9749 H2 Physics Topic 7

Gravitational Field

DUNMAN HIGH SCHOOL

Year 5 (2024)

Guiding Questions

- How do two masses interact? Do they need to be in physical contact to do so?
- What do field lines represent? Do field lines represent similar things for gravitational fields and electric fields?
- How can we understand the motion of planets and satellites? Are we at the centre of the universe?

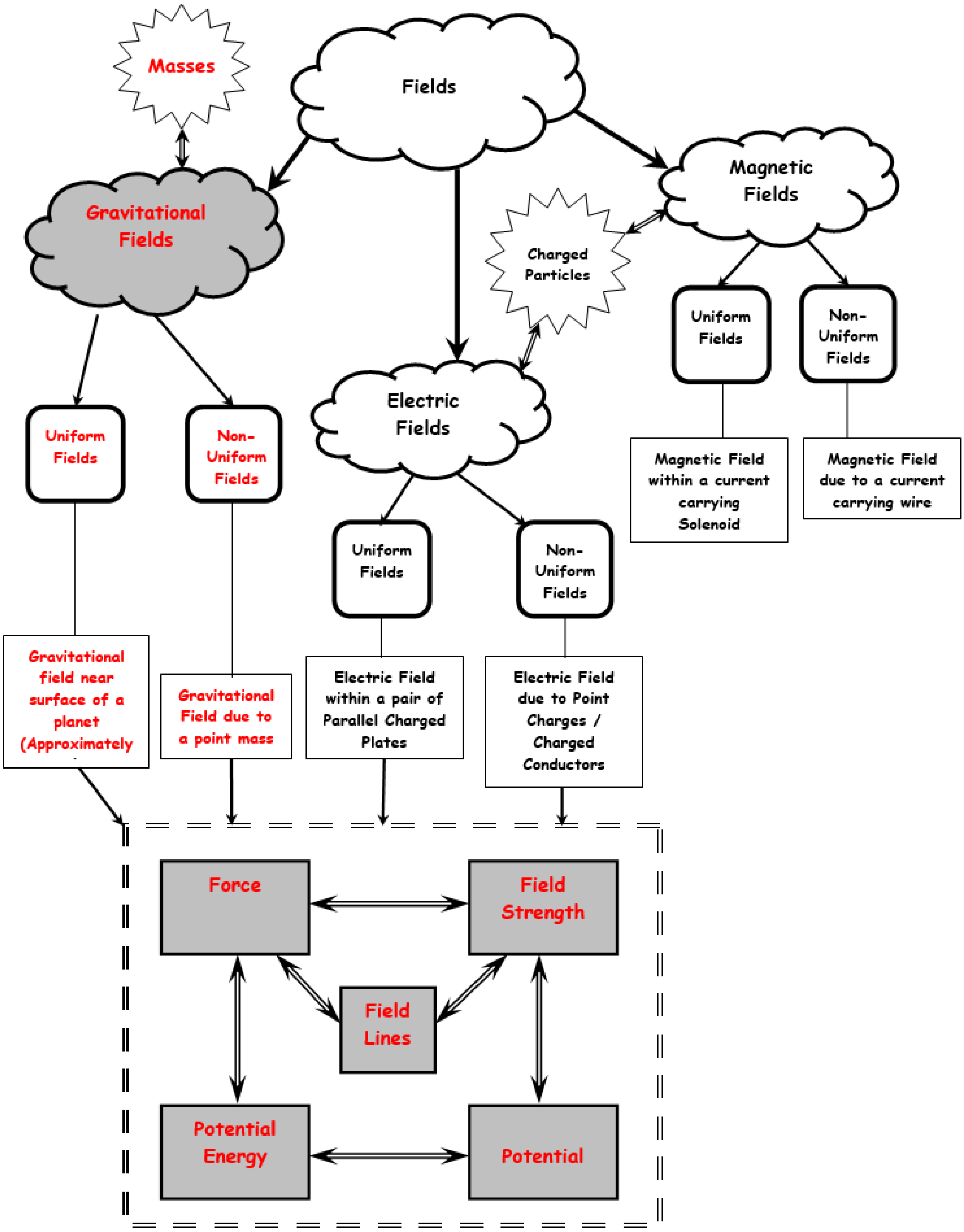
Content

- Gravitational field
- Gravitational force between point masses
- Gravitational field of a point mass
- Gravitational field near to the surface of the Earth
- Gravitation potential
- Circular orbits

