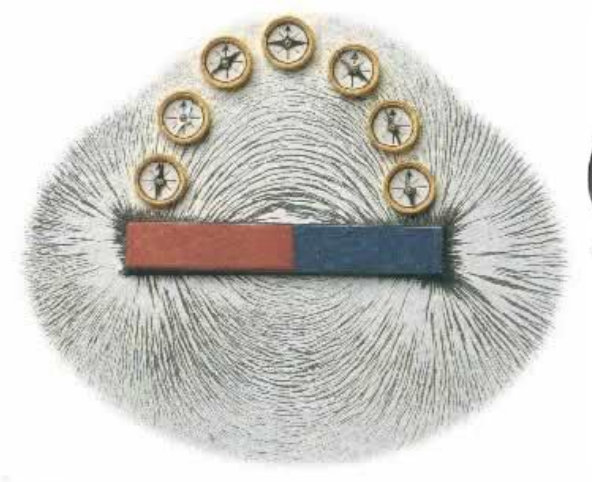
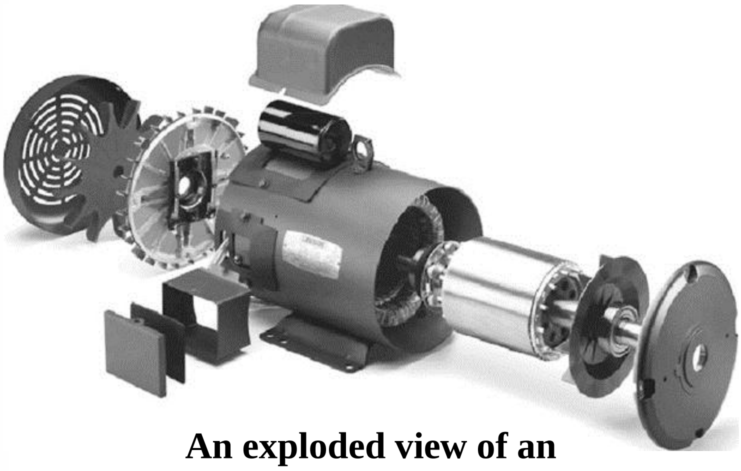
H2 Topic 15

Electromagnetism



Magnetic field lines showed by iron fillings



An exploded view of an electric motor

Large Hadron Collider (LHC) Tunnel:
A 17 mile long tunnel underlying the border between Switzerland and France

Learning Objectives

Content

- Force on a current-carrying conductor
- Force on a moving charge
- Magnetic fields due to currents
- Force between current-carrying conductors

Learning Outcomes

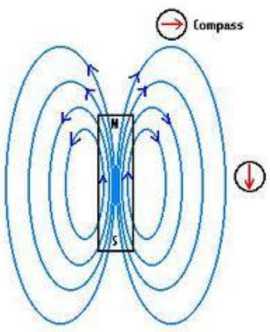
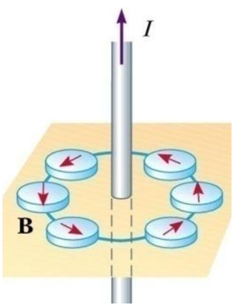
Candidates should be able to

a. Show an appreciation that a force might act on a current-carrying conductor placed in a magnetic field.
b. Recall and solve problems by using the equation $F = BIL \sin \theta$, with directions as interpreted by Fleming's left-hand rule.
c. Define magnetic flux density and the tesla.
d. Show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field, using a current balance.
e. Predict the direction of the force on a charge moving in a magnetic field.
f. Recall and solve problems by using $F = B q v \sin \theta$.
g. Describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields.
h. Explain how electric and magnetic fields can be used in velocity selection for charged particles.
i. Sketch flux patterns due to a long straight wire, a flat circular coil and a long solenoid.
j. Show an understanding that the field due to a solenoid may be influenced by the presence of a ferrous core.
k. Explain the forces between current-carrying conductors and predict the directions of the forces.

15.1 Magnetic Fields

Besides the gravitational field and the electric field, another field of force in nature is the magnetic field. A ***magnetic field*** is a region of space where a magnetic pole, a current-carrying conductor or a moving charge particle will experience a force.

A magnetic field can be produced by:

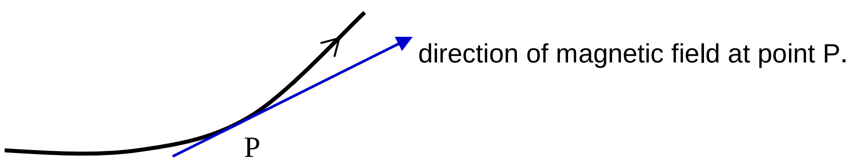
(i) *Permanent magnets*	(ii) *Current-carrying conductors*
 The magnetic field around a permanent magnet can be seen by sprinkling some iron filings around it.	 In 1820, the Danish physicist, Hans Christian Oersted first discovered that magnetic fields are produced by electric currents. It is the *motion of the electric charge* in the conductor that produces the magnetic field.

15.1.1 Magnetic Field Lines

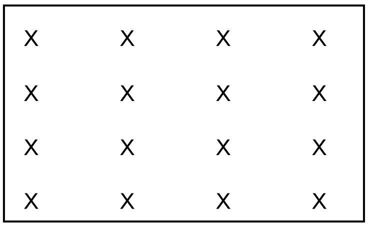
A magnetic field may be represented by a series of lines called *magnetic field lines*, each having a specific direction. The direction of a magnetic field line at any point in the field shows the direction of the force that a ‘free’ magnetic *north* pole would experience at that point.

The following are characteristics of magnetic field lines:

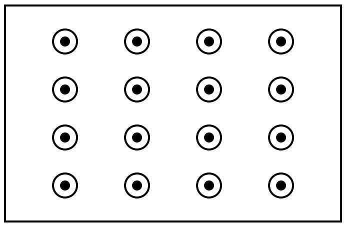
i. They are imaginary.
ii. By convention, magnetic field lines leave the north pole and enter the south pole of a magnet. Note that the field lines do not start or end at the north or south poles of a magnet, they continue to go from south pole to the north pole in the magnet.
iii. Do not touch or intersect one another.
iv. Can be straight lines or curves. The tangent to a curved field line at a point indicates the direction of the magnetic field at that point. This tangent also indicates the direction of the magnetic flux density B at that point.



v. They are represented by crosses or dotted circles in a 2-D plan view.



Magnetic field pointing into the paper.

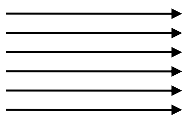


Magnetic field pointing out of the paper.

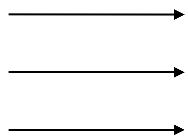
vi. If the lines are parallel and evenly spaced, the magnetic field is uniform. Otherwise, the field will vary in strength from one point to another.

vii. The closeness of the lines indicates the strength of the field; the field is said to be strong if the lines are crowded very closely together and weak when they are widely separated from one another.

Uniform Field



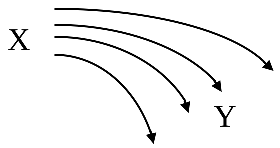
Strong Field



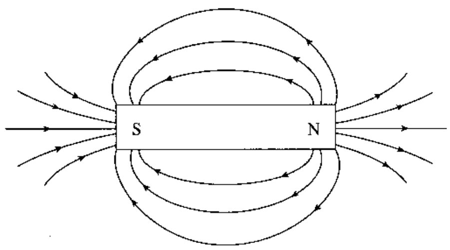
Weak Field

Non-Uniform Field

A non-uniform field



The field is stronger at X than at Y



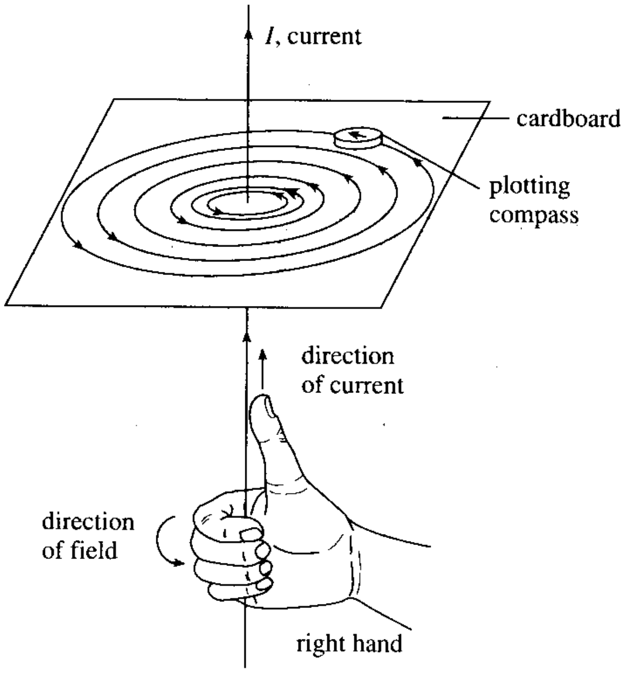
Magnet field around a bar magnet

15.2 Magnetic Flux Pattern

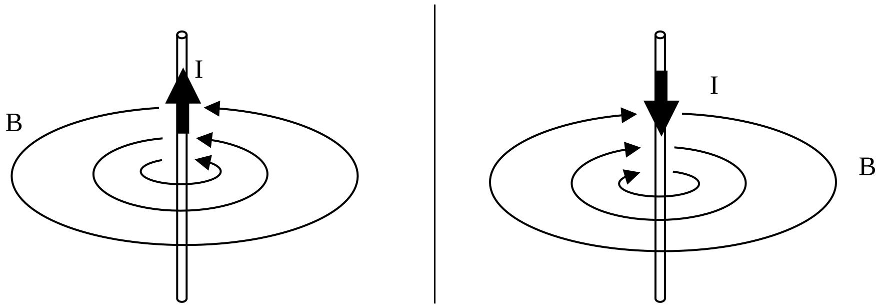
A conductor carrying an electric current produces a magnetic field. The pattern formed by the field lines is known as a *magnetic flux pattern*. Field patterns and the expressions for the associated magnetic flux densities for different current- carrying conductors are given below:

15.2.1 Magnetic Field Generated by a Long Straight Wire

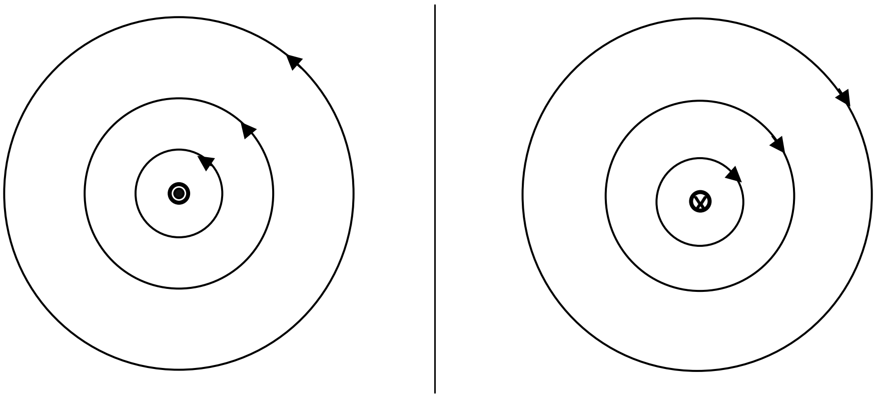
When a long straight wire carries a current, a magnetic field is generated around the wire in concentric circles as shown.



