2023 MI H2 Maths Pre-U 1 Exam Paper **Duration: 2 hr 15 mins** **Marks: 75**
Attempt all questions.

1 The sum, S_n, of the first n terms of the sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n = n^2 + 4n .$$

(i) Find u_n in terms of n. [2]

(ii) Hence show that the sequence is an arithmetic progression. [2]

2 A coffee vendor sells three types of coffee beverages: Latte, Cappuccino and Mocha. The cups of coffee come in three different sizes: Short, Tall and Grande. The number of cups for each type of coffee sold in a particular day is given in the following table.

	Short	Tall	Grande
Latte	3	12	8
Cappuccino	4	8	7
Mocha	2	5	4

The price of a cup of coffee for each size is the same regardless of the type of coffee. The total amount collected on that day from the sale of Latte, Cappuccino and Mocha is \$147.90, \$120.10 and \$70 respectively.

(i) Find the price of a short, tall and grande cup of coffee respectively. [3]

(ii) The vendor offers a buy-one-get-one-free promotion if a customer buys a grande cup of Latte and gives a 10% discount if a customer buys a short or tall cup of Latte. Andrew wishes to have 1 short cup, 4 tall cups, and 4 grande cups of Latte. How much does Andrew need to pay in total? [2]

3 **(i)** Sketch the curve C with equation $x^2 - (y+1)^2 = 1$, indicating clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) The curve C undergoes a sequence of three transformations as follows:

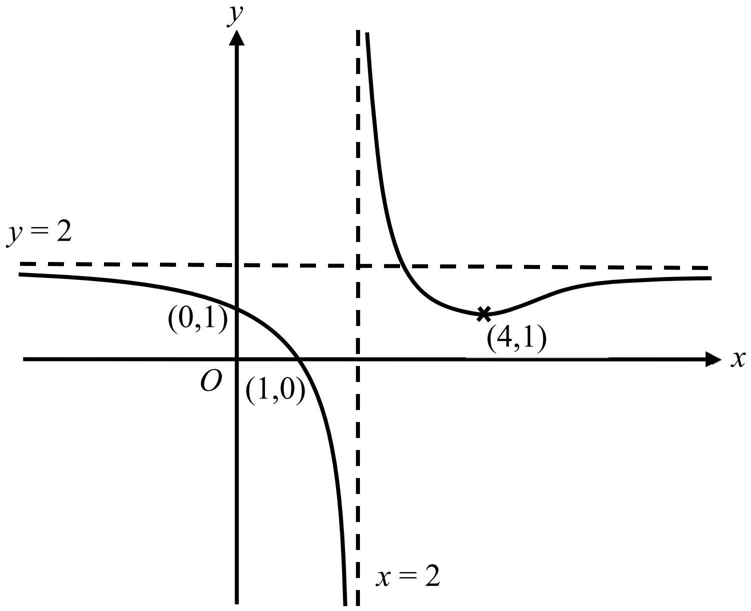
I: A scaling parallel to x-axis with a scale factor of 4,

II: A scaling parallel to y-axis with a scale factor of $\frac{1}{2}$,

III: A translation of 2 units in the positive y-direction.

Find the equation of the resulting curve. [3]

4 The diagram below shows the graph of $y = \mathrm{f}(x)$. The asymptotes of the graph are $x = 2$ and $y = 2$. The graph crosses the x-axis at $(1,\ 0)$ and the y-axis at $(0,\ 1)$, and has a turning point at $(4,\ 1)$.



On separate diagrams, sketch the following graphs, indicating clearly the coordinates of any points of intersections with both axes and any turning point(s), and the equations of any asymptotes where possible.

(i) $y = \mathrm{f}(|x|) + 1$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

5 With reference to the origin O, the position vectors of points A, B and C are expressed as $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively such that

$$\overrightarrow{OA} = (1-\lambda)\overrightarrow{OB} + \lambda\overrightarrow{OC} ,$$

where λ is a non-zero constant.

(i) Show that A, B, C are collinear. [2]

It is given that $\lambda = \dfrac{1}{6}$, $\overrightarrow{OA} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$.