Learning Outcomes
Students should be able to:

Gravitational field	(a)	show an understanding of the concept of a gravitational field as an example of field of force and define gravitational field strength at a point as the gravitational force exerted per unit mass placed at that point
	(b)	recognise the analogy between certain qualitative and quantitative aspects of gravitational and electric fields. [To be taught in the topic of *"Electric Field"*]
Gravitational force between point masses	(c)	recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$
Gravitational field of a point mass	(d)	derive, from Newton's law of gravitation and the definition of gravitational field strength, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass
	(e)	recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass to new situations or to solve related problems
Gravitational field near the surface of the Earth	(f)	show an understanding that near the surface of the Earth, gravitational field strength is approximately constant and is equal to the acceleration of free fall
Gravitational potential	(g)	define the gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to that point
	(h)	solve problems using the equation $\phi = -\frac{GM}{r}$ for the gravitational potential in the field of a point mass
Circular orbits	(i)	analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes
	(j)	show an understanding of geostationary orbits and their application

0. Field Concepts



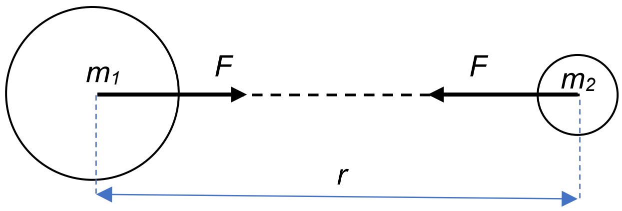
1. Introduction

Gravitational force is a force that is evident in our everyday lives and plays a crucial role in many processes on Earth. For instance, the ocean tides are caused by the gravitational attraction of both the Moon and Sun on the Earth's oceans. The falling of objects when released is also caused by the gravitational pull of the Earth on all objects. In terms of planetary motion, gravitational force is responsible for keeping the Earth in its orbit around the Sun, which in turn gives rise to four seasons in some countries, as Earth's tilted axis always points in the same direction when it orbits the Sun.

(c) recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$

2. Gravitational force acting between two point masses m_1 and m_2 with separation r

Newton's law of gravitation states that the gravitational force of attraction between two *point masses* is directly proportional to the product of their masses and inversely proportional to the square of the separation between their centres.



This means that if there are two point masses m_1 and m_2 and they are separated by distance r, the magnitude of the gravitational force attracting them to each other is

$$F = \frac{Gm_1m_2}{r^2}$$

where G = 6.67 × 10^{-11} N m^2 kg^{-2}, which is the constant of proportionality known as *gravitational constant* (provided in Data list).

Note:

(1) *Point masses* have non-zero mass and no volume. If two objects are placed sufficiently far apart such that their dimensions become negligible compared to the distance separating them, the two objects can be considered point masses.

(2) The gravitational forces between two masses are equal and opposite and constitute an action and reaction pair of forces (Newton's 3rd Law). The forces **always act along the line** joining the two point masses. To show its **attractive nature**, it is written (with negative sign) as:

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

The negative sign is ignored when only the magnitude of the force is required.

Example 1 **(Use of Formula for Newton's law of gravitation)**

There are (a) a boy and a girl, with masses 65 kg and 45 kg respectively, separated by 1.1 m, and (b) the Earth and its Moon, with masses 6.0 × 10^{24} kg and 7.4 × 10^{22} kg respectively, separated by 3.8 × 10^8 m. Determine the gravitational force between masses in (a) and (b).

Solution:

(a) $F = \frac{Gm_1m_2}{r^2} =$

(b) $F = \frac{Gm_1m_2}{r^2} =$

INQUIRY 01 :

What is the significance of the answers?

(a) *show an understanding of the concept of a gravitational field as an example of field of force and define gravitational field strength at a point as the gravitational force exerted per unit mass placed at that point.*

3.1. Gravitational field

Think about it: How can two objects exert attractive force on each other when they are not in contact with each other?

