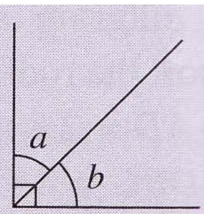
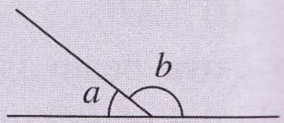
CHAPTER 9

9.1 ANGLES AND PLANE FIGURES

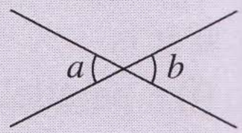
1. $a + b = 90°$ (comp. ∠s)



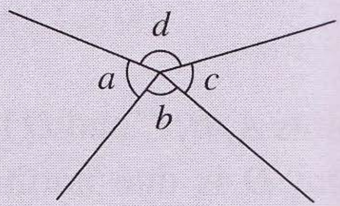
2. $a + b = 180°$ (adj. ∠s on a str. line)



3. $a = b$ (vert. opp. ∠s)



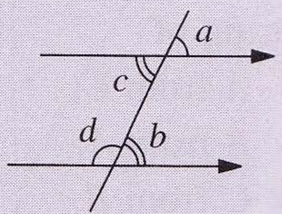
4. $a + b + c + d = 360°$ (∠s at a pt.)



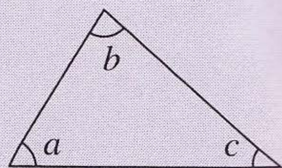
5. $a = b$ (corr. ∠s, // lines)

$b = c$ (alt. ∠s, // lines)

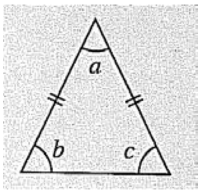
$c + d = 180°$ (int. ∠s, // lines)



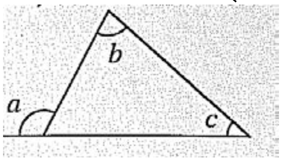
6. $a + b + c = 180°$ (sum of ∠s of a △)



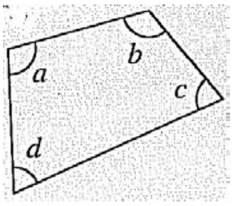
7. $b = \dfrac{180^o - a}{2} = c \quad (base\ \angle s\ of\ isos.\ \triangle)$



8. $a = b + c \quad (ext.\ \angle\ of\ \triangle = sum\ of\ int.\ opp.\ \angle s)$



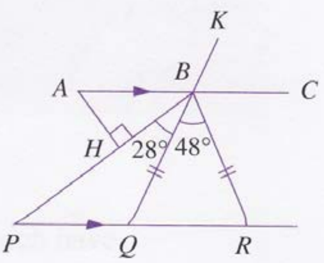
9. $a + b + c + d = 360^o \quad (sum\ of\ \angle s\ of\ a\ quad.)$



Example 1

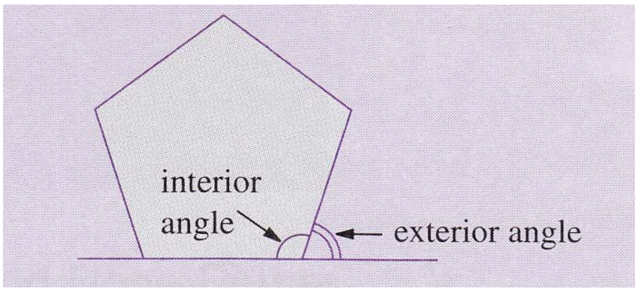
In the figure, ABC and PQR are parallel lines. Triangle BQR is isosceles with $BQ = BR$ and AH is perpendicular to BP. KBQ is a straight line. Given that $\angle PBQ = 28°$ and $\angle QBR = 48°$, calculate

(a) $\angle BQR$,
(b) $\angle KBC$,
(c) $\angle QPB$,
(d) $\angle BAH$.



9.2 ANGLE PROPERTIES OF POLYGONS

No. of sides (or vertices)	**Name of polygon**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
n	*n*-gon



1. Sum of **interior angles** of a polygon = $(n-2) \times 180^{\circ}$
   Where n = number of sides.

2. Sum of **exterior angles** of a polygon = 360°

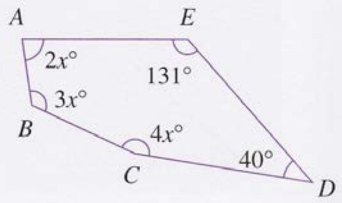
3. 1 Exterior Angle + 1 Interior Angle = 180° (adj. $\angle s$ on a str. line)

A **regular polygon** has all its sides equal and all its angles equal.