Isometric View



Plan view



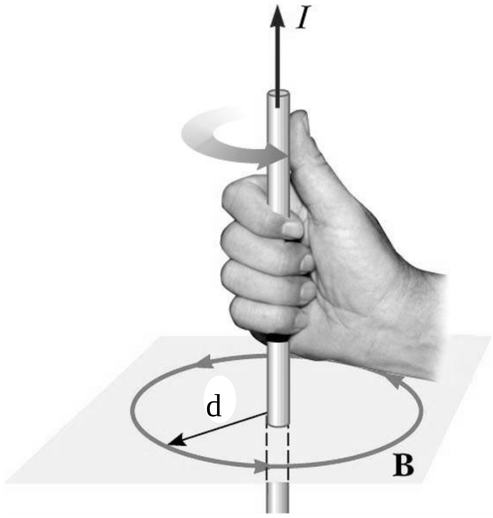
The direction of the magnetic field lines can be determined by the *right-hand grip rule* as illustrated in the diagram; when the thumb of the right hand points in the direction of the current, the fingers indicate the direction of the magnetic field lines.

The distance between successive circles increases as one move outwards from the wire; this indicates that the field is weaker as one moves away from the wire.

The magnetic flux density *B* at any point which is at a perpendicular distance *d* from the wire carrying current I is given by:

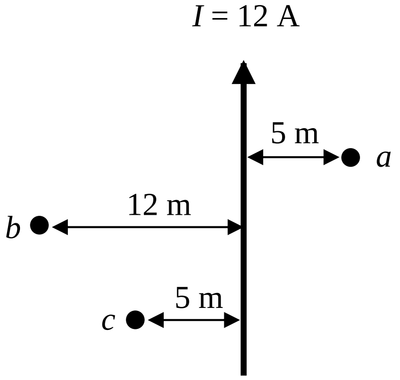
$$B = \frac{\mu_o I}{2\pi d}$$

where μ_o = permeability of free space (vacuum) = $4\pi \times 10^{-7}$ H m^{-1}



Example 1

The figures below shows 3 points close to a wire carrying current of 12 A. For point *a*, calculate the magnetic flux density and state whether the magnetic field points into or out of the plane of the page. Without calculation, compare the magnitude of the magnetic flux density at the three points.



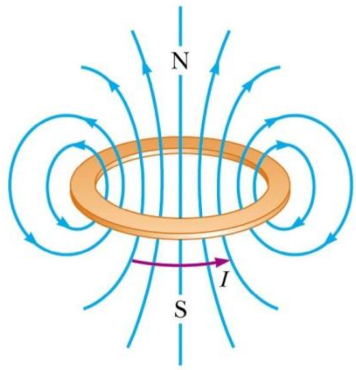
Using $B = \frac{\mu_0 I}{2\pi d}$, $B_a = \frac{4\pi \times 10^{-7} \times 12}{2\pi(5)}$ = 4.8 x 10^{-7} T

Using Right Hand Grip Rule, B_a points into the page.

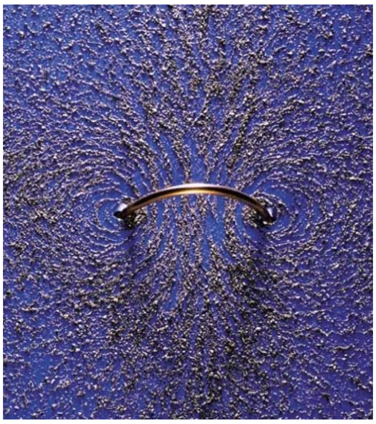
$B_a = B_c > B_b$

15.2.2 Magnetic Field Generated by a Flat Circular Coil

When a current passes through a flat circular coil, the magnetic flux pattern of the generated field is as shown below.

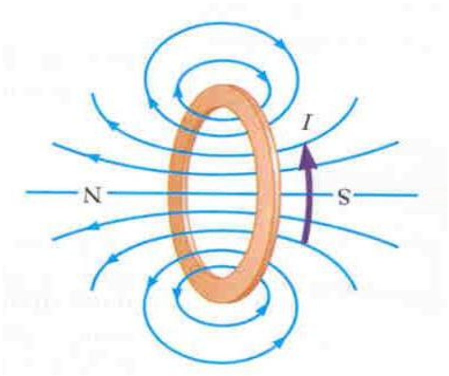


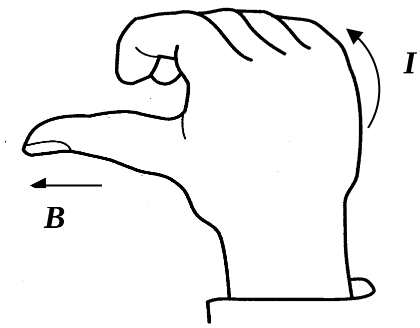
Magnetic field lines around a circular coil

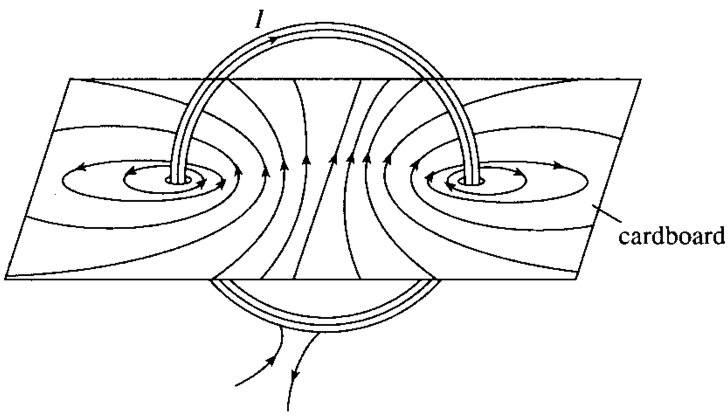


Iron filings sprinkled around a circular coil.

Field lines at the centre of the coil are straight and perpendicular to the plane of the coil. The direction of the field lines is given by the right-hand grip rule, which in this case the direction of the current is represented by the curled fingers and the thumb indicates the direction of the magnetic field acting through the centre of the coil.

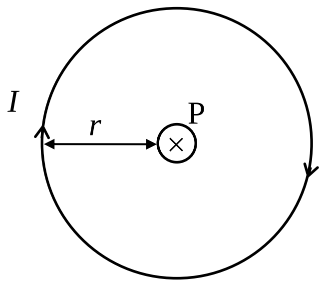






The magnetic flux density at the centre, P, for a current-carrying flat circular coil of N turns and radius r is:

$$B = \frac{\mu_0 NI}{2r}$$

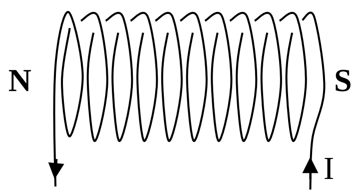


15.2.3 Magnetic Field Generated by a Solenoid

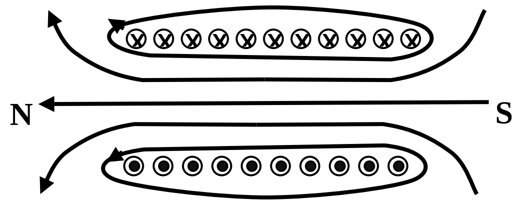
If a long straight wire is bent into a coil of several closely spaced loops, the resulting device is called a *solenoid*. A solenoid is also known as an electromagnet, because it acts like a magnet only when a current passes through its wire.

Field lines within a solenoid are straight and parallel to the axis of the coil, indicating that the field is uniform. The flux pattern outside the solenoid is similar to that of a bar magnet.

For a current carrying solenoid, the magnetic flux pattern may be illustrated as shown.



Current flow through a solenoid.



Magnetic flux around a solenoid. (Plan View)

Using the right hand grip rule, the direction of the current is represented by the curled fingers and the thumb indicates the direction of the magnetic field through the centre of the solenoid

The strength of the generated magnetic field can be increased (by a factor of 3) through adding a ferrous (iron) core inside the solenoid. This is because a ferrous material has a higher permeability than air. An alternative explanation is that iron, being a ferromagnetic material, becomes magnetized when placed into the solenoid, thus contributing to the overall magnetic field strength of the solenoid.

When a current flows in a solenoid, the magnetic field pattern outside the solenoid is identical to that of a bar magnet. Hence the solenoid has poles just like a bar magnet. To identify the pole at each end of the solenoid, we recall that the field lines of a magnet exits from the north pole and enters into the south pole.

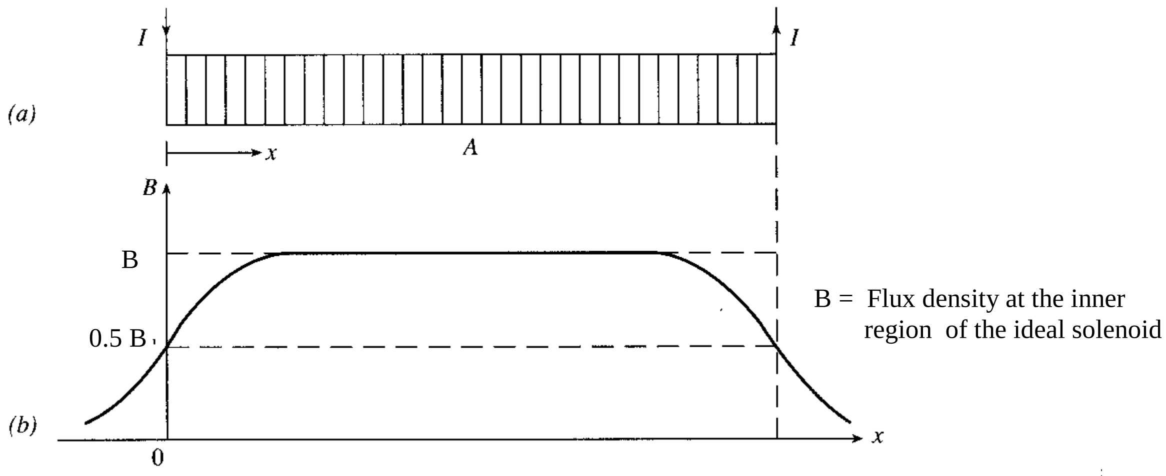
- The magnetic flux density <u>within the central region</u> of an infinitely long solenoid is:

$$B = \mu_0 n I$$

where n = number of turns per unit length

- The magnetic flux density at <u>either end</u> of a solenoid is:

$$B = \frac{1}{2}\mu_0 n I$$



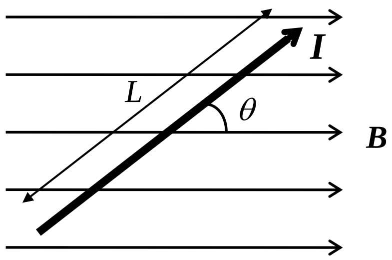
Note:

1. For an ideal solenoid where the loops are closely spaced and the length is much greater than the radius of the loops, the *B*-field is uniform over a great volume in the whole inner region, and the *B*-field is almost zero outside the solenoid, except near the ends.

