2 + 4k + 4)$ $= \frac{1}{4}(k+1)^2(k+2)^2$ = RHS $\therefore$ P(k+1) is true. Since P(1) is true, P(k) is true implying P(k+1) is true, by Mathematical Induction, P(n) is true for all $n \in Z^+$.	
	$\sum_{r=6}^{n+3} (r-4)^3$ Let $k = r - 4$, then $\sum_{k=2}^{n-1} k^3$ $= \sum_{k=1}^{n-1} k^3 - (1)^3$	

	$= \frac{1}{4}n^2(n-1)^2 - 1$	

Question 10 [13 Marks]		
i	$p_1 : \mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix} = 0$ and z-axis: $\mathbf{r} = \lambda\begin{pmatrix}0\\0\\1\end{pmatrix}, \lambda \in \Re$ Let θ be the acute angle between p_1 and the z-axis. $\left\|\begin{pmatrix}0\\2\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right\| = \left\|\begin{pmatrix}0\\2\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|\sin\theta$ $\sin\theta = \frac{1}{\sqrt{5}}$ $\theta = 26.6^o$ --- Alternatively Let θ be the acute angle between p_1 and the z-axis. Let α be the acute angle between the normal of p_1 and the z-axis. $\left\|\begin{pmatrix}0\\2\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right\| = \left\|\begin{pmatrix}0\\2\\-1\end{pmatrix}\right\|\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|\cos\alpha$ ---- (*) $\cos\alpha = \frac{1}{\sqrt{5}}$ $\alpha = 63.4^o$ $\theta = 90^o - \alpha = 26.6^o$ Method 1  The origin is the point of intersection of the z-axis and $p_1 : y - z = 0$ & A(0,0,4) is a point on the z-axis. Let d be the distance from the point A(0,0,4) to p_1. $d = 4\sin\theta$ $d = 4\left(\frac{1}{\sqrt{5}}\right) = \frac{4\sqrt{5}}{5}$ ---	

Method 2

Let N be the foot of perpendicular from A on p_1.

$$l_{AN}: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \lambda \in \Re$$

$$\overrightarrow{ON} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \text{ for some } \lambda$$

Since N lies on $p_1 : \mathbf{r} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0$

$$\left[\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right] \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0$$

$$4 + 5\lambda = 0$$

$$\lambda = \frac{4}{5}$$

$$\overrightarrow{AN} = \overrightarrow{ON} - \overrightarrow{OA} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \text{ ----(1)}$$

Sub $\lambda = 2$ into (1): $\overrightarrow{AN} = \frac{4}{5} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

$$d = \left| \overrightarrow{AN} \right| = \frac{4\sqrt{5}}{5}$$

Method 3

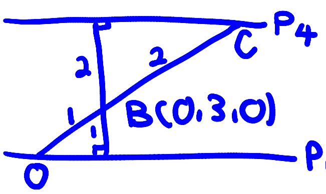
Since the origin O is a point on $p_1 : 2y - z = 0$

$$d = \left| \overrightarrow{OA} . \mathbf{n} \right|$$

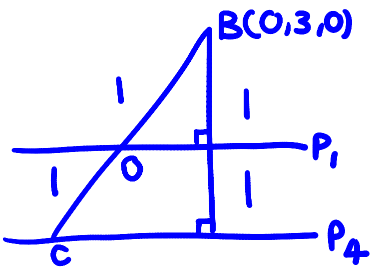
$$d = \left| \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \right| = \frac{4\sqrt{5}}{5}$$

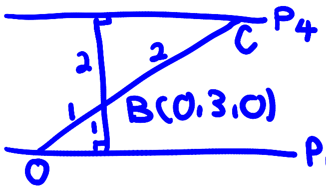
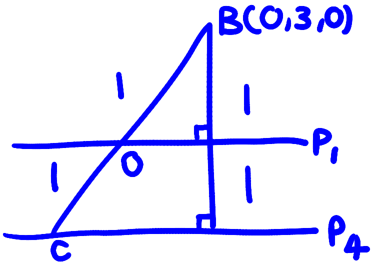
ii

Case 1: p_4 on opposite side of B to p_1



The origin O is a point on p_1.