Example 2 Each interior angle of a regular polygon is 140°. How many sides does it have?

Example 3

ABCDE is a pentagon. The angles *A*, *B*, *C*, *D* and *E* are $2x°$, $3x°$, $4x°$, $40°$ and $131°$. Find the value of *x*.



9.3 SIMILAR TRIANGLES

Two triangles are **similar** if one of the following is true:

1. Corresponding angles are equal (**AAA)**.
   (In fact ,if two of the pairs of corresponding angles are equal, hen the third pair must be equal.)

2. Corresponding sides are in the same ratio.

3. Two pairs of corresponding sides are in the same ratio and the angles included between them are equal.

9.4 AREAS AND VOLUMES OF SIMILAR FIGURES

For two similar figures, F_1 and F_2, we have the following:

1. The ratio of their areas is the square of the ratio of their corresponding engths.

   ie. $\dfrac{A_1}{A_2} = \left(\dfrac{l_1}{l_2}\right)^2$

2. The ratio of their volumes is the cube of the ratio of their corresponding lengths.

   ie. $\dfrac{V_1}{V_2} = \left(\dfrac{l_1}{l_2}\right)^3$

Example 4

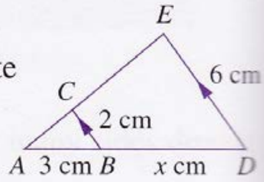
In the diagram, BC is parallel to DE.

(a) Calculate x.

(b) Given that the area of $\triangle ABC$ is 1.5 cm², calculate

(i) the area of $\triangle ADE$,

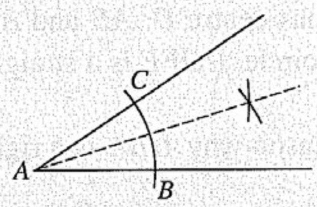
(ii) the area of the quadrilateral $BCED$.



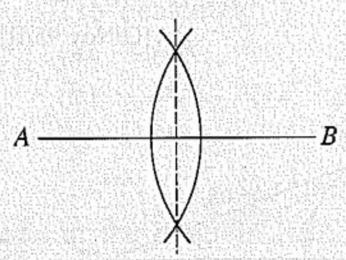
9.5 ANGLE BISECTORS AND PERPENDICULAR BISECTORS

In the following constructions, A, B and C give the order of the points at which the leg of a pair of compasses is placed.

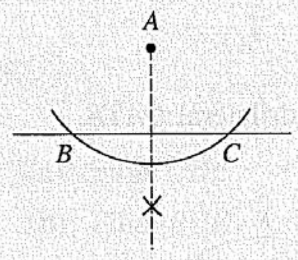
1. Bisecting an angle:



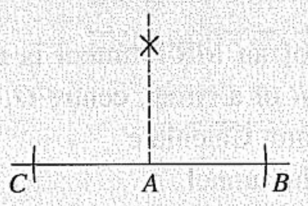
2. Bisecting a line:



3. Dropping a perpendicular onto a line from an external point:



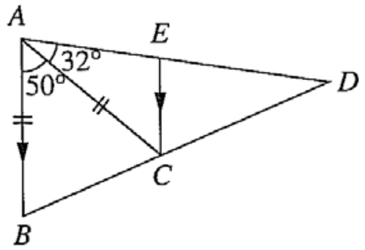
4. Drawing a perpendicular to a line from an internal point:



TUTORIAL 9

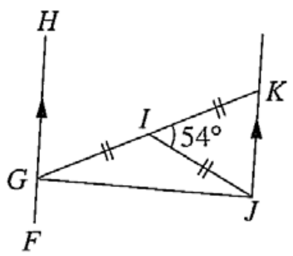
1. In the diagram, AED and BCD are straight lines. AB and EC are parallel and $AB = AC$. Given that $\angle BAC = 50°$ and $\angle CAE = 32°$, calculate

(a) $\angle DEC$,
(b) $\angle ABC$,
(c) $\angle CDE$.