2. The *B*-field can be greatly increased by the insertion of a ferromagnetic material such as iron, cobalt or nickel into its core.

15.3 Force Acting on a Current-Carrying Conductor

Consider a metallic conductor carrying a current I and inclined at an angle θ to a magnetic field *B*. The conductor will experience a force into the plane of the paper.



Experiments show that the magnitude of the force is given by:

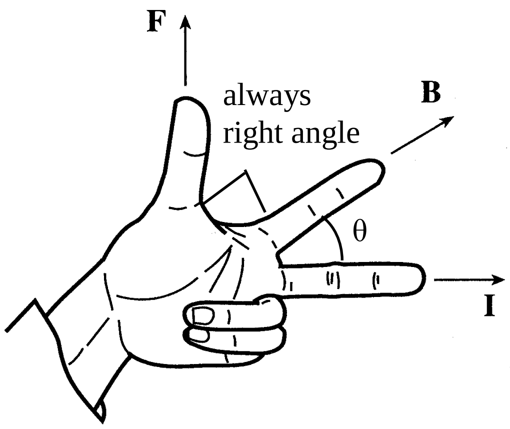
$$\boxed{F = BIL\sin\theta}$$

where *F* is the force acting on a conductor of length *L,* carrying a current I , inclined at an angle θ to a magnetic field *B*.

It should be noted that magnetic flux density is a vector; the resultant magnetic flux density at a point must be calculated as a vector sum of all the magnetic flux densities due to various sources acting at that point.

15.3.1 Direction of Magnetic Force

The direction of the force is always perpendicular to the plane containing both the current and the magnetic field. The direction of the force can be found by using *Fleming's left-hand rule* as shown in the diagram.



15.3.2 Definition of Magnetic Flux Density and the Tesla

From the above relation, we have $B = \dfrac{F}{IL\sin\theta}$.

If $\theta = 90^o$, then $B = \dfrac{F}{IL}$.

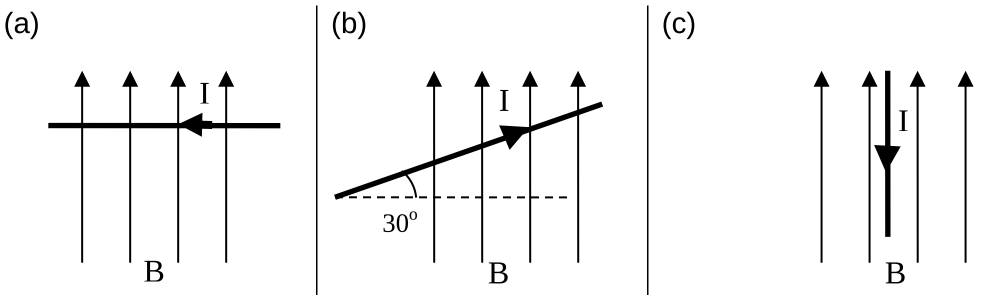
Nikola Tesla
(1856 – 1943)

> ➢ The **magnetic flux density** of a magnetic field is defined as the force per unit length per unit current acting on a straight conductor placed normal to the field.
>
> ➢ **Tesla** is the SI unit of magnetic flux density. If a long straight conductor carrying a current of one ampere is placed normal to a uniform magnetic field of flux density one tesla, then the force per unit length on the conductor is one newton per metre.

- A field of about 0.1 T can be achieved with big magnets.
- With electromagnets, fields of about 1 T can be attained.
- Earth's magnetic field is about 50 μT.
- Gigantic field values of up to 20 T can be found in many stars and in the interiors of many kinds of atoms.

Example 2

The 3 diagrams below each show a magnetic field of flux density 2 T that lies in the plane of the page. In each case, a 0.5 m length of wire carrying a 10 A current is placed in the field as shown. For each case, state the direction of the force acting on the wire and determine its magnitude.



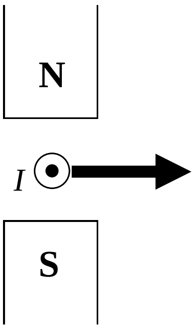
(a)
$F = BIL\sin\theta$
$= 2\times10\times0.5\times\sin 90^o$
$= 10$ N
(Into the Page)

(b)
$F = BIL\sin\theta$
$= 2\times10\times0.5\times\sin 60^o$
$= 8.7$ N
(Out of the Page)

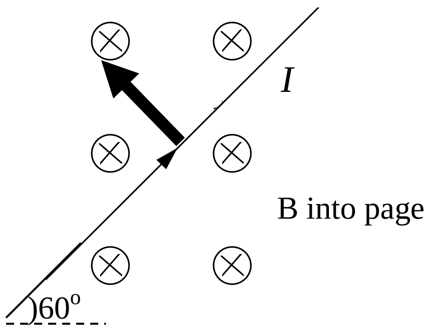
(c)
$F = BIL\sin\theta$
$= 2\times10\times0.5\times\sin 180^o$
$= 0$ N

Example 3

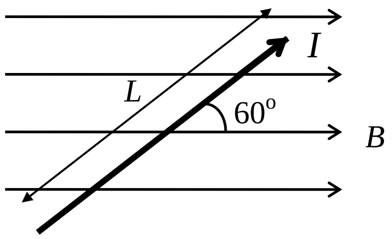
Determine the force acting on a conductor of length 1 m carrying a current of 2 A in a magnetic field of flux density 3 T in each of the three cases below:



case (a)



case (b) [wire on plane of page]



case (c)

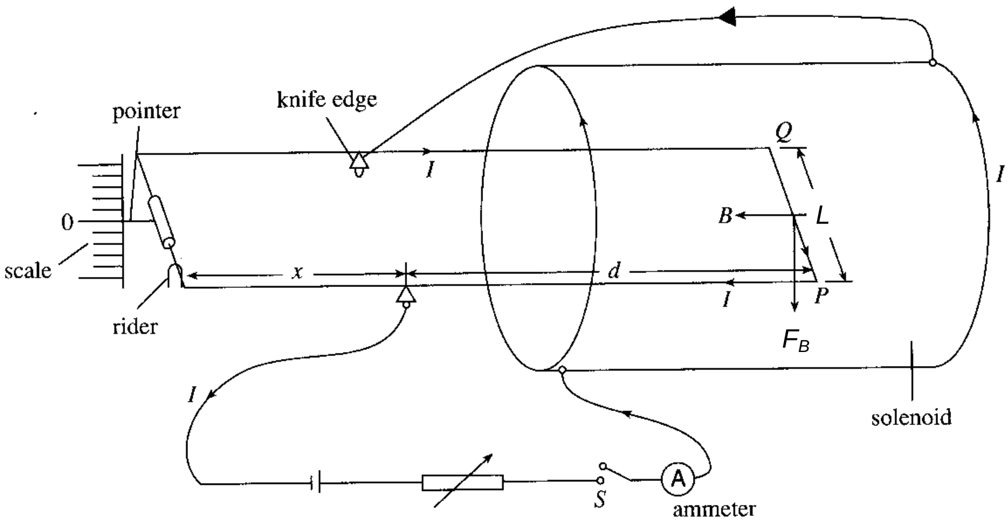
For case (a) and case (b), the force F has the same magnitude:

$F = BIL \sin\theta$ = 3 x 2 x 1 x sin 90° = 6 N (Direction of forces are indicated in bold arrows)

For case (c):

$F = BIL \sin\theta$ = 3 x 2 x 1 x sin 60° = 5.2 N (Direction of force is INTO the page)

15.3.3 Measurement of Magnetic Flux Density via a Current Balance



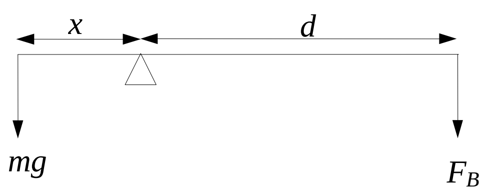
The diagram above shows a simple form of current balance. A rectangular copper loop, freely pivoted on the knife edges, is placed such that the side PQ is perpendicular to the magnetic field in the solenoid whose flux density is to be measured. The knife edges are adjusted such that the loop rests horizontally with pointer indicating zero.

When switch S is closed, a current I flows through the loop. Magnetic field is produced in the centre of the solenoid and current flows from Q to P. PQ experiences a downward force (Fleming's Left Hand Rule) , F_B, due to the magnetic field, B, where $F_B = BIL$ *where L is the length of PQ.*

The clockwise moment about the pivot (knife edge) for F_B is

$$F_B \times d = BILd$$

The loop is restored to the horizontal by placing a rider of mass m on the opposite edge parallel to PQ, which produces an anticlockwise moment of mgx.



By Principle of moments, anticlockwise moments = clockwise moments

$$mg\,x = F_B\,d$$

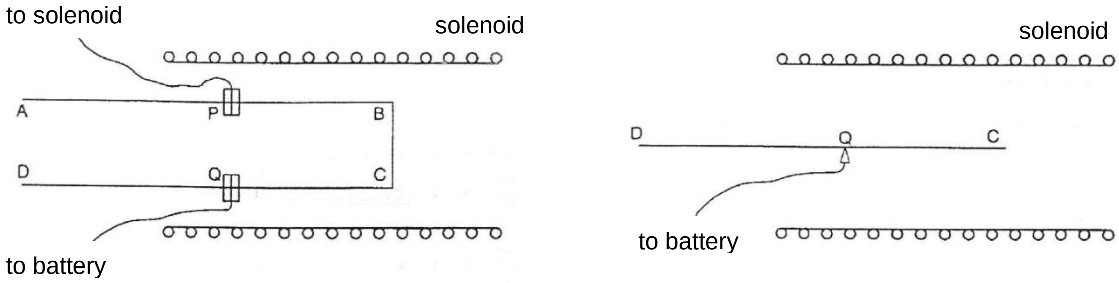
$$mg\,x = BIL\,d$$

$$B = \frac{mgx}{ILd}$$

By measuring the current I, the mass m, the distances x, d and L, the magnetic flux density B can be found.

Example 4

A wire frame ABCD is supported on two knife-edges P and Q so that the section PBCQ of the frame lies within a solenoid, as shown in the figure below. Electrical connections are made to the frame through the knife-edges so that part PBCQ of the frame and the solenoid can be connected in series with a battery. When there is no current in the circuit, the frame is horizontal.



