# H2 Mathematics (9758)
# Chapter 4 Equations and Inequalities
# Discussion Solutions

**Level 1 Questions**

**1** Solve the equation $21x^2 - 11x - 2 = 0$ algebraically.

| Q1 | | |
|---|---|---|
| | $21x^2 - 11x - 2 = 0$<br>$(7x+1)(3x-2) = 0$<br>$x = -\frac{1}{7}$ or $x = \frac{2}{3}$ | 3 methods to solve quadratic equations algebraically:<br>1) Factorisation (shown in solution)<br>2) Complete the square (try on your own)<br>3) Quadratic Formula (try on your won) |

**2** Using a graphing calculator, solve the following equations

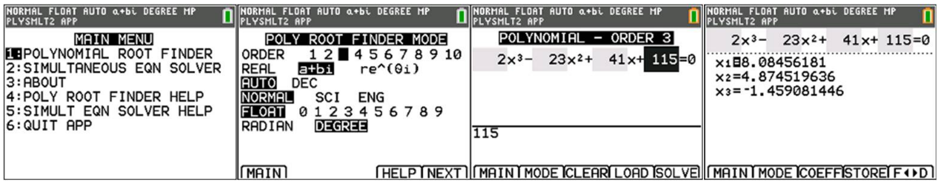
**(a)** $2x^3 - 23x^2 + 41x + 115 = 0$,

**(b)** $\frac{1}{x} - 2 + 2\ln x = 0$.

<table>
<tr><th>Q2</th><th></th></tr>
<tr><td>(a)</td><td>

$2x^3 - 23x^2 + 41x + 115 = 0$

Using GC, $x = -1.46$ (3 s.f.) or $x = 4.87$ (3 s.f.) or $x = 8.08$ (3 s.f.)

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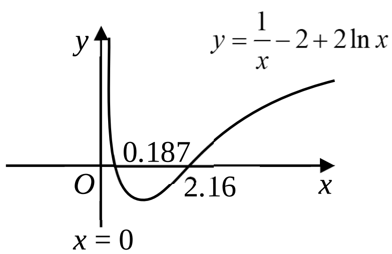
<tr><td>(b)</td><td>

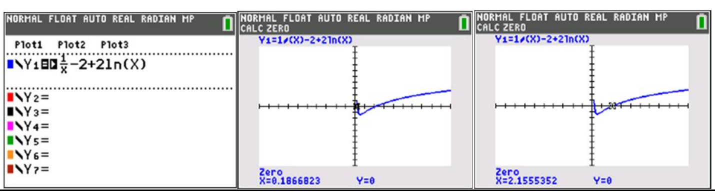
$\frac{1}{x} - 2 + 2\ln x = 0$




Solving equations using a GC:

- For polynomial equations (up to degree 10) we can use the PolyRootFinder in the APP.
- We can use the graphical method (sketching one or two graphs) to determine the roots (*x*-value of the intersection points).

Using GC,
$x = 0.187$ or $x = 2.16$




</td></tr>
</table>

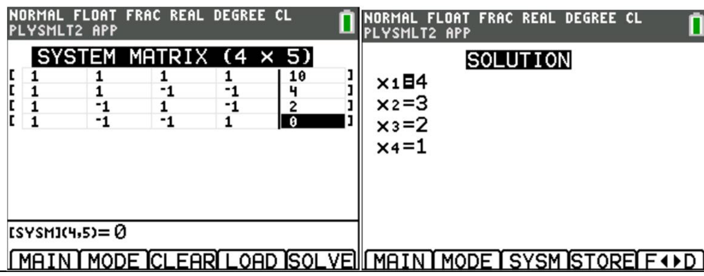
**3** Solve the system of linear equations:

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 10 \\ x_1 & + & x_2 & - & x_3 & - & x_4 & = & 4 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 2 \\ x_1 & - & x_2 & - & x_3 & + & x_4 & = & 0 \end{array}$$

**Q3**

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 10 \\ x_1 & + & x_2 & - & x_3 & - & x_4 & = & 4 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 2 \\ x_1 & - & x_2 & - & x_3 & + & x_4 & = & 0 \end{array}$$

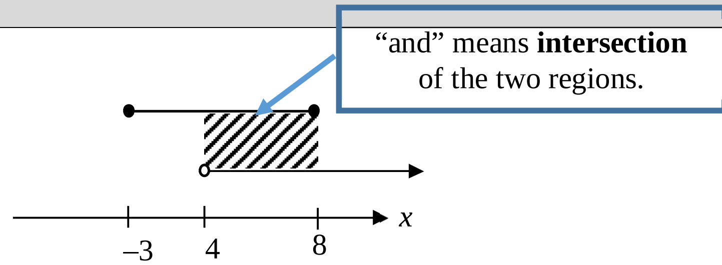
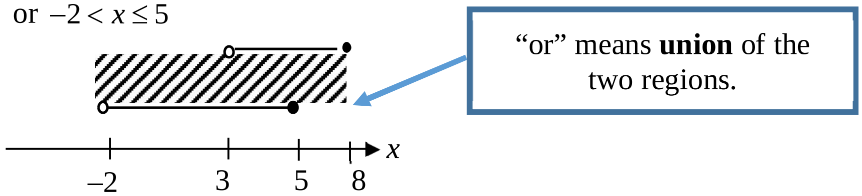
Using GC, .$x_1 = 4$, $x_2 = 3$, $x_3 = 2$, $x_4 = 1$.