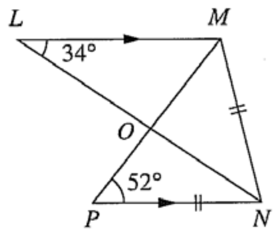


2. In the diagram, GIK is a straight line and $GI = IJ = IK$. The line FGH is parallel to JK. $\angle JIK = 54°$.
Calculate

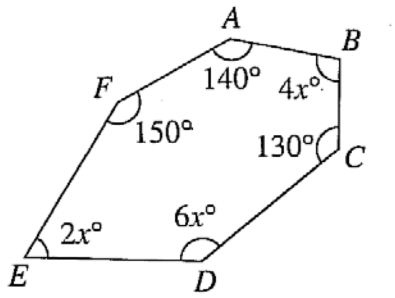
(a) $\angle IJK$,
(b) $\angle IJG$,
(c) $\angle FGJ$.



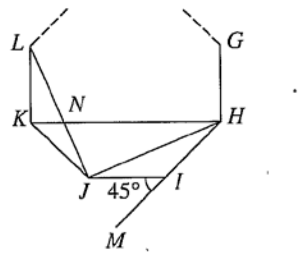
3. In the diagram, LM is parallel to PN and $MN = PN$. $\angle MLN = 34°$ and $\angle MPN = 52°$.
Calculate
(a) $\angle MNP$,
(b) $\angle LMP$,
(c) $\angle LOP$.



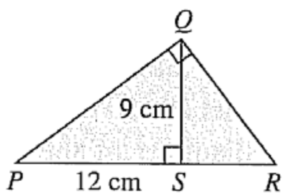
4. ABCDEF is a hexagon. The angles A, B, C, D,E and F are 140°, $4x^{\circ}$, 130°, $6x^{\circ}$, $2x^{\circ}$ and 150°. Calculate the value of x.



5.
G, H, I, J, K, L … are some vertices of a regular polygon.
HIM is a straight line and LJ cuts KH at N.
$\angle JIM = 45°$.
Calculate
(a) $\angle JHI$,
(b) $\angle HKJ$,
(c) $\angle LNH$.



6.



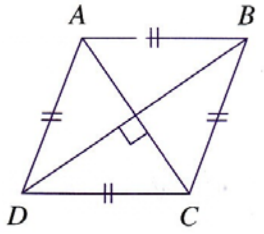
In the figure, S is the foot of the perpendicular from Q to PR and $\angle PQR = 90°$.
Using similar triangles, calculate

(a) the ratio $\dfrac{\text{area of } \triangle PQS}{\text{area of } \triangle QRS}$,

(b) the length of SR.

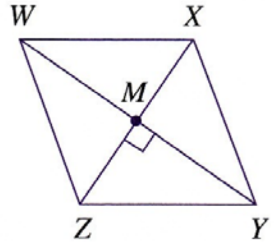
7.*

The point P which lies inside the rhombus $ABCD$ is such that $AP \leqslant PC$ and $\angle BCP \geqslant \angle DCP$. Copy the diagram and indicate clearly, by shading, the region in which P must lie.



8.*

The point Q which lies inside the rhombus $WXYZ$ is such that $\angle WXQ \geqslant \angle YXQ$ and $XQ \leqslant XM$. Copy the diagram and indicate clearly, by shading, the region in which Q must lie.



CHALLENGING QUESTION

1.

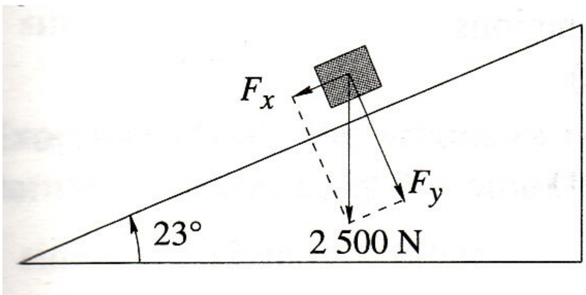


FIGURE 1

A 2,500 N weight is resting on an inclined plane that makes an angle of 23^{O} with the horizontal. Find the component F_x and F_y of the weight parallel to and perpendicular to the surface of the plane, as shown in Figure 1.

2.

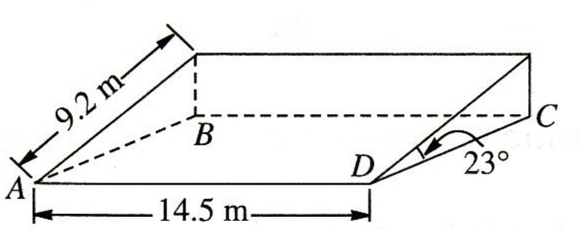


FIGURE 2

A roof that slopes at 23^{O} to the horizontal is 14.5 m long and has a slant height of 9.2 m.
How large an area does the roof actually cover? (Hint: Find the area of rectangle *ABCD in Figure 2)*