Top View Side View

(a) When the frame is horizontal and a current passes through the frame and the solenoid, what can you say about the direction of the force, if any, due to the magnetic field of the solenoid acting on
i. side BC, ii. side PB?

(b) (i) The solenoid has 700 turns m^{-1} and carries a current of 3.5 A. Given that the magnetic flux density *B* on the axis of a long solenoid is $B = \mu_0 nI$, calculate the magnetic flux density in the region of side BC of the frame.
(ii) Side BC has length 5.0 cm. Calculate the force acting on BC due to the magnetic field in the solenoid.
(iii) A small piece of paper of mass 0.10 g is placed on the side DQ and positioned so as to keep the frame horizontal. Given that QC is of length 15.0 cm, how far from the knife-edge must the paper be positioned?

Solution

(a) (i) On side BC, which is perpendicular to the magnetic field of the solenoid, a vertical force is expected. The direction, up or down, depends on the direction of the current BC and the direction of the magnetic field.
(ii) On side PB, which is parallel with the magnetic field of the solenoid, no force is expected.

(b) (i) magnetic flux at BC, $B = \mu_0 nI$
$= 4\pi \times 10^{-7} \times 700 \times 3.5$
$= 3.1 \times 10^{-3}$ T

(ii) Force on BC, $F = BIL$
$= 3.08 \times 10^{-3} \times 3.5 \times 5.0 \times 10^{-2}$
$= 5.4 \times 10^{-4}$ N

(iii) Let d = distance of paper, of mass m = 0.10 g, from knife-edge.
mgd = F x (length QC)
$0.10 \times 10^{-3} \times 9.81 \times d = 5.39 \times 10^{-4} \times 15.0 \times 10^{-2}$
d = 8.2 cm

15.3.4 Forces between Parallel Current-Carrying Conductors

In 1821, Andre-Marie Ampere discovered that current-carrying conductors exert a force on each other; if two current-carrying conductors are close enough, then each conductor is in the magnetic field which the *current in the other creates*.

Each conductor should thus experience a force (F = BIL $\sin\theta$), either of attraction or repulsion, depending on the direction of the currents.

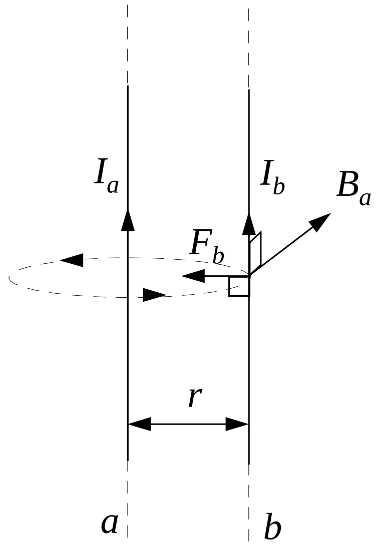


Figure (a):
Magnetic force on wire *b*

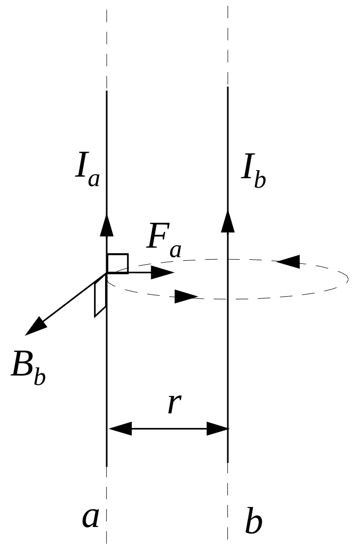


Figure (b):
Magnetic force on wire *a*

Consider two wires *a* and *b* (both of length *L*) carrying currents I_a and I_b flowing upwards. By using the right-hand grip rule, it can be determined that the magnetic field due to current I_a at the position of *b* is B_a directed perpendicularly into the plane of the paper.

By using Fleming's Left-Hand Rule, the force acting on wire *b* is F_b, *towards* the left.

$$F_b = B_a I_b L_b$$
$$= B_a I_b L$$

But since $B_a = \dfrac{\mu_0 I_a}{2\pi r}$, where *r* is the distance between the two wires,

$$F_b = \frac{\mu_0 I_a I_b L}{2\pi r}$$

Similarly, it can be shown that the force F_a is directed towards the right and that

$$F_a = \frac{\mu_0 I_b I_a L}{2\pi r}$$

Hence, $|F_A| = |F_B|$

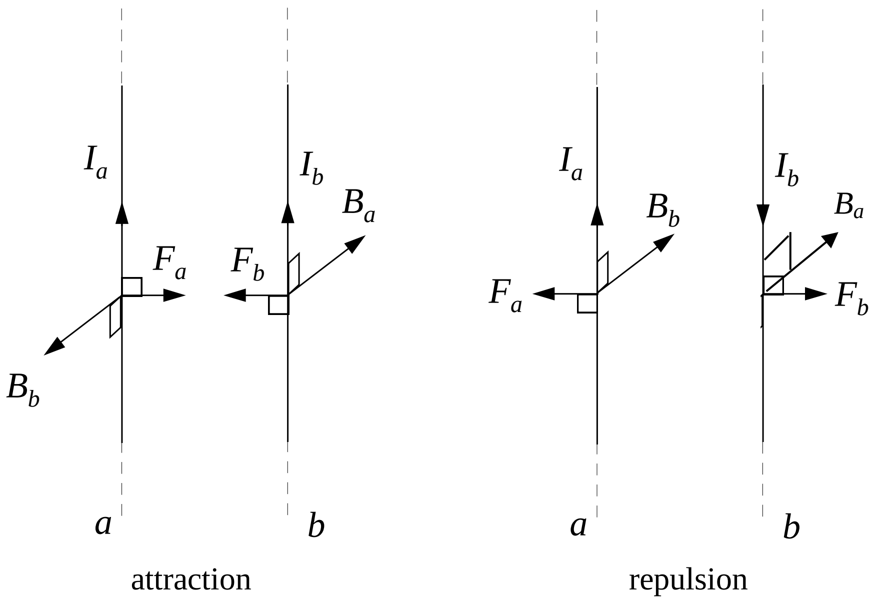
Note:

1. F_a and F_b are equal in magnitude, but are in opposite directions. They constitute an action-reaction pair.

2. Note that despite the difference in the amount of the current in both wires, they experience the same force.

3. For wires carrying currents in the same direction, there will be attraction to each other.

4. For wires carrying currents in the opposite directions, there will be repulsion from each other.



For parallel current-carrying wires:
Current Flowing in the Same Direction – Attractive Forces
Current Flowing in the Opposite Directions – Repulsive Forces

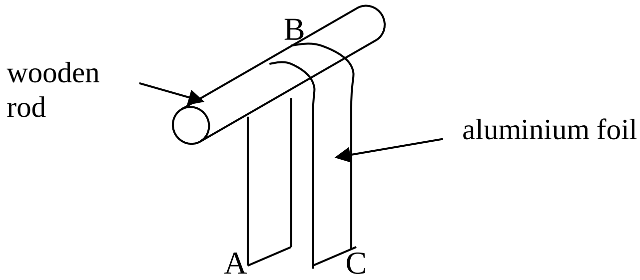
Example 5

A long length of aluminium foil ABC is hung over a wooden rod as shown below. A large current is momentarily passed through the foil in the direction ABC, and the foil moves.

(i) Draw arrows to indicate the directions in which AB and BC move

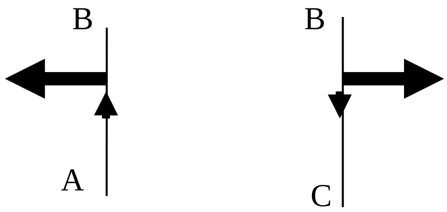
(ii) Explain why the foil moves in this way

(iii) Calculate the force given that the total length of the foil is 10 m, radius of the rod is 8 cm and current is 3 A.



Ans:

(i) Since currents in AB and BC are 'unlike' currents (they are flowing in opposite directions), the two foil sections AB and BC will repel each other.



(ii) Assume the current flows from A to B to C. The current in the left foil AB produces a magnetic field into the page on BC according to the Right Hand Grip Rule. From Fleming's Left Hand Rule, the force on BC is to the right i.e. away from AB. By a similar consideration, the force on AB is also away from BC. Thus the forces between the foils are repulsive.

(iii) F= $\dfrac{\mu_o I_A I_B L}{2\pi d} = \dfrac{4\pi x 10^{-7}(3)(3)(5)}{2\pi(16\times10^{-2})}$ = 5.63 x 10^{-5} N

15.4 Force on a Moving Charge

As current essentially consists of moving charges, it can be deduced that a *moving* charged particle also experiences an electromagnetic force. Consider a charge q traveling at a constant speed v at an angle θ to a magnetic field of flux density B. Assume the charge travels a distance L in time t, so its speed $v = \frac{L}{t}$ and $L = v\,t$.

The equation of the force on the conductor $F = BIL \sin\theta$ can be rearranged as

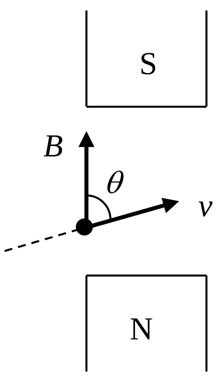
$$F = B\left(\frac{q}{t}\right)(vt)\sin\theta$$

Hence the force on a moving charge q is:

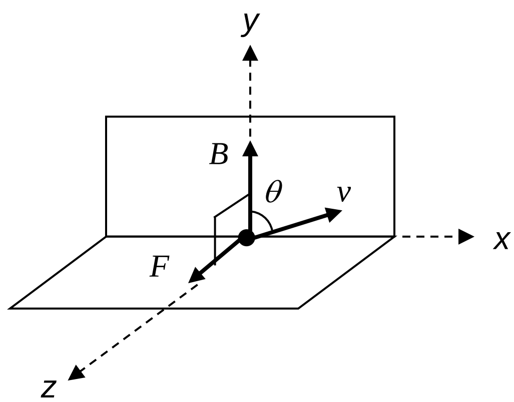
$$\boxed{F = Bqv\sin\theta}$$

where B = magnetic flux density of the magnetic field
q = amount of charge on the charge carrier
v = velocity of the charge carrier
θ = angle between the motion of the charged particle and the magnetic field.
i.e. angle between B and v.