**4** By drawing number lines, simplify the following ranges of values of $x$:

**(a)** $-3 \le x \le 8$ and $x > 4$,

**(b)** $3 < x \le 8$ or $-2 < x \le 5$.

| **Q4** | |
|---|---|
| **(a)** | $-3 \le x \le 8$ and $x > 4$ <br>   <br> $\therefore 4 < x \le 8$ |
| **(b)** | $3 < x \le 8$ or $-2 < x \le 5$ <br>   <br> $\therefore -2 < x \le 8$ |

**5** Without the use of a calculator, solve the following inequalities

**(a)** $2x \ge 5 + 4x$,

**(b)** $9x^2 - 6x - 8 > 0$,

**(c)** $\lvert x+2 \rvert < 7$,

**(d)** $\lvert 2x-1 \rvert > 3$.

| **Q5** | |
|---|---|
| **(a)** | $2x \ge 5 + 4x$ <br> $-2x \ge 5$ <br> $x \le -\frac{5}{2}$ |
| **(b)** | $9x^2 - 6x - 8 > 0$ <br> $(3x+2)(3x-4) > 0$ <br> $x < -\frac{2}{3}$ or $x > \frac{4}{3}$ |
| **(c)** | $\lvert x+2 \rvert < 7$ <br> $-7 < x + 2 < 7$ <br> $-9 < x < 5$ |
| **(d)** | $\lvert 2x-1 \rvert > 3$ <br> $2x - 1 < -3$ or $2x - 1 > 3$ <br> $2x < -2$ or $2x > 4$ <br> $x < -1$ or $x > 2$ |

**6** By sketching appropriate graphs, solve the following inequalities

**(a)** $\frac{1}{x} < \ln x$,

**(b)** $\left|x^2 - 4x + 1\right| - 1 \leq 0$.

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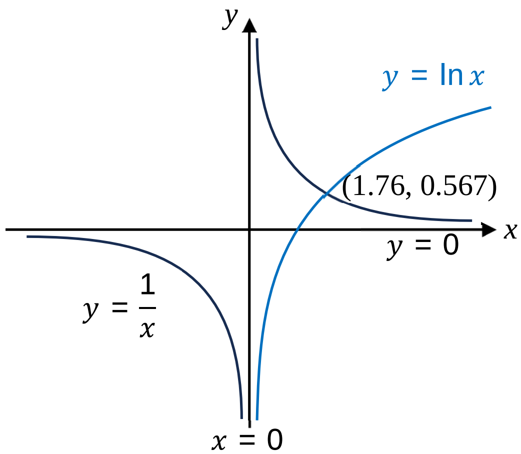
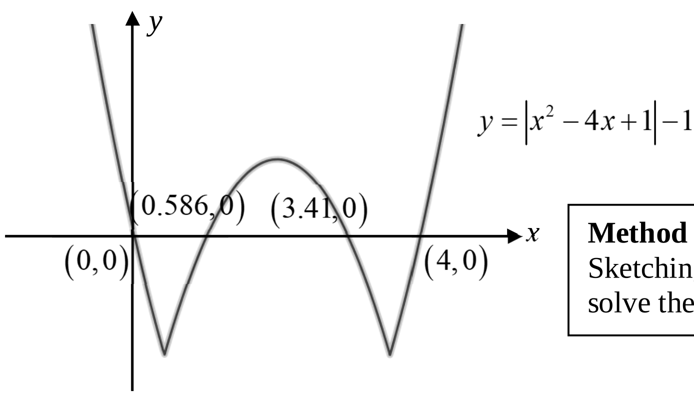
<tr><th>Q6</th><th></th></tr>
<tr><td>(a)</td><td>

Method 1:<br>Sketching of two graphs (both LHS and RHS) to solve the inequality.<br><br>For $\frac{1}{x} < \ln x$, $x > 1.76$ (3 s.f.)</td></tr>
<tr><td>(b)</td><td>

Method 2:<br>Sketching of one graph to solve the inequality.<br><br>For $\left|x^2 - 4x + 1\right| - 1 \leq 0$, $0 \leq x \leq 0.586$ or $3.41 \leq x \leq 4$ (3 s.f.)</td></tr>
</table>

## Level 2 Questions

**7** A VCR manufacturing company produces three models: model *A*, model *B* and model *C*. The time taken for the production, assembly and testing of each model is given in the table below. To minimize costs, the company decides that 385 hours should be allocated for production, 557 hours for assembly, and 128 hours for testing. How many of each model can the company produce if it wants to use up all the allocated time for each phase of the process?

| Model | Production time in hrs | Assembly time in hrs | Testing time in hrs |
|---|---|---|---|
| *A* | 1.8 | 3.0 | 0.5 |
| *B* | 2.2 | 3.2 | 0.8 |
| *C* | 3.0 | 3.5 | 1.0 |

| **Q7** | |
|---|---|
| | Let $a$, $b$, and $c$ represents the number of VCRs models *A*, *B*, and *C* respectively.<br>1) Identify unknowns<br>2) Form equations with the unknowns.<br>(3 unknowns, 3 distinct equations)<br>$1.8a + 2.2b + 3.0c = 385$ no. of hrs for production<br>$3.0a + 3.2b + 3.5c = 557$ no. of hrs for assembly<br>$0.5a + 0.8b + 1.0c = 128$ no. of hrs for testing<br>Using GC, $a = 60$, $b = 85$, $c = 30$.<br>The company has to produce 60 of model *A*, 85 of model *B* and 30 of model *C*. |

**8 2008/HCI prelim/I/1**

A student has been saving 10 cent, 20 cent and 50 cent coins in a moneybox. When she opened the box after one month, she found the amount saved is $15 and the number of 10 cent coins equals the total number of 20 cent and 50 cent coins. She also found that only half as many coins is needed to save the same amount using just 50 cent coins. Find the number of 10 cent, 20 cent and 50 cent coins in the moneybox. [4]

| **Q8** | **2008 HCI Prelim/P1/1** |
|---|---|

Let *x*, *y* and *z* be the number of 10 cent, 20 cent and 50 cent coins respectively.