Every object with **mass** sets up a gravitational field in its surrounding space. When two objects enter each other's gravitational fields, they will be attracted towards each other. Hence, when an object with **mass** is placed in a **gravitational field** (an example of field of force[1]), there would be a **gravitational force** acting on it.

Graphs play an important role in this topic. Let us investigative various key graphs through the "Inquiry" questions.

INQUIRY 02 :

Gravitational field is invisible and is represented by imaginary field lines. How would the Earth's gravitational field (both over large distances from Earth and near Earth) looks like?

(a) In **Fig. 1**, several small masses are placed **far** from the Earth. Draw (using pencil) the direction of gravitational forces acting on them by Earth;

(b) In **Fig. 2**, several small masses are placed equidistant from each other **near** the Earth's surface. Draw the direction of gravitational forces acting on them by Earth.

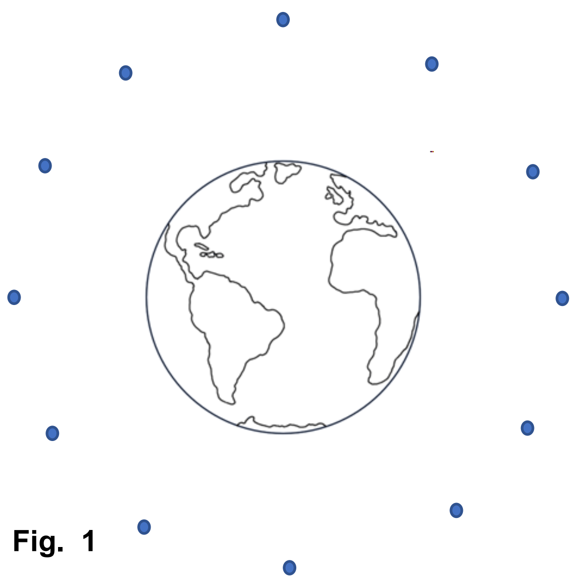


Fig. 1

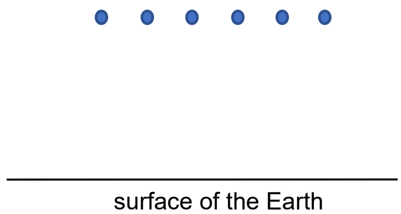


Fig. 2

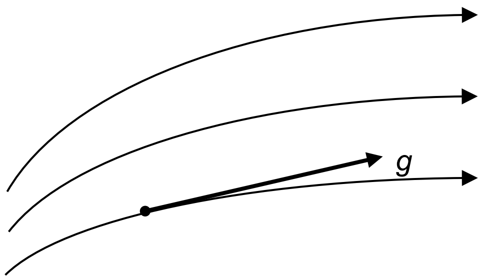
Fig. 1:
- The gravitational field around Earth is ____________.
- The Earth is usually assumed to be a **point mass** with all its mass concentrated at its centre. The field lines should be drawn radially ____________________________.

Fig. 2:
- The gravitational field near Earth's surface is ________. Within a **uniform field**, the **field strength is the same at all points**.
- The field lines should be drawn ________________ and of ____________.

[1] A **field of force** is a region of space where there is a force acting on an object placed in that field. An object placed in an ordinary space (like in deep outer space, which is not a field) would not have any force acting on it.

The **direction of a field at a point** is along a ***tangent*** to the field line at that point, as shown in **Fig. 3**.

Fig. 3



Note:
- Field lines never touch or cross.
- Field lines are perpendicular to the surface of the mass.

The density of the field lines at a point (number of lines per unit area) corresponds to the strength of the field at that point. A denser arrangement of field lines indicates greater gravitational field strength and vice versa.

Gravitational field strength at a point g is defined as the **gravitational force per unit mass** exerted on a *small test mass* placed at that point.

g is a **vector** quantity and it is in **the same direction as the gravitational force.**
Its SI unit is N kg^{-1}.