2D view



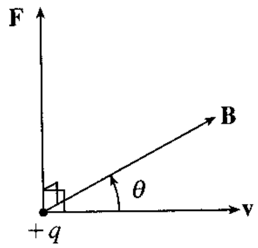
3D view

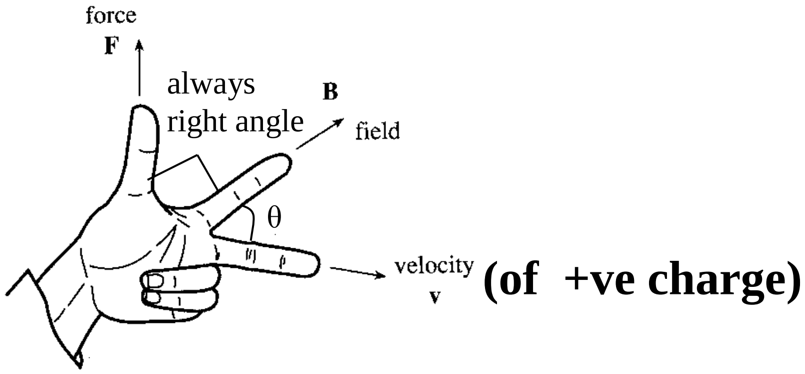


Both B and v are on x-y plane
F is on the x-z plane, parallel to z-axis.

Direction of Force:

The direction of force on the charged particle can be predicted by using Fleming's Left-Hand Rule and is at right angle to both the field and the velocity of the positive charge (equal to the direction of the conventional current I).





<u>Negative charge</u> moving one way is equivalent to conventional current flowing in the opposite direction. Thus direction of middle finger needs to be reversed for negative charge.

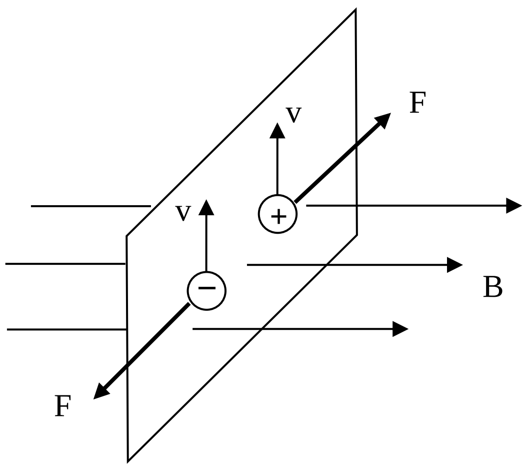
Note:

1. $F = 0$ if
(a) the charge is stationary, i.e. $v = 0$, or
(b) the charge is moving parallel to the field, i.e. $\theta = 0^o$ or 180^o.

2. The force which the charged particle experiences in a magnetic field is a deflecting force. In contrast to the electric and gravitational forces, this magnetic force is not in the direction of the field.

3. The magnetic force causes no change in speed and thus performs no work. It merely causes a change in direction.

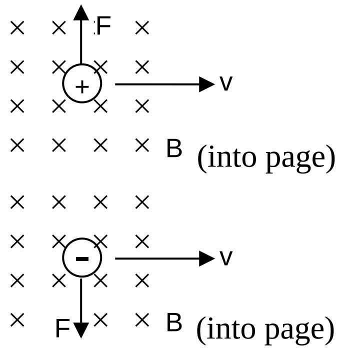
4. The force on a positive charge is in the direction opposite to the force on a negative charge moving in the same direction in the same magnetic field.



More illustrations

The direction of this force may also be found by using Fleming's left hand rule. Here, the middle finger points in the direction of the velocity if q is *positive*.

i.e. if q is negative, then the middle finger points in the opposite direction to v.

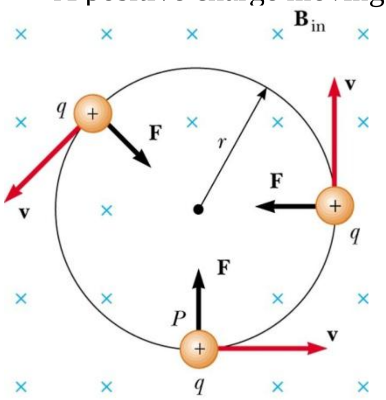


The angle θ determines the type of path the charged particle will take when moving through a uniform magnetic field:

- If θ **= 0°**, the charged particle takes a **straight path** since it is not deflected (F = 0)
- If θ **= 90°**, the charged particle takes a **circular path** since the force at every point in the path is perpendicular to the motion of the charged particle.

Circulating Charges

A positive charge moving in a magnetic field



Note: Magnetic field is pointing INTO the paper

Consider a positive charge moving at velocity v in a magnetic field which is directed downwards into the plane of the paper. For discussion purpose, we consider the velocity v is perpendicular to field B. If the magnetic field is extensive enough, the charge will describe a circular path inside the magnetic field. This is because the force F is directed towards the centre of the circle at any point of the charge's path.

Magnetic force $F = Bqv \sin\theta$ ------------------ (1)

For circular motion, $F = \dfrac{mv^2}{r}$ ------------------ (2)

Combine (1) and (2),

$$Bqv \sin\theta = \frac{mv^2}{r} \qquad \text{where } \theta = 90^{o}$$

Where:
r is the radius of the circular orbit of the charge,
m is the mass of the charge,
v is the velocity of the charge.

Sub in θ = 90°, we get $\boxed{r = \frac{mv}{Bq}}$

The period of revolution, $T = \frac{2\pi r}{v}$

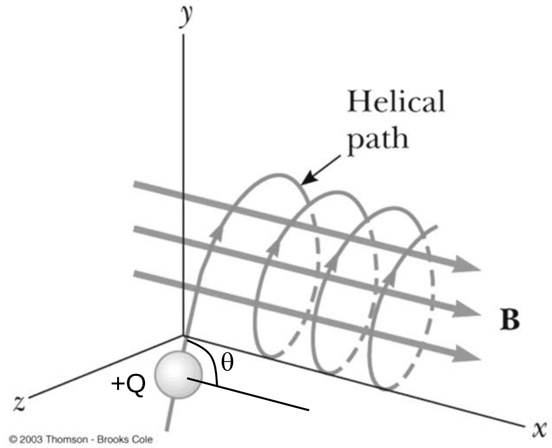
$$= \frac{2\pi(\frac{mv}{Bq})}{v}$$

$$= \frac{2\pi m}{Bq},$$ which is independent of *v*.

Note:

Since the orbital period does not depend on the speed of the charge, fast and slow moving charges take the same time to complete one cycle. Fast moving charges move in a large circle and slow moving charges move in a small circle.

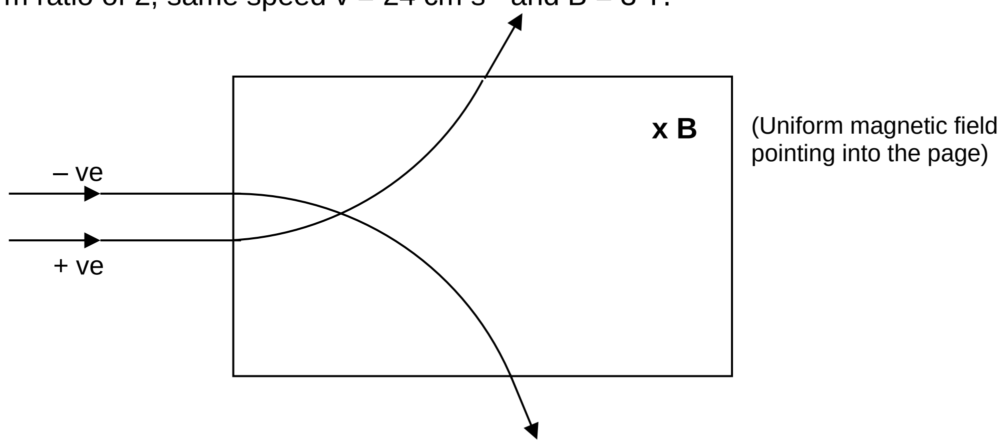
- If **0° < θ < 90°**, the charged particle will move in a **helical path** (i.e. it spirals forward).



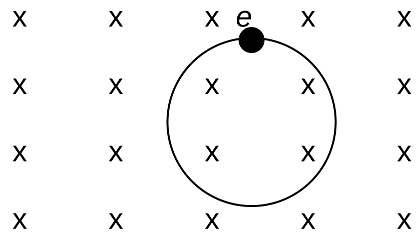
charged particles in a helical path.

Example 6

Predict the path (to scale) that will be taken by the two different charges as they enter the region of uniform field as shown. Assume the two charges got the same q/m ratio of 2, same speed v = 24 cm s^{-1} and B = 3 T.



Example 7
An electron moves in a circular path in vacuum under the influence of a magnetic field.



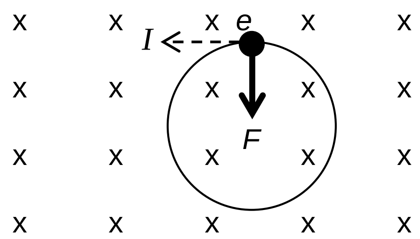
The radius of the path is 0.010 m and the magnetic flux density is 0.020 T. Given that the mass of the electron is 9.11 x 10^{-31} kg and the charge on the electron is –1.6 x 10^{-19} C, determine

(i) whether the motion is clockwise or anticlockwise;

(ii) the velocity of the electron.

Ans:

(i) The magnetic force on the electron points towards the centre of the circular path; hence using Fleming's left hand rule, we deduce that the current I points to the left. The electron is negatively charged, it must be moving clockwise.



(ii) The magnetic force provides centripetal force,

$F_B = F_C$

$$Bqv = \frac{mv^2}{r}$$

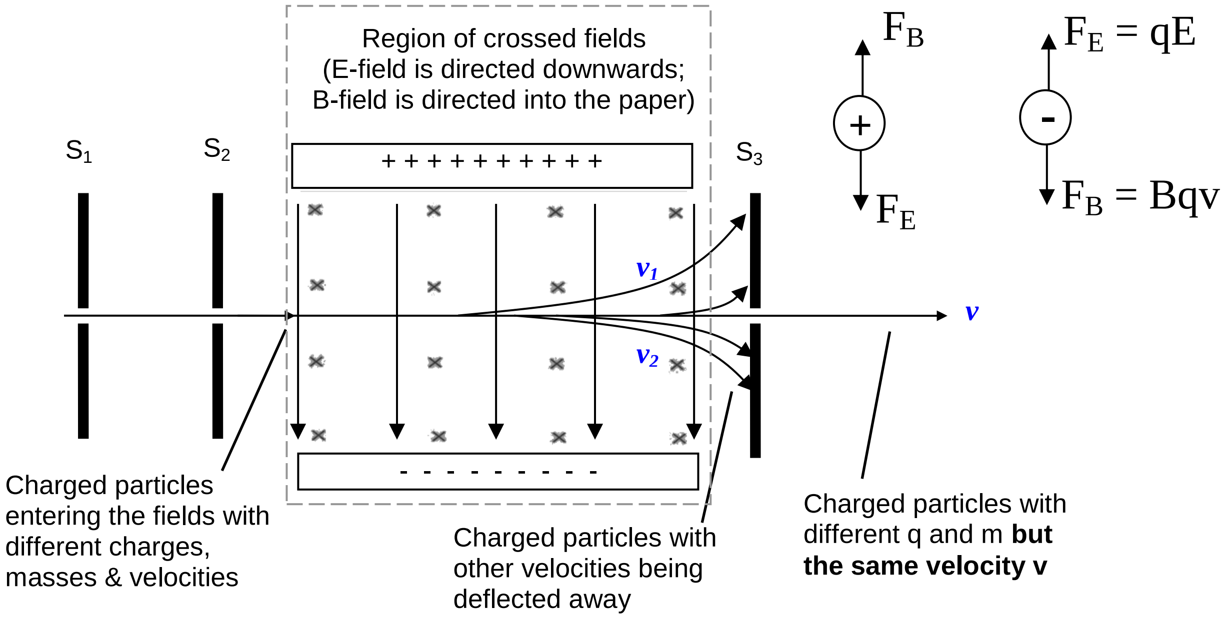
$$\Rightarrow v = \frac{Bqr}{m} = \frac{(0.020)(1.6\times10^{-19})(0.010)}{9.11\times10^{-31}}$$

$$\Rightarrow v = 3.51\times10^{7}\ \text{ms}^{-1}$$

15.5 Use of Crossed Fields in Velocity Selection

Uniform E and B fields (electric and magnetic fields respectively) could be set up **perpendicular** to each other such that they exert equal forces of opposite directions on a moving charged particle. This setup may be referred to as ***crossed* fields**, as shown below.