1) Identify unknowns
2) Form equations with the unknowns.
(3 unknowns, 3 distinct equations)

<u>Equation 1</u>

$10x + 20y + 50z = 1500 \quad ---(1)$

<u>Equation 2</u>

$x = y + z$

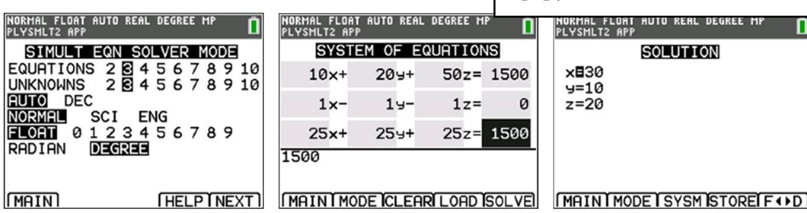
$\Rightarrow x - y - z = 0 \quad ---(2)$

<u>Equation 3</u>

$\left(\frac{x+y+z}{2}\right)50 = 1500$

$\Rightarrow 25x + 25y + 25z = 1500 \quad ---(3)$

Remember to arrange your equations in terms of $ax + by + cz = d$, where *a*, *b*, *c*, *d* are constants, before keying into GC.




Conclude your answer.

Using GC, $x = 30, \quad y = 10, \quad z = 20$

There are 30 10-cent coins, 10 20-cent coins and 20 50-cent coins.

**9** **2015(9740)/I/1**

A curve $C$ has equation

$$y=\frac{a}{x^2}+bx+c$$

where $a$, $b$ and $c$ are constants. It is given that $C$ passes through the points with coordinates $(1.6,-2.4)$ and $(-0.7,3.6)$, and that the gradient of $C$ is 2 at the point where $x=1$.

**(i)** Find the values of $a$, $b$ and $c$, giving your answers correct to 3 decimal places. [4]

**(ii)** Find the $x$-coordinate of the point where $C$ crosses the $x$-axis, giving your answer correct to 3 decimal places. [2]

**(iii)** One asymptote of $C$ is the line with equation $x=0$. Write down the equation of the other asymptote of $C$. [1]

| **Q9** | **2015(9740)/I/1** |
| --- | --- |

**(i)**

$\frac{\mathrm{d}y}{\mathrm{d}x}=-2ax^{-3}+b$ --- (*)

Substitute $x=1$ into (*),

$\frac{\mathrm{d}y}{\mathrm{d}x}=-2a(1)^{-3}+b=2$

When $x$ = 1, gradient of $C$ is 2

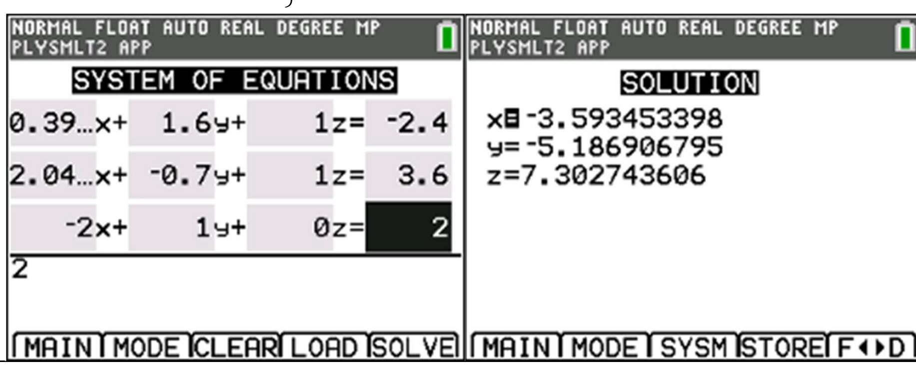
$-2a+b=2$ --- (1)

Substitute $(1.6,-2.4)$ into $y=\frac{a}{x^2}+bx+c$, $\frac{1}{1.6^2}a+1.6b+c=-2.4$ --- (2)

Substitute $(-0.7,3.6)$ into $y=\frac{a}{x^2}+bx+c$, $\frac{1}{(-0.7)^2}a-0.7b+c=3.6$ --- (3)

Solving (1), (2) and (3) using GC,

$$\left.\begin{aligned}a&=-3.59345\approx-3.593\\ b&=-5.18691\approx-5.187\\ c&=7.30274\approx7.303\end{aligned}\right\}\text{ correct to 3 d.p.}$$



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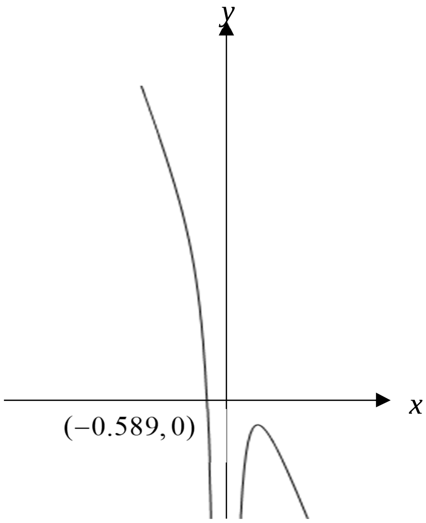

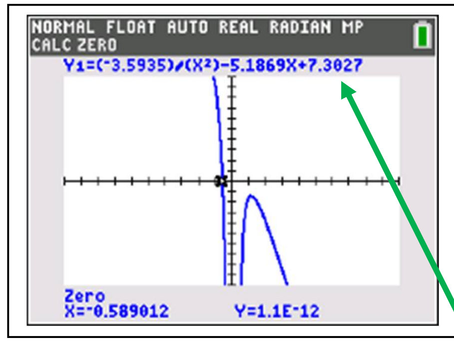

Use 5 s.f. values of a, b and c.<br>(intermediate step use 5 s.f. )<br><br>
Using GC, $x = -0.589$ (3 d.p.)
</td>
</tr>
<tr>
<td>(iii)</td>
<td>
$y = -5.187x + 7.303$<br>
$y = -5.19x + 7.30$ (correct to 3 s.f.)<br><br>
The equations of asymptote of $y = \frac{a}{x^2} + bx + c$ are $y = bx + c$ and $x = 0$
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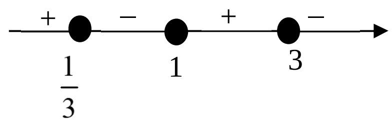
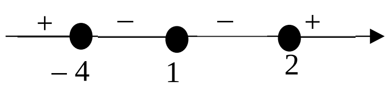
</table>