(ii) Find the position vector of the point D that lies on the line segment AC such that $AD : DC = 1 : 4$. [3]

It is given that the point E has coordinates $(1, -1, 2)$.

(iii) Find the exact area of the triangle ABE. [3]

(iv) Find the length of projection of $\overrightarrow{OE}$ onto $\overrightarrow{AB}$. [2]

6 **(i)** Without using a calculator, solve the inequality

$$\frac{3x^2+9x-8}{x-1}<2\,. \qquad [4]$$

Hence solve the following inequalities

(ii) $\dfrac{3e^{2x}+9e^{x}-8}{e^{x}-1}<2$, [2]

(iii) $\dfrac{3+9x-8x^2}{x-x^2}<2$. [3]

7 Relative to the origin *O*, the point *A* has position vector $3\mathbf{i}+7\mathbf{j}+\mathbf{k}$. The plane π contains *A* and is parallel to the vector $-2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and the line $\mathbf{r}=\alpha(-\mathbf{i}+2\mathbf{j}+3\mathbf{k})$, where $\alpha\in\mathbb{R}$.

(i) Show that the cartesian equation of plane π can be expressed as $4x+5y-2z=45$.

[3]

The point *B* has coordinates $(2,\ 0,\ 4)$.

(ii) Find the position vector of the point *F*, the foot of the perpendicular from *B* to the plane π. [3]

(iii) Hence find the position vector of the point B', the reflection of *B* in the plane π. [2]

(iv) Find angle BAB'. [3]

8 The functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2-2x+5 \quad \text{for } x\in\mathbb{R},$$

$$\mathrm{g}: x \mapsto \frac{1}{x-1} \quad \text{for } x\in\mathbb{R},\ x\neq 1.$$

(i) Show that the composite function fg exists. [2]

(ii) Find an expression for $\mathrm{fg}(x)$ and state its domain. (There is no need to express $\mathrm{fg}(x)$ as a single algebraic fraction.) [2]

(iii) Find the range of fg. [2]

For the rest of the question, the domain of f is now restricted to $(-\infty,\ 1]$.

(iv) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(v) Find the range of values of *x* that satisfies the equation $\mathrm{ff}^{-1}(x)=x$. [1]

(vi) It is given that $\mathrm{g}^{-1}(a)=b$, where *a* and *b* are constants and $a\neq 0,\ b\neq 1$. Without finding an expression for $\mathrm{g}^{-1}(x)$, find *a* in terms of *b*. [2]

9 In a computer simulation, each player uses an identical pail to pour water repeatedly into his assigned tank, which is initially empty. Two players, Chris and Elliot, participate in this simulation and pour water into their assigned tanks as follows.

- Chris pours 5 litres of water into the tank using the pail for the first time. Subsequently, he pours 0.3 litres less than the preceding volume of water poured.

- Elliot pours 5 litres of water into the tank using the pail for the first time. Subsequently, he pours 85% of the preceding volume of water poured.

The simulation stops when the amount of water that can be poured from the pail by either player is no longer positive.

(i) Determine the number of times Chris can pour water into his tank before the simulation stops. Hence find the volume of water in the tank at this instant. [4]

The simulation also stops when any player reaches or exceeds the total target volume of water in their respective tanks. The total target volume of water in each tank is given to be 33.5 litres.

(ii) Elliot comments that the target is unfair because he will not reach it. Explain if Elliot's comment is justified. [2]

The total target volume of water in each tank is now changed to 30 litres.

Chris and Elliot start the simulation at the same time. The simulation is designed such that all players pour water into their assigned tanks at the same time each time.

(iii) Determine which player reaches or exceeds the total target volume of water in the tank first, showing your workings clearly. [4]

(iv) Suppose that Gary is a new player. He pours 5 litres into his assigned tank using the pail the first time. Subsequently, he pours r% of the preceding volume of water poured. Find the smallest value of r such that Gary reaches or exceeds the total target volume of water of 30 litres in the tank after pouring for 7 times. Leave your answer to the nearest integer. [2]

End of Paper