NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME	

CT CLASS	1	9			Centre Number/ Index Number					/				

MATHEMATICS **9758/01**

Paper 1 **28 August 2020**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For examiner's use only	
Question number	Marks
1	
2	
3	
4	
5	
6	
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8	
9	
10	
11	
12	
Total	

This document consists of **5** printed pages and **0** blank page.

1 A household's monthly utility bill is calculated by summing up the charges it incurs for its usage of electricity, gas and water based on the prevailing price rate per unit. The table below shows Mrs Ang's household utilities usage for the months of April, May, June and July in a particular year.

	Monthly usage (units)			
	April	**May**	**June**	**July**
Electricity	23	33	49	35
Gas	12	16	22	17
Water	16	21	33	26

The prevailing price rate per unit were unchanged for the period of April to June. However, in the month of July, the electricity rate was increased by 7%, the gas rate was reduced by 5% and the water rate remained unchanged. Given that Mrs Ang's monthly utility bills for April, May and June were \$103.00, \$142.00 and \$209.00 respectively, find her monthly utility bill for the month of July. [4]

2 **(i)** Sketch the graph of $y=\dfrac{7-2x}{3x^2+x-14}$. Give the equations of the asymptotes and the coordinates of the turning points and the point(s) where the curve crosses the axes. [4]

(ii) Find the range of values of k such that $\dfrac{7-2x}{3x^2+x-14}=k$ has only positive root(s). [1]

3 A curve C has parametric equations

$$x=t-\frac{a}{t},\quad y=t+\frac{a}{t}, \text{ where } t\in\mathbb{R},\ t\neq 0 \text{ and } a \text{ is a positive constant.}$$

(i) Find a cartesian equation of C. [2]

(ii) The point P on C has parameter p. Show that the equation of the tangent at P can be expressed in the form $y\left(p^2+a\right)-x\left(p^2-a\right)=kap$, where k is a constant to be determined. [4]

4 **(a)** By writing $\sin^3 x$ as $\sin^2 x(\sin x)$, find $\int \sin^3 x\cos^3 x\ \mathrm{d}x$. [3]

(b) Find $\displaystyle\int \frac{5-6x}{\sqrt{5-3x^2+2x}}\ \mathrm{d}x$. [4]

5 **(i)** On the same axes, sketch the curves with equations $y=\left|\dfrac{ax+1}{1-ax}\right|$ and $y=\dfrac{1}{a}x+1$, where $a>1$, giving the equations of the asymptotes and the coordinates of the points where the curves meet the axes. [3]

(ii) Solve the inequality $\left|\dfrac{ax+1}{1-ax}\right|>\dfrac{1}{a}x+1$, giving your answers in terms of a. [4]

6 The tangent at the point (x, y) on the curve with equation $y = \mathrm{f}(x)$ passes through the point (a, x), where a is a positive constant and $x < a$.

(i) Write down a differential equation relating $\frac{\mathrm{d}y}{\mathrm{d}x}$, a, x and y. [1]

(ii) Using the substitution $y = (a - x)z$, show that the differential equation can be transformed to $\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{x}{(a-x)^2}$. By using partial fractions, find the general solution for y in terms of a and x. [5]

(iii) Given that the curve $y = \mathrm{f}(x)$ passes through the origin, show that $\mathrm{f}(x) = (a - x)\ln\left(\frac{a-x}{a}\right) + x$. [2]

7 A sequence of numbers u_1, u_2, u_3, … has a sum S_n where $S_n = \sum_{r=1}^{n} u_r$. It is given that $S_n = An^2(n+1) - Bn$, where A and B are non-zero constants.

(i) Find a simplified expression for u_n in terms of A, B and n. [2]

(ii) It is also given that the first and second term of the sequence are 1 and 9 respectively. Show that $A = 1$ and $B = 1$. [3]

(iii) Hence, find $\sum_{r=1}^{n}\left[3(r-1)^2 - r\right]$, giving your answer in the form $an^3 - bn^2$, where a and b are constants to be determined . [3]

8 You are given that $C = \int_0^{\pi} \mathrm{e}^{-2x}\cos x \,\mathrm{d}x$ and $S = \int_0^{\pi} \mathrm{e}^{-2x}\sin x \,\mathrm{d}x$. Use integration by parts to show $C = 2S$ and $S = -2C + 1 + \mathrm{e}^{-2\pi}$.