**10** Solve the following inequalities

**(a)** $(1-3x)(3-x)(1-x)\ge 0$,

**(b)** $(x+4)(x-1)^2(x-2)\ge 0$,

**(c)** $(x^2-4x+5)(x-1)\le 0$.

| **Q10** | |
|---|---|
| **(a)** | $(1-3x)(3-x)(1-x)\ge 0$ <br>   <br> Put "closed circles" for the roots for inequalities of the form: $\ge 0$ and $\le 0$ <br> $\therefore x\le \frac{1}{3}$ or $1\le x\le 3$ |
| **(b)** | $(x+4)(x-1)^2(x-2)\ge 0$ <br>   <br> Put "closed circles" for the roots for inequalities of the form: $\ge 0$ and $\le 0$ <br> $\therefore x\le -4$ or $x=1$ or $x\ge 2$ |
| **(c)** | $(x^2-4x+5)(x-1)\le 0$ <br> Since $x^2-4x+5=(x-2)^2+1>0$ for all real values of *x*, <br> $(x-1)\le 0$ <br> $\therefore x\le 1$ |

**11** Solve the following inequalities algebraically

**(a)** $\dfrac{x-1}{3x+4}<0$,

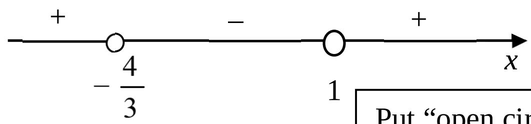
**(b)** $x+2\le\dfrac{30}{x+1}$.

**Q11**

**(a)**

$\dfrac{x-1}{3x+4}<0,\ x\neq-\dfrac{4}{3}$

Put "closed circles" for the roots for inequalities of the form: $\ge 0$ and $\le 0$
For this question, there are no "closed circles" since it is $< 0$




Put "open circles" to exclude values of $x$ where the denominator equals to 0.

$\therefore -\dfrac{4}{3}<x<1$

**(b)**

$x+2\le\dfrac{30}{x+1},\qquad x\neq-1$

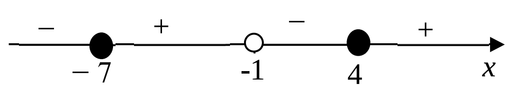
$x+2-\dfrac{30}{x+1}\le 0$

$\dfrac{(x+2)(x+1)-30}{x+1}\le 0$ Combine into one fraction

$\dfrac{x^2+3x-28}{x+1}\le 0$ Simplify/ factorise /completing the square

$\dfrac{(x+7)(x-4)}{x+1}\le 0$

Put "closed circles" for the roots for inequalities of the form: $\ge 0$ and $\le 0$



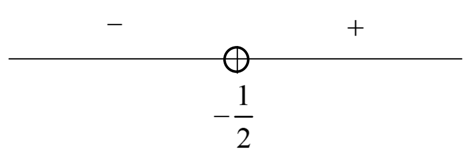

Put "open circles" to exclude values of $x$ where the denominator equals to 0.

Hence $\dfrac{(x+7)(x-4)}{x+1}\le 0 \quad \Leftrightarrow x\le-7$ or $-1<x\le 4$.

**12 2010/TPJC/I/4**

Given that $x$ is real, prove that $4x^2+2x+1$ is always positive.

Hence, without using a calculator, solve the inequality $2x+\dfrac{1}{1+2x}>0$. [4]

| **Q12** | **2010/TPJC/I/4** |
|---|---|
| | **<u>Method 1:</u> Completing the square**<br>$4x^2+2x+1=4\left(x^2+\frac{1}{2}x\right)+1$<br>$=4\left[\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2\right]+1$<br>$=4\left(x+\frac{1}{4}\right)^2+\frac{3}{4}$<br>$\therefore 4x^2+2x+1=4\left(x+\frac{1}{4}\right)^2+\frac{3}{4}>0$ for all real values of $x$. (Proved)<br>**<u>Method 2:</u>** Discriminant Method<br>Discriminant = $2^2-4(4)(1)=-12<0$<br>Coefficient of $x^2=4>0$<br>$\therefore$ Since coefficient of $x^2=4>0$ and discriminant $=-12<0$, $4x^2+2x+1>0$ for all real values of $x$. (Proved) |
| | $2x+\frac{1}{1+2x}>0 \quad , x\neq -\frac{1}{2}$<br>1) "Move" all expressions to one side.<br>2) Combine into one fraction<br>3) Factorize numerator and denominator<br>$\frac{4x^2+2x+1}{1+2x}>0$<br>Since $4x^2+2x+1>0$, $\therefore \frac{1}{1+2x}>0$<br>If a quadratic factor is **always positive**, you can **remove** the factor from the inequality.<br><br><br>Put "open circles" for<br>1) values of $x$ for which denominator = 0<br>2) if inequalities are **<u>not</u>** strict (no equal)<br>Choose solutions based on $\frac{1}{1+2x}>0$, ">" means choose "+" regions.<br>$\therefore x>-\frac{1}{2}$ |

**13 2002(9740)/I/4 (modified)**

Without using a calculator, find the solution set of the following inequalities.

**(a)** $x-\dfrac{1}{x}\le 1$, **(b)** $-1<\dfrac{2x+3}{x-1}<1$.

| Q13 | 2002(9740)/I/4 (modified) |
|---|---|

**(a)**

$$x-\frac{1}{x}\le 1,\ x\ne 0$$

$$x-\frac{1}{x}-1\le 0,\ \ x\ne 0$$

$$\frac{x^2-x-1}{x}\le 0$$

Check denominator and exclude the values from the solution. In this case, $x\ne 0$.
Bring all terms to one side and combine to a single fraction.
Note that we cannot multiply the inequality by $x$ since it is not always positive or always negative.

$$\frac{\left(x-\frac{1}{2}\right)^2-\frac{5}{4}}{x}\le 0$$

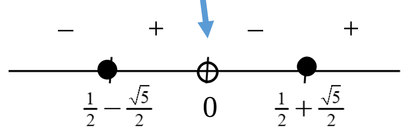
$$\frac{\left(\left(x-\frac{1}{2}\right)-\frac{\sqrt{5}}{2}\right)\left(\left(x-\frac{1}{2}\right)+\frac{\sqrt{5}}{2}\right)}{x}\le 0$$

$$\frac{\left(x-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{x}\le 0$$

Factorise numerator and denominator completely.