	Let C be the point on p_4, along the line segment OB. Since the distance of p_4 from the point B is twice that of the distance of p_1 from the point B. Ratio of $OC:OB=3:1$ $\overrightarrow{OC}=3\overrightarrow{OB}=9\mathbf{j}$ $p_1:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ Since C lies on p_4 and $\begin{pmatrix}0\\9\\0\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=18$ $p_4:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=18$ Case 2: p_4 on same side of B as p_1  Ratio of $OC:OB=1:1$ $\overrightarrow{OC}=-\overrightarrow{OB}=-3\mathbf{j}$ $p_1:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ Since C lies on p_4 and $\begin{pmatrix}0\\-3\\0\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=-6$ $p_4:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=-6$	
	Alternative Solution Let q be the plane passing through B(0,3,0) parallel to	

	p_1, where $p_1 : \mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$. Since $\begin{pmatrix}0\\3\\0\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=6 \Rightarrow q:\mathbf{r}\cdot\left[\frac{1}{\sqrt{5}}\begin{pmatrix}0\\2\\-1\end{pmatrix}\right]=\frac{6}{\sqrt{5}}$ Case 1: p_4 on opposite side of B to p_1  $p_4:\mathbf{r}\cdot\left[\frac{1}{\sqrt{5}}\begin{pmatrix}0\\2\\-1\end{pmatrix}\right]=\frac{18}{\sqrt{5}} \Rightarrow p_4:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=18$ Case 2: p_4 on same side of B as p_1  $p_4:\mathbf{r}\cdot\left[\frac{1}{\sqrt{5}}\begin{pmatrix}0\\2\\-1\end{pmatrix}\right]=-\frac{6}{\sqrt{5}} \Rightarrow p_4:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=-6$	
(iii)	$p_1:\mathbf{r}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=0$ and $p_2:\mathbf{r}\cdot\begin{pmatrix}\beta\\0\\1\end{pmatrix}=2$ $\begin{pmatrix}0\\1\\2\end{pmatrix}\cdot\begin{pmatrix}0\\2\\-1\end{pmatrix}=2-2=0 \Rightarrow (0,1,2)$ lies on p_1 $\begin{pmatrix}0\\1\\2\end{pmatrix}\cdot\begin{pmatrix}\beta\\0\\1\end{pmatrix}=0+0+2=2 \Rightarrow (0,1,2)$ lies on p_2 Let d be the direction vector of l.	

	$\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2$ $\mathbf{d} = \begin{pmatrix} \beta \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ \beta \\ 2\beta \end{pmatrix}$ $l : \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ \beta \\ 2\beta \end{pmatrix}, \ \gamma \in \Re$ **----(*)**	
(iv)	Method 1 Using GC to solve $2y - z = 0$ and $2x + z = 2$, then $l : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \ \gamma \in \Re$ l is parallel to p_3. $\mathbf{d}$ parallel to $\mathbf{n}_3$ $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} . \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix} = 0$ $\lambda = 5$ The point $(1,0,0)$ does not lie on p_3. $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} . \begin{pmatrix} 1 \\ \lambda \\ -2 \end{pmatrix} \neq \mu$ $\mu \neq 1$ --- Method 2 Substituting $\beta = 2$ into (*) $l : \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} -2 \\ \beta \\ 2\beta \end{pmatrix}, \ \gamma \in \Re$ **----(*)** $l : \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \ \gamma \in \Re$ **----(*)** l is parallel to p_3. $\mathbf{d}$ parallel to $\mathbf{n}_3$	

	$\begin{pmatrix}-1\\1\\2\end{pmatrix}.\begin{pmatrix}1\\\lambda\\-2\end{pmatrix}=0$ $\lambda=5$ The point $(0,1,2)$ does not lie on p_3. $\begin{pmatrix}0\\1\\2\end{pmatrix}.\begin{pmatrix}1\\\lambda\\-2\end{pmatrix}\neq\mu$ $\mu\neq1$	