Example 2 [76/II/6][Modified] **(Synoptic Question: Units)**
The SI unit for gravitational field strength is

A m s^{-2} **B** J kg^{-1} **C** kg N^{-1} **D** N m^{-1}

Solution: (Ans:)
$g = \dfrac{F}{m}$ implies that the SI unit of g is .
Since 1 N = 1 kg m s^{-2}, N kg^{-1} =

The equation $g = F/m$ can also be related to Newton's 2nd Law when applied to the acceleration, a, of an object of constant mass: $a = F/m$. Hence the gravitational field strength g near the surface of the Earth is 9.81 N kg^{-1}. It is numerically equal to the *acceleration of free fall, a, near the surface of the Earth,* with the value 9.81 m s^{-2}.

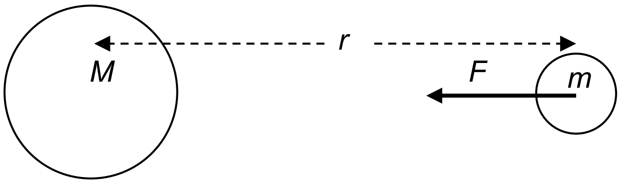
Example 3 **(Gravitational Field Strength: Small Test Mass)**
Gravitational field strength at a point is defined as the gravitational force per unit mass exerted on a small test mass placed at that point.
Why is it necessary for the test mass to be small?

A so that the test mass does not distort the gravitational field
B so that the force on the test mass is small
C so that the test mass does not create any forces on nearby masses
D so that Newton's law for point masses is obeyed

Solution: (Ans:)

(d) derive, from Newton's law of gravitation and the definition of gravitational field strength, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass.

3.2. Gravitational field strength of a point mass *M*



Consider two point masses *M* and *m* separated by a distance *r*.

1) From Newton's law of gravitation, the magnitude of the attractive gravitational force *F* acting on the mass *m* due to (the gravitational field of) *M* is expressed as

$$F = \frac{GMm}{r^2} \qquad \text{- - - - - (1)}$$

2) Based on definition of gravitational field strength, the magnitude of *g* at the point where *m* is situated is expressed as

$$\boxed{g \equiv \frac{F}{m}} \qquad \text{- - - - - (2)}$$

3) From equations (1) and (2),

$$\boxed{g = \frac{GM}{r^2}}$$

which is an expression for the **magnitude** of the gravitational field strength *g at a point* a distance *r* measured from a point mass *M*. (This point mass ***M* is *the one* which creates the gravitational field** in its surrounding region of space.).

In vector form, a negative sign indicates that the direction of the field strength is towards decreasing *r*: $\vec{g} = -\frac{GM}{r^2}\hat{r}$, $\hat{r}$ is a unit vector in the outward radial direction.

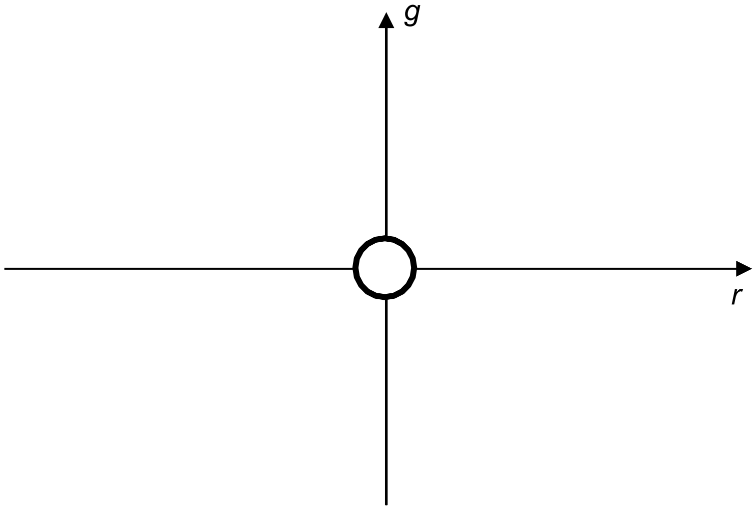
Example 4 [J81/II/8][Modified] **(Use of Formula for Gravitational Field Strength)**
At a point on the surface of a uniform sphere of diameter *d*, the gravitational field due to the sphere is *X*. What would be the corresponding value on the surface of a uniform sphere of the same density but of diameter 2*d*?