Identify the roots on the number line.

Open circle since $x\ne 0$




Closed circle since $x=\dfrac{1}{2}+\dfrac{\sqrt{5}}{2}$ and $x=\dfrac{1}{2}-\dfrac{\sqrt{5}}{2}$ results in $\dfrac{\left(x-\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{x}=0$.

Solution set $=\left\{x\in\mathbb{R}:\ x\le\dfrac{1}{2}-\dfrac{\sqrt{5}}{2}\ \text{ or }\ 0<x\le\dfrac{1}{2}+\dfrac{\sqrt{5}}{2}\right\}$

Remember to give answer in set notation since question requires 'solution set'.

**(b)**

$-1<\dfrac{2x+3}{x-1}<1, \qquad x\neq 1$

$-1<\dfrac{2x+3}{x-1}$ and $\dfrac{2x+3}{x-1}<1$

Split inequality into two and solve them separately.

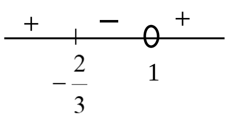
The solution for the original inequality is the intersection of the solution for both inequalities, hence 'and' is used.

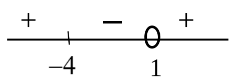
Check denominator and exclude the values from the solution. In this case, $x\neq 1$. Bring all terms to one side and combine to a single fraction. Note that we cannot multiply the inequality by $x-1$ since it is not always positive or always negative.

$\dfrac{2x+3}{x-1}+1>0$ and $\dfrac{2x+3}{x-1}-1<0$

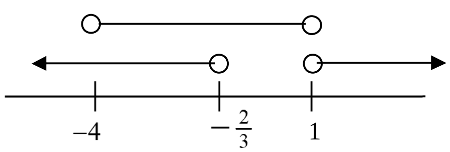
$\dfrac{2x+3+x-1}{x-1}>0$ and $\dfrac{2x+3-x+1}{x-1}<0$

$\dfrac{3x+2}{x-1}>0$ and $\dfrac{x+4}{x-1}<0$




$\therefore x<-\dfrac{2}{3}$ or $x>1$ and $\therefore -4<x<1$




Find the intersection of the two solution above to find the solution for the required inequality.

Solution set: $\left\{x\in\mathbb{R}:-4<x<-\dfrac{2}{3}\right\}$

Remember to give answer in set notation since question requires 'solution set'.

<u>**Alternative Method:**</u>

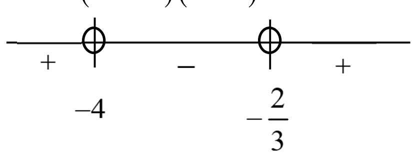
$$\left|\frac{2x+3}{x-1}\right|<1, \qquad x\neq 1$$

$$|2x+3|^2-|x-1|^2<0$$

$$(2x+3)^2-(x-1)^2<0$$

$$[(2x+3)+(x-1)][(2x+3)-(x-1)]<0$$

$$(3x+2)(x+4)<0$$




$\left\{x\in\mathbb{R}:-4<x<-\dfrac{2}{3}\right\}$

**14 2006(9740)/II/1**

Solve the inequality $\dfrac{x-9}{x^2-9} \le 1$.

Hence, solve the inequalities **(i)** $\dfrac{|x|-9}{x^2-9} \le 1$, **(ii)** $\dfrac{x+9}{x^2-9} \ge -1$.

| Q14 | 2006(9740)/II/1 |
|---|---|

$\dfrac{x-9}{x^2-9} \le 1 \quad x \ne \pm 3$

$\dfrac{x-9}{x^2-9} - 1 \le 0$

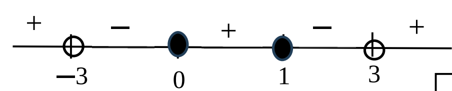
$\dfrac{x-9-\left(x^2-9\right)}{x^2-9} \le 0$

$\dfrac{-x^2+x}{x^2-9} \le 0$

$\dfrac{x(x-1)}{(x-3)(x+3)} \ge 0$

1) "Move" all expressions to one side.
2) Combine into one fraction
3) Factorize numerator and denominator

Remember to exclude values of $x$ for which denominator = 0.




Put "open circles" for
1) values of $x$ for which denominator = 0
2) if inequalities are **<u>not</u>** strict (no equal)

Choose solutions based on $\dfrac{x(x-1)}{(x-3)(x+3)} \ge 0$,
"≥" means choose "+" regions, including critical values.

$\therefore \quad x < -3 \text{ or } 0 \le x \le 1 \text{ or } x > 3$

**(i)**

Hence (Use results from previous parts)
<u>Aim:</u> Relate $\dfrac{|x|-9}{x^2-9} \le 1$ to $\dfrac{x-9}{x^2-9} \le 1$ by finding replacement for $x$.

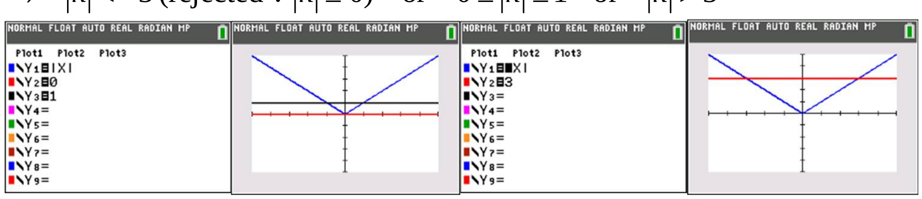
$\dfrac{|x|-9}{x^2-9} \le 1 \Rightarrow \dfrac{|x|-9}{|x|^2-9} \le 1$ as $|x|^2 = x^2$

$\therefore \dfrac{x-9}{x^2-9} \le 1 \Rightarrow \dfrac{\square-9}{\square^2-9} \le 1$, find the replacement.