Hence, find the exact values of C and S. [6]

The finite region bounded by the axes and the curve $y = \mathrm{e}^{-x}\cos\left(\frac{1}{2}x\right)$, for $0 \le x \le \pi$, is rotated through four right angles about the x-axis. Find the exact volume of the solid of revolution. [4]

9 **(i)** Using standard series from the List of Formula (MF26), expand $\ln(k+2x)^n$ in ascending powers of x, as far as the term in x^3, where k and n are real constants, $k > 0$, $n < 0$ and $x > -\frac{k}{2}$. [2]

(ii) State the range of values of x for which the expansion in part **(i)** is valid. [1]

(iii) Find the exact value of $\int_0^1 \ln(1+2x)^{-2}\,\mathrm{d}x$. [3]

(iv) Use your series from part **(i)** to estimate $\int_0^1 \ln(1+2x)^{-2}\,\mathrm{d}x$. [2]

(v) Calculate the percentage error of the estimate in part **(iv)**, and comment on the accuracy of your approximation using your answer from part **(ii)**. [2]

10 Referred to the origin O, the points P and Q have position vectors $4\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ and $8\mathbf{i} + 8\mathbf{j} - \mathbf{k}$ respectively.

(i) Find the position vector of the point R where the line PQ meets the x-y plane. [3]

(ii) The plane Π contains R and is perpendicular to PQ. Show that a cartesian equation of Π is

$$x + y - z = c,$$

where c is a constant to be determined. [2]

(iii) Find the acute angle between Π and the x-y plane. [2]

(iv) Point A has position vector $\mathbf{i} + \mathbf{j}$. Show that $\overrightarrow{AR}$ is perpendicular to the line of intersection of Π and the x-y plane. [2]

(v) Find the exact shortest distance from A to Π . [2]

11 A farmer has a plot of the land, OAB, with adjacent sides OA and OB. With reference to O as the origin, OA and OB are parallel to the x-axis and y-axis respectively. The arc AB is decribed by the parametric equations

$$x = \cos\theta, \;\; y = \theta - \frac{1}{2}\sin 2\theta, \;\; 0 \le \theta \le \frac{\pi}{2}.$$

(i) Show that the area of the plot of land is $\frac{2}{3}$ units^2. [4]

Due to a strong worldwide demand for latex, the farmer plans to use a portion of his land to grow rubber trees. He divides his land into three portions. One portion is a rectangle with vertices at O and along arc AB, and sides parallel to OA and OB. He intends to use only this rectangular plot of land which has an area of K units^2 to plant rubber trees.

(ii) By using differentiation, find the largest value of K. [4]

In the midst of his planning, the futures contract for latex shot to historical high. To maximise his returns, the farmer decides to redistribute his plot of land to plant more rubber trees. He divides his plot of land into two portions using the line $y = 2x + 0.1$ and allocates the larger plot to plant rubber trees.

(iii) Find the area of this larger plot of land meant for rubber trees. [4]

12 The reproduction number, R_0, of a disease is a mathematical term that indicates how contagious an infectious disease is. It gives the average number of people who will contract a contagious disease from a person with that disease. That replication will continue if no one has been vaccinated against the disease or is already immune to it in his or her community.

The infectious period of a disease is the time period during which an infected person can transmit the disease to any susceptible person he/she contacts. For example, if a disease has an R_0 of 3, a person who has the disease will transmit it to an average of 3 other people during the infectious period. Given that the infectious period is **2 weeks** and initially x people were infected, the average number of people infected is $3x$ in week 2, 3^2x in week 4, 3^3x in week 6, and so on.

In a large population of people with no immunity to a virus, initially **one** person was infected with the virus. It is given that this virus has an R_0 of 2.5 and the infectious period is **2 weeks**.

(i) Show that the least number of weeks it would take for the virus to infect 1000 people is 16. [2]

(ii) Determine the least number of weeks it would take for the virus to infect the next 1000 people. [2]

(iii) A proposed model claimed the number of people infected, u_n, at the end of the *n*th week is given by $u_n = u_{n-1} + 2^n + n - 1$. By letting $u_0 = 1$, and considering $u_n - u_{n-1}$, find in terms of *n*, the number of people infected at the end of the *n*th week. [4]

The health authority took immediate action when they learnt of the spread of the virus through an index case and began selective testing of the population. Initially they were only able to conduct 300 tests a day. They were able to ramp up the number of daily tests by 100 in every 7 days (1 week). In this case, 300 tests were conducted daily in week 1, 400 tests were conducted daily in week 2, and so on.

(iv) Determine the least number of weeks it would take to test 200 000 people. [2]

An experimental vaccine is found to be effective against the virus and the whole population is to be inoculated with the vaccine.

(v) It is suggested that initially 10% of the population would be inoculated with the vaccine and then every week, the number of people inoculated would be 90% of the previous week. Comment on the feasibility of this proposal, justifying your answer. [2]