The velocity selector is an instrument that makes use of such crossed fields to select and emit a stream of charged particles (e.g. electrons) of a *specific velocity*.



velocity selector

A beam of charged particles with a range of velocities ($v_1, v_2, \ldots, v_n$) is made to pass through a region where there is a crossed field.

If the charged particles were electrons, then each electron in the crossed field experiences an upward electric force, and a downward magnetic force. (For positively charged particles: a downward electric force and an upward magnetic force)

For the particles to pass through undeflected, the electric force and magnetic force must be **equal in magnitude**:

Magnetic Force = Electric Force

$$Bqv = qE$$

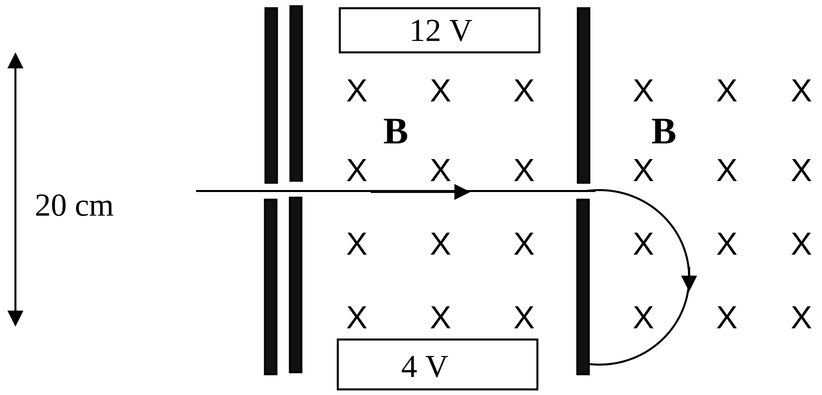
i.e. $$v = \frac{E}{B}$$

Particles which emerge from S_3 are hence those which are **undeflected**, and therefore are those with a specific velocity v, as given by the equation above. Charged particles of a particular desired speed v can thus be emitted from slit S_3 by adjusting the ratio E/B. This can be done by adjusting the potential difference V across the plates and the magnetic field strength B.

For the case of electrons, those moving with a slower speed v_1, ($v_1 < \frac{E}{B}$) or higher speed v_2 ($v_2 > \frac{E}{B}$) (as shown in Figure in previous page), will be deflected and blocked by the collimator plate S_3. The slits S_1 and S_2 (called a collimator) help confine the particles to a narrow beam.

Example 8

After passing through the velocity selector, the electrons move in a circular path of radius 5 cm. Determine the magnetic flux density B. (Given: mass of electron = 9.11×10^{-31} kg, charge of electron = -1.6×10^{-19} C)



Answer

In the velocity selector, for electron to travel undeflected, $F_E = F_B$

$$qE = qvB$$

$$v = \frac{E}{B}$$

After the velocity selector, $qvB \sin\theta = \frac{mv^2}{r}$ where $\theta = 90^o$

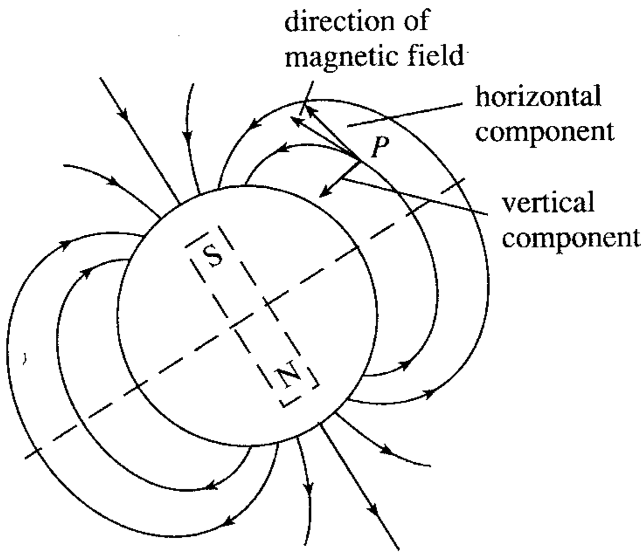
$$r = \frac{mv}{Bq} = \frac{m}{Bq}\left(\frac{E}{B}\right) = \frac{mE}{B^2 q} = \frac{m}{B^2 q}\left(\frac{V}{d}\right)$$

$$B = \sqrt{\frac{mV}{qdr}} = \sqrt{\frac{(9.11\times10^{-31})(12-4)}{(1.6\times10^{-19})(20\times10^{-2})(5\times10^{-2})}} = 6.75 \times 10^{-5}\ \text{T}$$

Annex

A.1 Magnetic Field of the Earth

Earth is a large magnet. The magnetic north pole of the magnet lies beneath the geographic south pole of the earth. The axis of rotation (along the geographic north-south line) and the magnetic axis (along the magnetic north-south line) are off by 11^0. The earth's magnetic field lines are almost parallel to the surface near the equator, and are almost perpendicular near the magnetic poles; at the geographic south pole (magnetic north pole) of the earth, the direction of the field is vertically upwards whereas at the geographic north pole (magnetic south pole), it is vertically downwards.

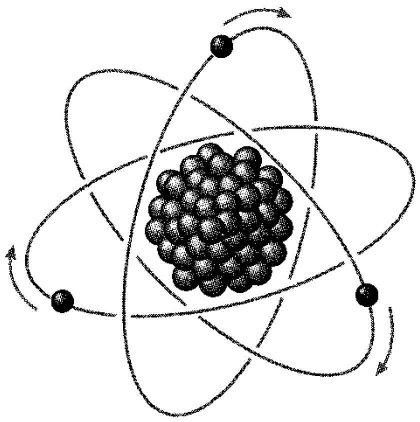


The earth's magnetic field at a point such as *P* can be resolved into two mutually perpendicular components: the horizontal component and the vertical component. The angle that the magnetic field lines make to the surface is called the angle of dip or inclination.

A.2 Magnetic Materials

We are familiar with the fact that magnets attract iron but not most other materials. It turns out that only a *few ferromagnetic* materials (iron, nickel, cobalt, and their alloys) are greatly influenced by a steady magnetic field.

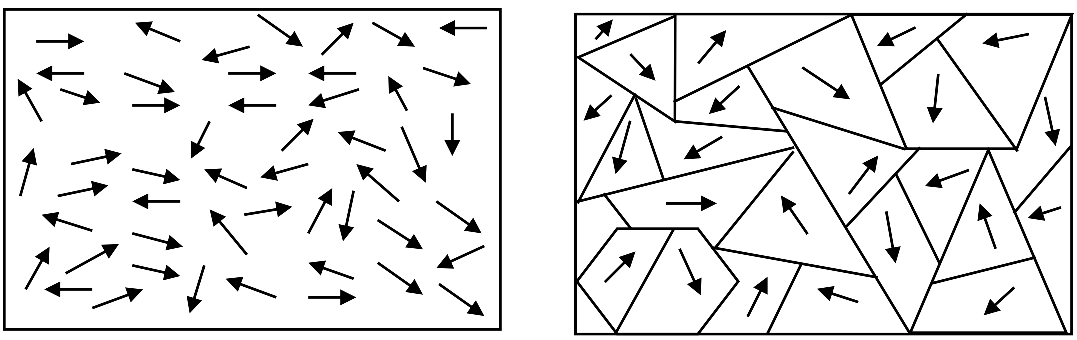
Some atoms act like tiny bar magnets. Since an electron in its orbit constitutes a circular current loop, each orbiting electron in an atom generates a magnetic field similar to that of the plane circular coil.



There is a second phenomenon that causes atoms to act like magnets. Particles such as the electron and proton act as though they are spinning on an axis through their centers; we say that such particles have *spin.* Any spinning charge acts like a current loop and generates a magnetic field.

In many atoms, the magnetic effects of the various electrons cancel each other. In other atoms, the cancellation is nearly, but not quite, complete. Only in the transition-element atoms, which are the ferromagnetic elements mentioned earlier, do the contributions of enough electrons add to give each atom a significant total magnetic field. These atoms therefore act like tiny compass needles. If a majority of these atoms become aligned in a bulk sample of ferromagnetic material, the sample becomes *magnetized.* Let us examine this condition more closely.

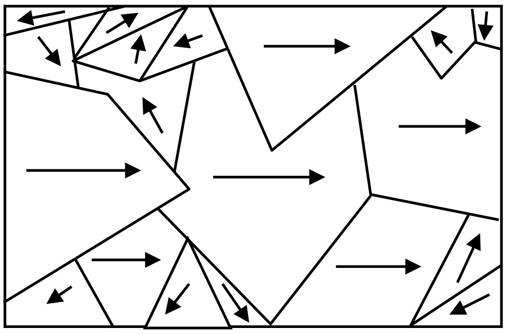
We know that if we place a group of tiny magnets close to one another, insofar as possible they arrange themselves in such a way that each south pole is close to a north pole. This is the result of the attraction of unlike poles and the repulsion of like poles. However, if the magnets are strongly agitated (perhaps by someone shaking the board on which they rest), they will break loose from this alignment.



(a) **(b)**

Most materials have their atomic magnets, if there are any, randomly oriented, as in figure (a). The ferromagnetic materials, however, consist of little regions within which the atoms are all aligned. Each of these oriented regions is called a *domain* (Fig. (b)). In an ordinary piece of iron, each domain may contain as many as 10^{16} atoms and consist of a region a small fraction of a millimeter in linear dimension. However, the domains in an unmagnetized piece of iron are randomly oriented, as in Fig. (b). For a bar of iron to be magnetized, the domains within it must be lined up. This can be done in the following way.