$\therefore$ replacing $x$ by $|x|$ on $\dfrac{x-9}{x^2-9} \le 1$.

$\dfrac{|x|-9}{x^2-9} \le 1, \quad x \ne 3$

$\Rightarrow \quad |x| < -3 \text{ (rejected} \because |x| \ge 0) \quad \text{or} \quad 0 \le |x| \le 1 \quad \text{or} \quad |x| > 3$



$\Rightarrow \quad -1 \le x \le 1 \quad \text{or} \quad x < -3 \quad \text{or} \quad x > 3$

Use GC to solve.

**(ii)**

**Method 1:** Work towards expression similar to $\frac{x-9}{x^2-9} \le 1$

$\frac{x+9}{x^2-9} \ge -1$

$\frac{-(x+9)}{x^2-9} \le 1$

$\frac{(-x)-9}{(-x)^2-9} \le 1$

Ensure RHS is the same.

$\therefore \frac{x-9}{x^2-9} \le 1 \Rightarrow \frac{\square-9}{\square^2-9} \le 1$, find the replacement.

Hint: use coefficients as a guide.

Therefore, replacing $x$ with $-x$, we have

$\Rightarrow -x < -3 \quad \text{or} \quad 0 \le -x \le 1 \quad \text{or} \quad -x > 3$

$\Rightarrow x > 3 \quad \text{or} \quad -1 \le x \le 0 \quad \text{or} \quad x < -3$

**Method 2:** "Trial and Error" Method by observation (Might be difficult)

Replacing $x$ with $-x$ in $\frac{x+9}{x^2-9} \ge -1$, we have

$\frac{(-x)+9}{(-x)^2-9} \ge -1 \Rightarrow -\frac{x-9}{x^2-9} \ge -1 \Rightarrow \frac{x-9}{x^2-9} \le 1$ (same as previous part)

Therefore, replacing $x$ with $-x$, we have

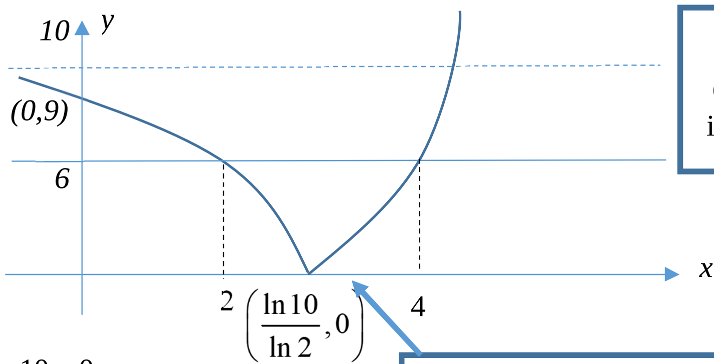
$\Rightarrow -x < -3 \quad \text{or} \quad 0 \le -x \le 1 \quad \text{or} \quad -x > 3$

$\Rightarrow x > 3 \quad \text{or} \quad -1 \le x \le 0 \quad \text{or} \quad x < -3$

**15 2019(9758)/I/4**

**(i)** Sketch the graph of $y=\left|2^x-10\right|$, giving the exact values of any points where the curve meets the axes. [3]

**(ii)** Without using a calculator, and showing all your working, find the exact interval, or intervals, for which $\left|2^x-10\right|\le 6$. Give your answer in its simplest form. [3]

| **Q15** | **2019(9758)/I/4** |
|---|---|
| **(i)** | <br>Check ASMILE! Good habit to label in coordinates form.<br>Show SHARP point!<br>$2^x-10=0$<br>$2^x=10$<br>$\ln 2^x=\ln 10$<br>$x=\dfrac{\ln 10}{\ln 2}$<br>Read the question carefully – EXACT! Show sufficient workings, but use GC to check your answer. |
| **(ii)** | $2^x-10=6$ OR $2^x-10=-6$<br>$2^x=16$ $\quad$ $2^x=4$<br>$2^x=2^4$ $\quad$ $2^x=2^2$<br>$x=4$ $\quad$ $x=2$<br>Solve the equality first to get the solutions. $\left\|2^x-10\right\|=6$<br>2 possible solutions because of the modulus sign. $\left\|2^x-10\right\|=6\Rightarrow 2^x-10=6$ or $2^x-10=-6$<br>$\left\|2^x-10\right\|\le 6$<br>$0\le\left\|2^x-10\right\|\le 6$<br>From part (i), $2\le x\le 4$<br>Refer to graph in (i) to find the correct region where *y*-values range from 0 to 6 inclusive. |

Read the question carefully – SIMPLEST FORM!
i.e. $\dfrac{\ln 4}{\ln 2}$ and $\dfrac{\ln 16}{\ln 2}$ not allowed.

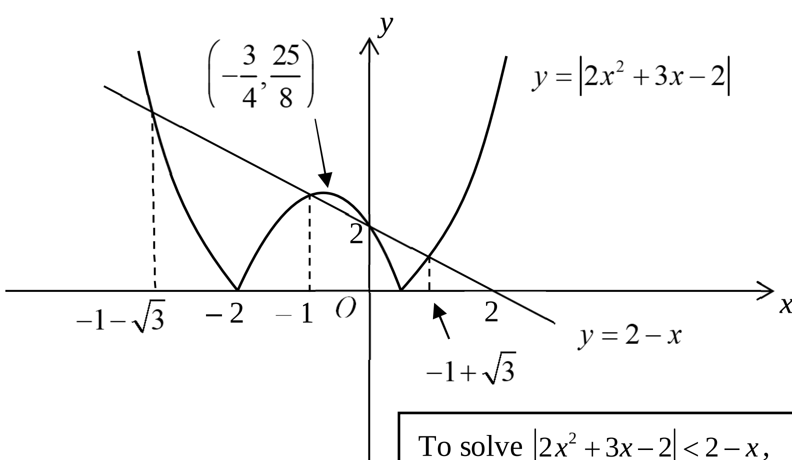
**16 2018(9758)/I/4**

**(i)** Find the exact roots of the equation $\left|2x^2+3x-2\right|=2-x$. [4]

**(ii)** On the same axes, sketch the curves with equations $y=\left|2x^2+3x-2\right|$ and $y=2-x$.

Hence solve exactly the inequality

$$\left|2x^2+3x-2\right|<2-x.$$ [4]