A 2*X* **B** 4*X* **C** 8*X* **D** 16*X*

Solution: (Ans:)

INQUIRY 03 :

Sketch the gravitational field strength, g against distance, r graph for a 500 kg mass below, by treating it as a point mass:



(e) *recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass to new situations or to solve related problems.*

Example 5 [12/I/13] **(Synoptic Question: Dynamics)**

A meteorite falls freely towards the centre of an isolated spherical planet of radius R. When it reaches the surface of the planet, its acceleration is g_s. What is the acceleration when at a height $2R$ above the surface?

A $\frac{1}{9}g_s$ **B** $\frac{1}{4}g_s$ **C** $\frac{1}{3}g_s$ **D** $\frac{1}{2}g_s$

Solution: (Ans:)

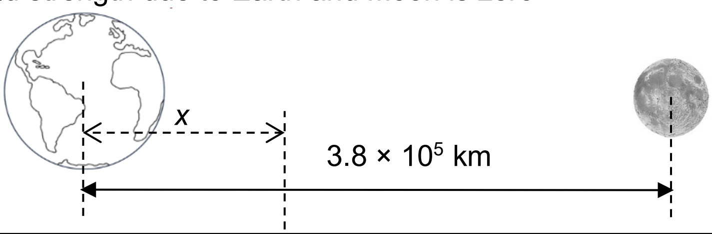
Acceleration = gravitational field strength due to the planet of mass M, given by $g = \frac{GM}{r^2}$.

On the surface, $r = R$, $g_s =$

At a height of $2R$, $r = 3R$, $a =$

Example 6 **(Synoptic Question: Vectors)**

Earth is a point mass of 6.0×10^{24} kg and Moon is a point mass of 7.4×10^{22} kg and the distance between them is 3.8×10^5 km. Determine the position of a point from Earth where the gravitational field strength due to Earth and Moon is zero.



INQUIRY 04 :

What if we draw the Earth and Moon to scale?

Solution:
Let x = distance from Earth to the point where resultant gravitational field strength is zero

At that point, gravitational field strength due to Earth, g_E directed toward Earth, is balanced by gravitational field strength due to Moon, g_M directed toward Moon.

i.e. $$g_E = g_M$$

→ $$\frac{GM_E}{x^2} = \frac{GM_M}{(3.8\times10^8 - x)^2}$$

→ $$\frac{6.0\times10^{24}}{x^2} = \frac{7.4\times10^{22}}{(3.8\times10^8 - x)^2}$$

→ $$x = $$

INQUIRY 05 :
How do we solve for x efficiently?

(f) *show an understanding that near the surface of the Earth, gravitational field strength is approximately constant and is equal to the acceleration of free fall.*

The *gravitational field strength g* near the surface of the Earth is *approximately constant* at 9.81 N kg^{-1}, which is also known as the *acceleration of free fall* with the value of 9.81 m s^{-2}. (Refer to **Example 2**)

The approximate constancy should be appreciated by considering the value of g at height h above the surface, where h is small compared to the radius R of the Earth of mass M.

$$g = \frac{GM}{(h+R)^2} \approx \frac{GM}{R^2} \qquad \text{since } h << R$$

Example 7 [J99/III/3] **(Gravitational Field Strength: < 1 km)**
Calculate the gravitational field strength at a height of 1 km above the Earth's surface. The radius of the Earth is 6400 km.

Solution:
Let M_E = mass of Earth and R_E = radius of the Earth.

At the surface of the Earth, using $g = \frac{GM}{r^2}$

$$9.81 = \frac{GM_E}{R_E^{\,2}} = \frac{GM_E}{(6400\times10^3)^2} \quad \text{- - - - - (1)}$$

At a height of 1 km, r = 6401 km,

$$g_1 = \frac{GM_E}{r^2} = \frac{GM_E}{(6401\times10^3)^2} \quad \text{- - - - - (2)}$$

(2) / (1): $$\frac{g_1}{9.81} = \frac{(6400\times10^3)^2}{(6401\times10^3)^2}$$ → g_1 = 9.81 N kg^{-1} (3 s.f.)