Suppose you start with the unmagnetized bar of iron. A solenoid with a current in it possesses a weak magnetic field within its windings. If now you place the iron bar in the solenoid, the magnetic field of the solenoid will exert forces on the domains. Those domains that are oriented along the field grow, and those oriented in other directions decrease in size. The net effect is to align the domains with the field, as shown in (c).



(c)

The iron is now a bar magnet, with north and south poles. In what is referred to as *soft iron,* the domains are easily oriented, but in what is called *hard iron* the external field must be made quite strong or the domains must be agitated by heat or mechanical means to allow them to grow in the direction of the field. (The designations *hard* and *soft* refer only to the magnetic properties, not to the physical hardness.) It is possible, however, to align the domains nearly perfectly and form a strong bar magnet.

Once the domains have been aligned, the resultant magnetic field consists of two parts: the original small field of the solenoid plus the field produced by the bar magnet, which is usually hundreds of times larger than the field of the solenoid. *The combination of a solenoid and a piece of soft iron is called an* **electromagnet.**

If the current in the solenoid is turned off, the domains in a bar of soft iron return nearly to their original random state. Thermal motion causes them to disarrange. This is a desirable situation in an electromagnet because it makes it possible to turn it on or off at will. A piece of hard iron used in the solenoid, on the other hand would retain most of its alignment when it was removed from the solenoid and would be a permanent bar magnet.

A.3 Electromagnet

As discussed in the above section, an electromagnet is a device that is designed to make best use of the magnetic effect of a current. It consists of a solenoid wound on a ferromagnetic core (e.g. soft iron) which becomes temporarily magnetised when a current flows through the solenoid. The magnetic effect disappears once the current is switched off.

Electromagnets form the basis of many types of electric motors, relay switches, generators and transformers. Because electric current can be switched on and off easily, and reversed at high speed, they have a great advantage in these applications over permanent magnets.

Many electromagnets make use of the way in which ferrous materials such as iron behave in a magnetic field. The soft iron core inside the electromagnet greatly increases the strength of the field. Soft iron must be used; hard steel would retain some of its magnetism when the current switched off, and energy is wasted in reversing the field in a hard magnetic material.

Questions

Self Attempt Questions

S1 Sketch magnetic flux patterns due to a long straight wire, a flat circular coil and a long solenoid.

S2 Sketch the resultant fields around two long wires carrying currents of the same magnitudes. The directions of the currents are as shown in the figures:

(a) Currents in the same direction

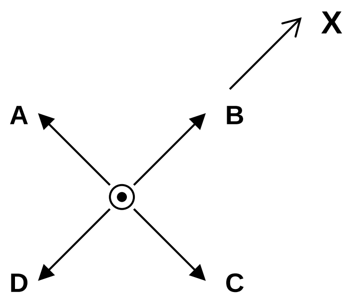
(b) Currents in opposite directions

S3 What is the size and direction of the force on 3.00 m of conductor wire which carries a current of 12.0 A due East (on the plane of the page) through a magnetic field of flux density 6.00×10^{-7} T whose direction is vertically out of the page?

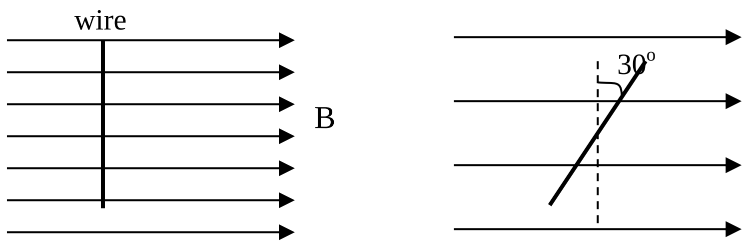
S4 An electron is moving along the axis of a solenoid carrying a current. Which of the following is a correct statement about the electromagnetic force acting on the electron? [N98/I/27]

A The force acts radially inwards
B The force acts radially outwards
C The force acts in the direction of motion
D No force acts

S5 The diagram shows cross-section of a straight wire that carries a steady current out of the plane of the paper towards the observer. The arrows represent the directions of four magnetic fields, A, B, C and D. Which field causes the wire to move towards the point X? [J01/I/18]



S6 A straight, horizontal, current-carrying wire lies at right angles to a horizontal magnetic field. The field exerts a vertical force of 8.0 mN on the wire. The wire is rotated, in its horizontal plane, through 30^o as shown. The flux density of the magnetic field is halved. What is the vertical force on the wire? [N02/I/25]

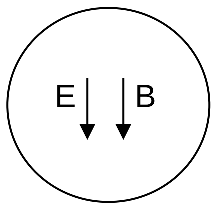


S7 Two long, straight, parallel wires carry currents of magnitudes 1.0 A and 2.0 A respectively. Both currents are pointing northwards. Draw a diagram showing the directions and relative magnitudes of the forces on each of the wires.

S8 In a C.R.O. tube, the electron beam passes through a region when there are electric and magnetic fields directed downwards as shown.

The deflections of the spot from the centre of the screen produced by E and B acting separately are equal in magnitude. Sketch the position of the spot on the screen when both fields are operating together.

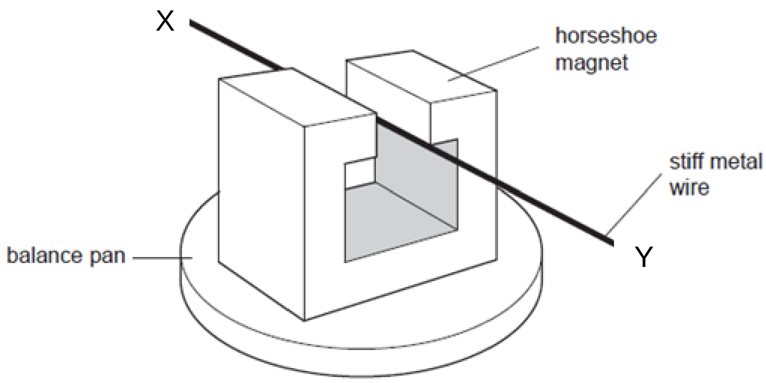
Front view of screen



S9 A charged particle enters a uniform magnetic field and describes a circular path. What factors affect the magnitude of the radius of the circular path?

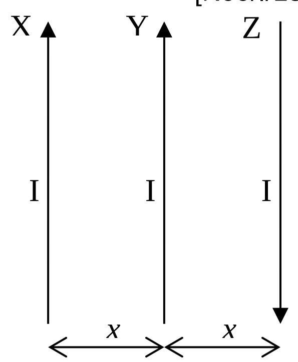
Tutorial Discussion

T1 A horseshoe magnet rest on a top-pan balance with a wire situated between the poles of the magnet. When no current flows in the wire, the reading on the balance is 142.0 g. When a current of 2.0 A flows in the wire in the direction XY, the reading on the balance changes to 144.6 g. What is the reading on the balance when there is a current of 3.0 A in the wire in the direction YX? [N04/I/24]



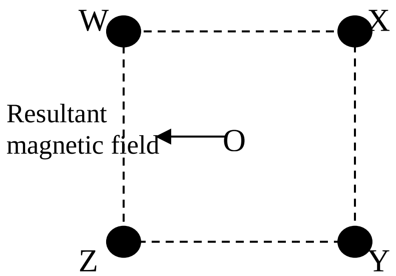
T2 The diagram shows 3 parallel wires X, Y, Z that carry currents of equal magnitude in the directions shown. The resultant force experienced by Y due to the currents in X and Z is [N99/I/18]

A perpendicular to the plane of the paper
B to the left
C to the right
D zero

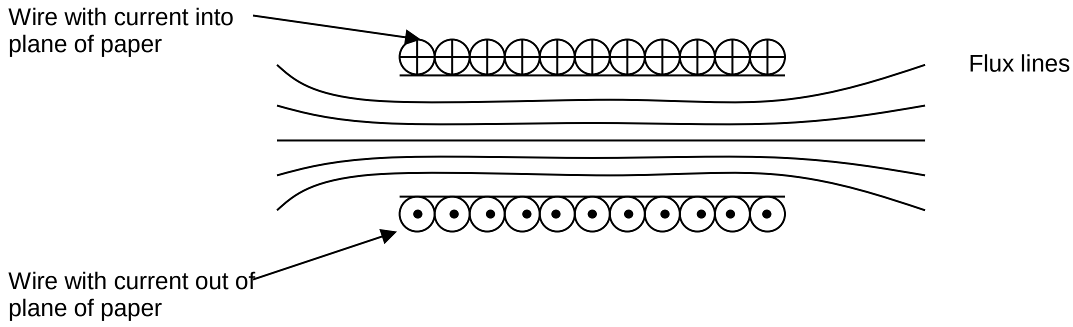


T3 Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square WXYZ. In two conductors, the current is flowing into the page, and in the other two, out of the page. In what directions must the current flow to produce a resultant magnetic field in the direction shown at O, at the centre of the square? [J96/I/18]

	Into the page	Out of the page
A	W & X	Y & Z
B	W & Y	X & Z
C	W & Z	X & Y
D	X & Z	W & X



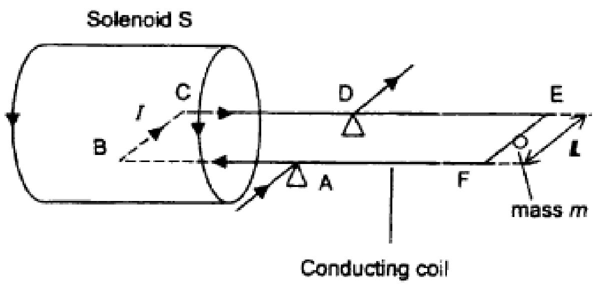
T4. The diagram below illustrates the pattern of the magnetic flux due to a current in a solenoid.



On the diagram above,

(i) Draw arrows to show the direction of the magnetic field in the solenoid.
(ii) Draw a line to represent a current-carrying conductor in the magnetic field which does not experience a force due to the magnetic field. Label the conductor C. [J97/II/3]

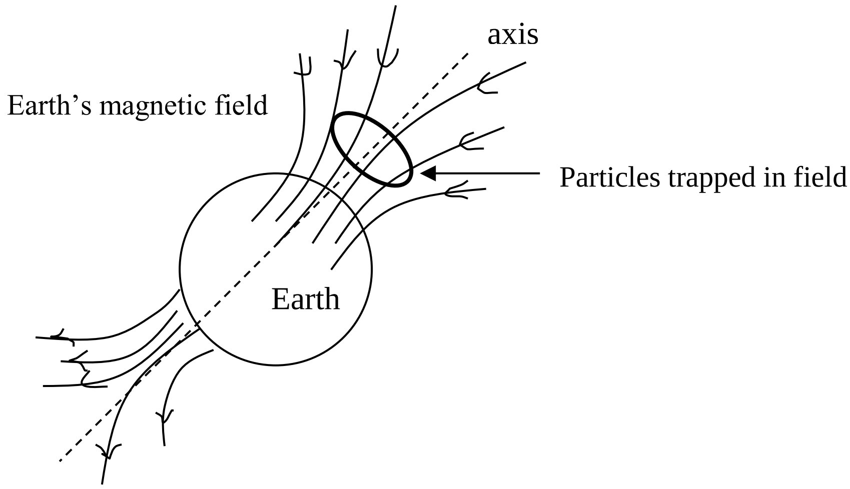
T5. A solenoid S which has n turns per unit length is connected in series with a horizontal rectangular conducting coil BCEF as shown below. The coil is pivoted freely at the midpoints of the side BF and CE. The side BC of length L is inside a solenoid S and perpendicular to the axis of the solenoid. When the current I flows in ABCD and the solenoid, a mass *m* hanging on the side EF is required to restore equilibrium.