| **Q16** | **2018(9758)/I/4** |
|---|---|
| **(i)** | $\left\|2x^2+3x-2\right\|=2-x$<br>$\left\|x\right\|=a \Rightarrow x=a$ or $x=-a$<br>$2x^2+3x-2=2-x$ or $2x^2+3x-2=-(2-x)$<br>$2x^2+4x-4=0$ or $2x^2+2x=0$<br>**Exact** roots (No GC)<br>$x^2+2x-2=0$ or $x^2+x=0$<br>$x=\dfrac{-2\pm\sqrt{2^2-4(1)(-2)}}{2(1)}$ or $x(x+1)=0$<br>$x=-1\pm\sqrt{3}$ or $x=0$ or $x=-1$<br>$\therefore$ The roots are $-1-\sqrt{3}$, $-1$, 0 and $-1+\sqrt{3}$. |
| **(ii)** | <br><br>To solve $\left\|2x^2+3x-2\right\|<2-x$,<br>We need to find the values of $x$ such that the graph of $y=\left\|2x^2+3x-2\right\|$ is "below (<)" the graph of $y=2-x$.<br>From the graph,<br>$-1-\sqrt{3}<x<-1$ or $0<x<-1+\sqrt{3}$. |

## Level 3 Questions

**17 2009(9740)/I/1**

**(i)** The first three terms of a sequence are given by $u_1 = 10$, $u_2 = 6$, $u_3 = 5$. Given that $u_n$ is a quadratic polynomial in *n*, find $u_n$ in terms of *n*. [4]

**(ii)** Find the set of values of *n* for which $u_n$ is greater than 100. [2]

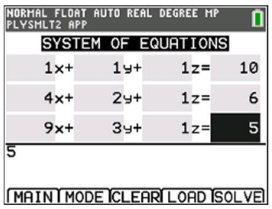
| **Q17** | **2009(9740)/I/1** |
|---|---|
| **(i)** | Let $u_n = An^2 + Bn + C$, where $A$, $B$ and $C$ are constants. |

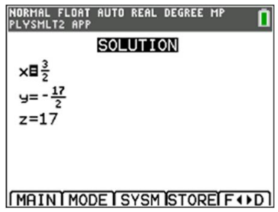
$u_1 = A + B + C = 10$

$u_2 = 4A + 2B + C = 6$

$u_3 = 9A + 3B + C = 5$

Be clear on how to write the general form for
- Quadratic Polynomial
- Cubic Polynomial
- Quartic Polynomial




Using GC,

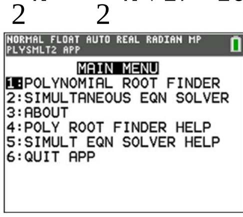
$$A = \frac{3}{2},\ B = -\frac{17}{2},\ C = 17$$

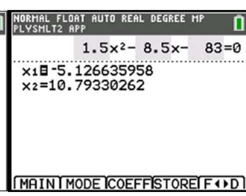
So, $u_n = \frac{3}{2}n^2 - \frac{17}{2}n + 17$

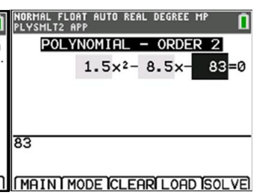
**(ii)**

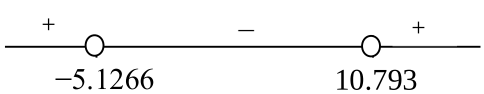
$$u_n = \frac{3}{2}n^2 - \frac{17}{2}n + 17 > 100$$

$$\frac{3}{2}n^2 - \frac{17}{2}n + 17 - 100 > 0$$




Positive integer

Using GC, $n > 10.793$ or $n < -5.1266$

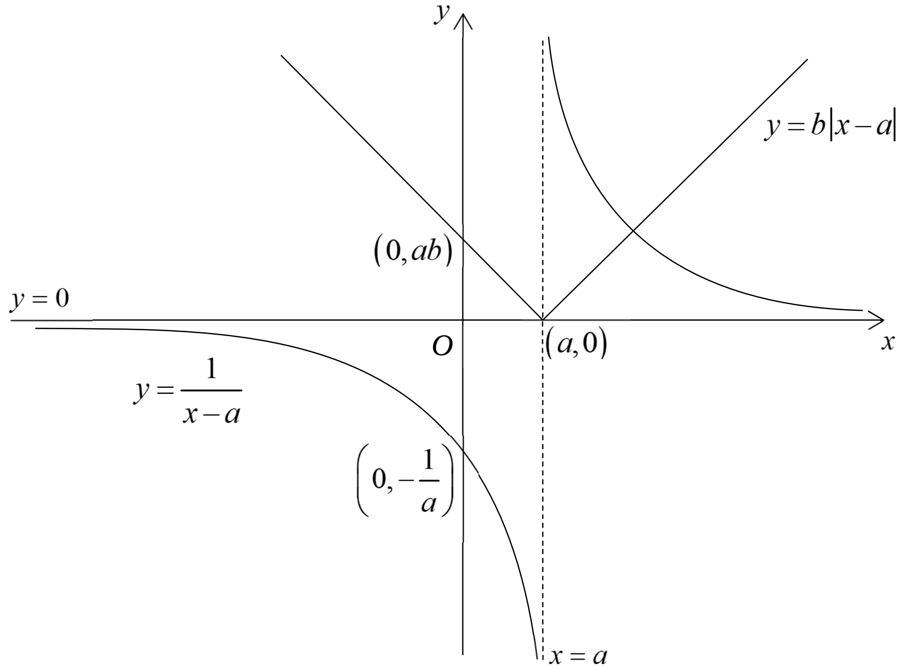
Since $n \in \mathbb{Z}^+$, the set of values of $n$ is $\{n \in \mathbb{Z}^+ : n \geq 11\}$.