INQUIRY 06 :
The above results show that for up to a height of 1000 m above the surface of the Earth, **g is approximately constant** i.e. 9.81 N kg^{-1} to 3 s.f.. This value is accurate and sufficient enough for practical purposes. Why?

Example 8 [91/I/8][Modified] **(Gravitation Field Strength & Force)**
A 20 kg mass is situated 4 m above the Earth's surface. Taking g as 10 m s^{-2}, what are the gravitational field strength and gravitational force acting on the mass?

	gravitational field strength / N kg^{-1}	gravitational force / N
A	0.5	10
B	10	10
C	10	200
D	40	200

Solution: **(Ans:)**

(g) define the gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to that point.

4.1. Gravitational Potential Energy Near the Surface of the Earth

Consider **Fig. 4**. Recall from the topic of "*Work, Energy and Power*" that when lifting a mass m through a vertical height h with gravitational field strength g (= 9.81 N kg^{-1}) near the surface of the Earth, the increase in gravitational potential energy = $\boldsymbol{mgh}$

or $$\boxed{\Delta U = mgh}$$

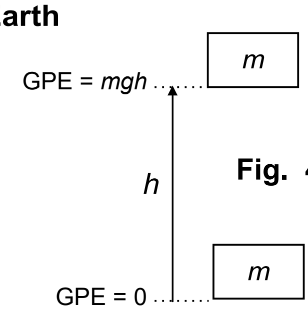


Fig. 4

Strictly speaking, the quantity mgh represents the *change* in potential energy when the mass is lifted through a vertical height h rather than the absolute amount it possesses at a height h. We conveniently choose a **lower** reference level (usually the **ground** level) as the zero potential energy level so that mgh is the extra energy a mass has when it is at height h compared to when it is at the **zero** potential energy level.

Recall as well that the increase in gravitational potential energy is actually the work you must do against the gravity to move the mass m from the GPE = 0 reference point through a vertical height h to the higher point. It is based on this understanding that the formula $\boldsymbol{\Delta U = mgh}$ is derived.

4.2. Formal Definition of Gravitational Potential Energy

The **gravitational potential energy** of a mass at a point is defined as the work done on the mass in moving it from infinity to that point.

Here, the work is done by an external force acting in the **opposite** direction to the gravitational attraction.

Alternatively, the gravitational potential energy of a mass at a point can also be understood as the work done **against** the gravity in bringing the mass from infinity to that point.

To illustrate the meaning of the gravitational potential energy U, consider moving a mass m from infinity to a point A, at distance r measured from centre of a mass M, at constant speed as shown in **Fig. 5**. $\overrightarrow{F_g}$ represents gravitational force acting on m by M while $\overrightarrow{F_{ext}}$ represents the force acting on m by an external agent such that $\left|\overrightarrow{F_{ext}}\right| = \left|\overrightarrow{F_g}\right|$.