(i) Derive an expression for the magnetic flux density *B* inside the solenoid S in terms of m, *g, L and* I, where *g* is the acceleration due to gravity.

(ii) If m = 0.10 g, L = 2.5 cm, and n = 1800 turns m^{-1}, what is the value of I, given that the magnetic flux density on the axis of a long solenoid is $B = \mu_o nI$.

T6. Charged particles from the sun, on approaching the Earth, may become trapped in the Earth's magnetic field near the poles, as shown below. This can cause the sky to glow. The phenomenon is called the aurora borealis.



Some of the charged particles travel in a circle of radius 50 km in a region where the magnetic flux density is 6.0×10^{-5} T.

(i) For a charged particle of charge to mass ratio *e/m*, deduce an expression for its speed *v* when traveling in a circle of radius *r* with a magnetic field of flux density B.

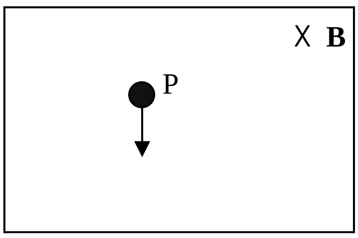
(ii) Use your answer to (i) and the information about the path of the particles to show that the charged particles causing the aurora cannot be electrons.

(iii) Suggest what could cause the aurora and, for your suggested particle, calculate its speed as it travels in a circle.

[N04/III/5(d)]

T7 An electron is traveling at right angles to a uniform magnetic field of flux density 1.5 mT, directed into the plane of the paper, as illustrated below. When the electron is at P, its velocity is 2.9×10^{7} ms^{-1} in the direction shown. This is normal to the magnetic field.

(a) On the figure, sketch the path of the electron, assuming that it does not leave the region of the magnetic field.



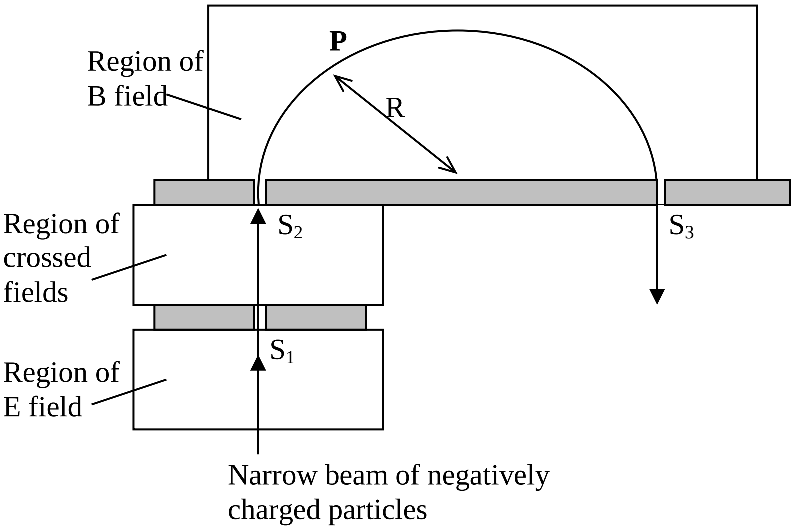
Region of magnetic field into the plane of the paper

(b) Calculate, for the electron,
(i) the force on it due to the magnetic field
(ii) the radius of its path
(iii) the time taken for it to complete half a revolution
(iv) its kinetic energy

(c) Determine the minimum potential difference required to accelerate the electron to this speed from rest.

[N03/2/6b mod]

T8 A narrow beam of negatively charged particles are accelerated by a potential difference V towards the slit S_1. Subsequently, the beam enters a region where a uniform electric field E and a uniform magnetic field B_1 exist simultaneously through slits S_1, before entering a region of magnetic field B_2 through the slit S_2 to exit through slit S_3.



(a) For a particle with charge Q and mass M, show that its speed just before entering S_1 is given by the expression $\sqrt{\frac{2QV}{M}}$.

(b) The uniform magnetic field in the region between S_1 and S_2 is applied in a direction out of the plane of the paper.

(i) State the direction of the electric field in that same region if there are to be some charged particles arriving at slit S_2.

(ii) Explain how this combination of magnetic and electric fields allows particles of only one speed v to pass through S_2.

(iii) Deduce an expression for v in terms of B_1 and E.

(iv) Sketch two possible paths, in the region of the crossed-fields, one of particles with a speed *greater than v* and one of particles with a speed *less than v*. Label them W and X respectively.

(c) When the particles enter the magnetic field B_2, particles with a charge to mass ratio of $\left(\frac{Q}{M}\right)_1$ travel in a circular path with a radius R and exit the field through slit S_3.

(i) Deduce the direction of the magnetic field B_2.

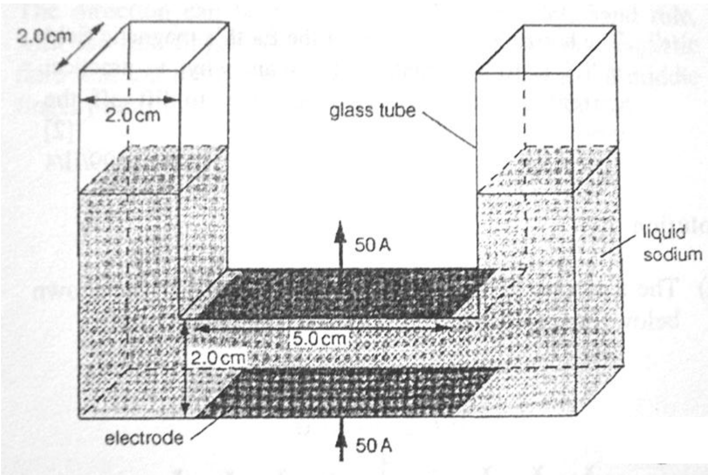
(ii) Indicate on the diagram, the direction of the velocity and resultant force acting on the particle at point P.

(iii) Write an expression for the radius R in terms of $\left(\frac{Q}{M}\right)_1$, B_2 and v.

(iv) Sketch two possible paths, in the region of the field B_2, one of particles with a charge to mass ratio larger than $\left(\frac{Q}{M}\right)_1$ and one of particles with a charge to mass ratio less than $\left(\frac{Q}{M}\right)_1$. Label them Y and Z respectively.

Assignment Questions

A1 A glass U-tube is constructed from hollow tubing having a square cross-section of side 2.0 cm, as shown below.

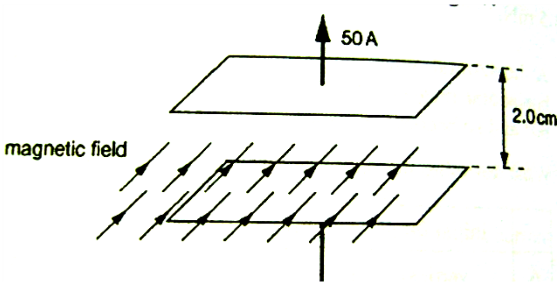


The U-tube has vertical arms and a horizontal section between the arms. Electrodes are set into the upper and lower faces of the horizontal section. Each electrode is of length 5.0 cm and width 2.0 cm. The U-tube contains liquid sodium of density 9.6 x 10^2 kg m^{-3} and of resistivity 4.8 x 10^{-8} Ωm.

(a) Assuming that the liquid sodium outside the electrodes has no effect on the resistance between the electrodes, calculate

(i) the resistance of the liquid sodium between the electrodes
(ii) the potential difference V between the electrodes required to maintain a current of 50 A in the liquid sodium

(b) A uniform horizontal magnetic field of flux density 0.12 T is now applied at right angles to the axis of the horizontal section of the tube in the region between the electrodes as shown below.



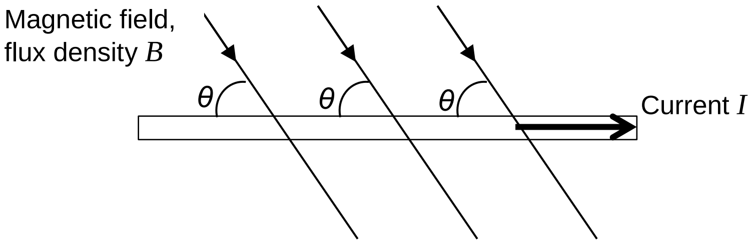
A force is exerted on the liquid due to the magnetic field. For this force,
(i) state and explain its direction,
(ii) calculate its magnitude.

(c) By considering the pressure difference caused by the force, determine the difference in height of the surfaces of the liquid sodium in the vertical arms of the U-tube.

(d) The technique outlined above could be used as a means to pump liquid. Suggest one advantage and one disadvantage of such a system.

[N97/III/4 part]

A2 (a) A straight conductor carrying a current I is at an angle θ to a uniform magnetic field of flux density B, as shown below.

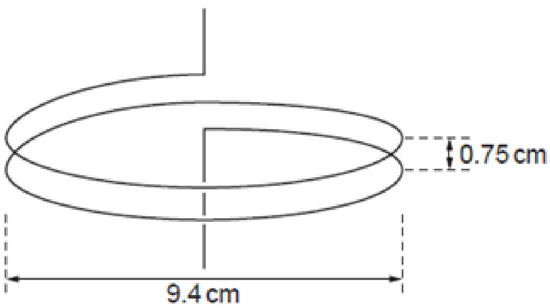


The conductor and the magnetic field are both in the plane of the paper. State

(i) an expression for the force per unit length acting on the conductor due to the magnetic field,

(ii) the direction of the force on the conductor

(b) A coil of wire consisting of two loops is suspended from a fixed point as shown below.



Each loop of wire has a diameter 9.4 cm and the separation of the loops is 0.75 cm. The coil is connected into a circuit such that the lower end of the coil is free to move.

(i) Explain why, when a current is switched on in the coil, the separation of the loops of the coil decreases.

(ii) Each loop of the coil may be considered as being a long straight wire. In SI units, the magnetic flux density B at a distance x from a long straight wire carrying a current I is given by the expression

$$B = 2.0 \times 10^{-7}\, I / x$$

When the current in the coil is switched on, a mass of 0.26 g is hung from the free end of the coil in order to return the loops of the coil to their original separation. Calculate the current in the coil.