Set of values

**18 2017(9758)/I/2**

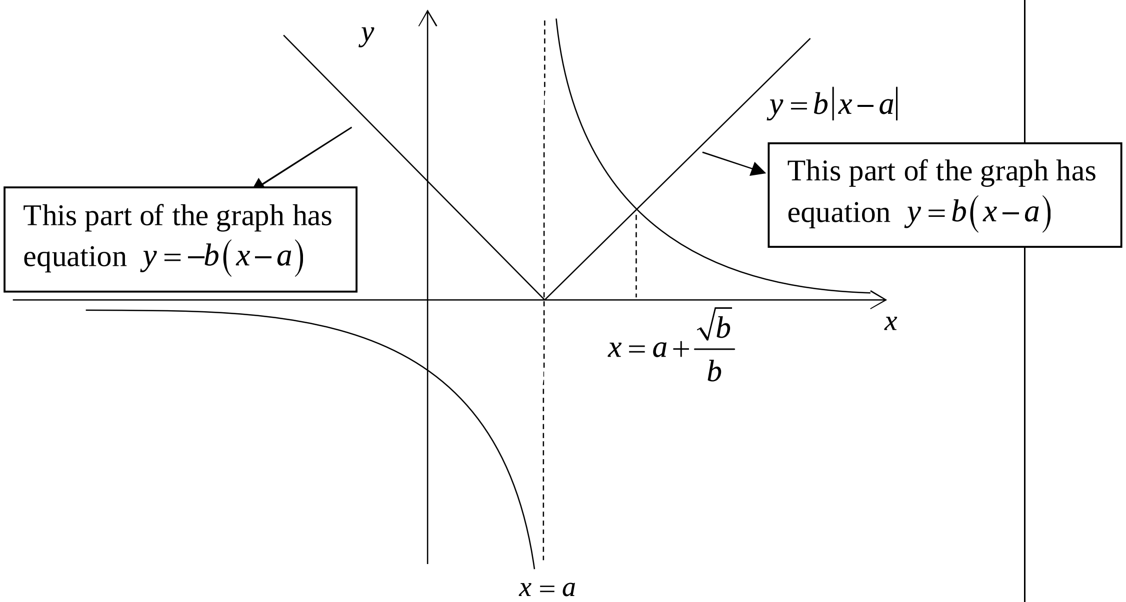
**(i)** On the same axes, sketch the graphs of $y = \dfrac{1}{x-a}$ and $y = b|x-a|$, where $a$ and $b$ are positive constants. [2]

**(ii)** Hence, or otherwise, solve the inequality $\dfrac{1}{x-a} < b|x-a|$. [4]

| **Q 18** | **2017(9758)/I/2** |
|---|---|
| **(i)** | <br>**Tips when sketching graphs with unknown constants:**<br>Given that *a* and *b* are positive constants, students are encouraged to input any positive real value of *a* and *b* into the GC to see the general shape of the graphs. Note that all axial intercepts and equation of asymptote must still be labelled with unknown constants *a* and *b.* |

**(ii)**

To solve $\dfrac{1}{x-a} < b|x-a|$, students are strongly encouraged to make use of the graphs sketched in part (i)




Recall:

$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

Therefore:

$$|x-a| = \begin{cases} x-a & \text{if } x \ge a \\ -(x-a) & \text{if } x < a \end{cases}$$

To find intersection between the 2 graphs,

$$\frac{1}{x-a} = b(x-a) \qquad (\text{since } x > a)$$

$$(x-a)^2 = \frac{1}{b}$$

$$x = a + \frac{\sqrt{b}}{b} \quad \text{or} \quad x = a - \frac{\sqrt{b}}{b} \quad (\text{rej. } \because x > a)$$

$$\therefore \frac{1}{x-a} < b|x-a|,$$

$$x < a \quad \text{or} \quad x > a + \frac{\sqrt{b}}{b}$$

**19 2018/AJC Promo/Q1**

A dietitian wishes to plan a meal using three types of ingredients. The meal is to include 8800 microgram (μg) of vitamin $A$, 3380 μg of vitamin $C$ and 1020 μg of calcium. The amount of the vitamins and calcium in each unit of the ingredients is summarised in the following table:

| | **Ingredient I** | **Ingredient II** | **Ingredient III** |
|---|---|---|---|
| **Vitamin *A*** | 400 | 1200 | 800 |
| **Vitamin *C*** | 110 | 570 | 340 |
| **Calcium** | 90 | 30 | 60 |

Assuming that all the 3 ingredients were used, find the three possible combinations of the number of integer units of each ingredient the dietitian should include in the meal in order to meet the vitamins and calcium requirements. [5]

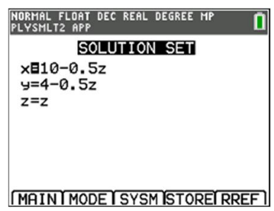
| **Q19** | **2018/AJC Promo/Q1** |
|---|---|

Let $x$, $y$ and $z$ be the number of units of Ingredients I, II and III used per meal, respectively.

$400x + 1200y + 800z = 8800$ ----(1)
$110x + 570y + 340z = 3380$ ----(2)
$90x + 30y + 60z = 1020$ ----(3)

1) Identify unknowns
2) Form equations with the unknowns.
(3 unknowns, 3 distinct equations)

From G.C.,




$x = 10 - \frac{1}{2}z$

$y = 4 - \frac{1}{2}z$

Since $x$, $y$, $z$ are positive integers,

$x = 10 - \frac{1}{2}z > 0 \Rightarrow z < 20$ and $y = 4 - \frac{1}{2}z > 0 \Rightarrow z < 8$

Since $z < 20$ and $z < 8$, this means that $z < 8$.

Since $x$, $y$, $z$ are positive **<u>integers,</u>**
and $z$ must be a multiple of 2 and at the same time less than 8.

$\therefore z = 2, 4$ or $6$

When $z = 2$, $x = 9$, $y = 3$.
When $z = 4$, $x = 8$, $y = 2$.
When $z = 6$, $x = 7$, $y = 1$.