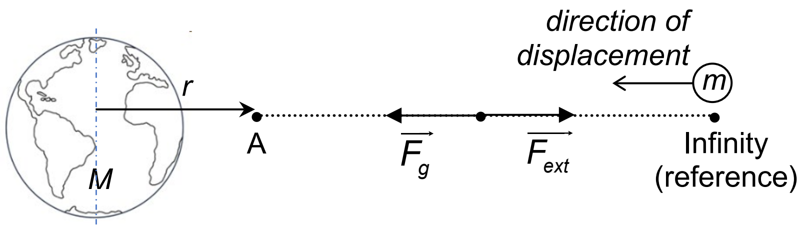


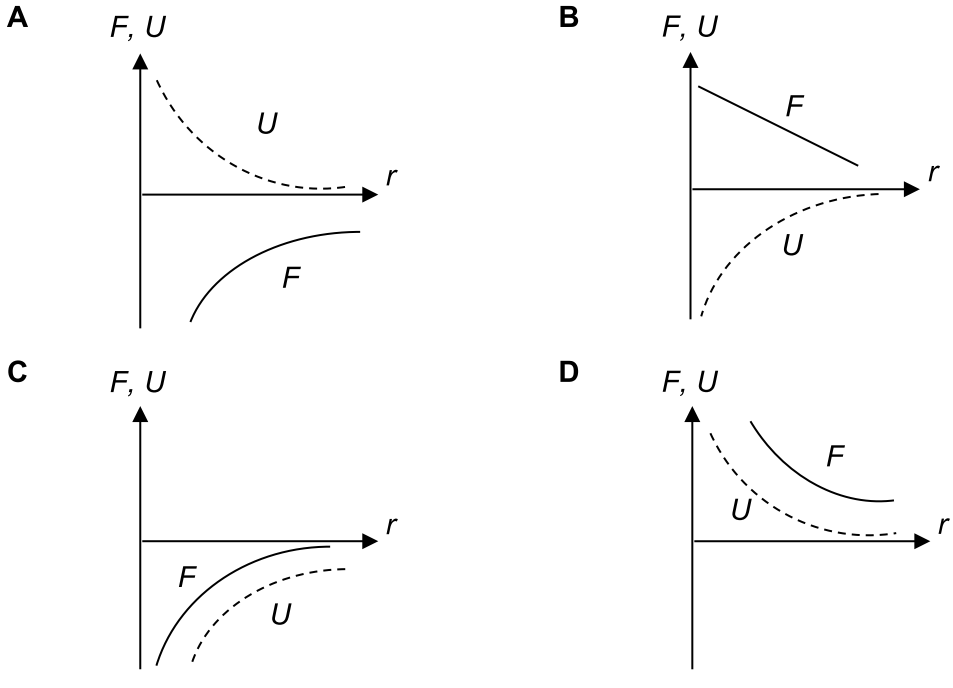
Fig. 5

Mathematically, in combination with Newton's 3rd Law, the definition can be written as

$$U = \int_{\infty}^{r} \overrightarrow{F_{ext}}\, dr = \int_{\infty}^{r} -\overrightarrow{F_g}\, dr = \int_{\infty}^{r} \left(\frac{GMm}{r^2}\right) dr = \left[-\frac{GMm}{r}\right]_{\infty}^{r} = -\frac{GMm}{r}$$

Example 9 [82/II/8; 95/I/7] **(Gravitation Force & Potential Energy)**

Which diagram shows the variation of gravitational force F on a point mass, and of gravitational potential energy U of the mass, with its distance r from another point mass?



Solution: (Ans:)

INQUIRY 07 :

Can the graph of U be above F?

Example 10 **(Gravitational field strength for height > 1 km)**
A 3.0 kg mass is projected to a height h above Earth's surface. Taking the radius of the Earth as 6400 km and the mass of the Earth as 6.0×10^{24} kg. Find the change in the gravitational potential energy for the 3.0 kg mass if
(i) h = 1000 m
(ii) h = 1000 km

Solution:

(i) Using $\Delta U = mgh$, $\Delta U = (3.0)(9.81)(1000) \approx 29$ kJ

Note:

Using $\Delta U = U_f - U_i = \left(-\frac{GMm}{r+h}\right) - \left(-\frac{GMm}{r}\right) = GMm\left(\frac{h}{r(r+h)}\right)$,

$$\Delta U = (6.67\times10^{-11})(6.0\times10^{24})(3)\left(\frac{1000}{6400(6401)\times10^{6}}\right) \approx 29 \text{ kJ}$$

(ii) Using $\Delta U = mgh$, $\Delta U = (3.0)(9.81)(1000000) \approx 29$ MJ

Note:

Using $\Delta U = U_f - U_i = \left(-\frac{GMm}{r+h}\right) - \left(-\frac{GMm}{r}\right) = GMm\left(\frac{h}{r(r+h)}\right)$,

$$\Delta U = (6.67\times10^{-11})(6.0\times10^{24})(3)\left(\frac{1000\times10^{3}}{6400(7400)\times10^{6}}\right) \approx 25 \text{ MJ}$$

The gravitational potential energy of a mass depends on both the value of the mass as well as the position of the mass. We would like to define an energy-related quantity that is only dependent on the position (just like gravitational field strength) in a gravitational field. This quantity is the *gravitational potential*.

The **gravitational potential** at a point is defined as the work done per unit mass (by an external force) in bringing a small test mass from infinity to that point.

Expressed mathematically, $\boxed{\phi \equiv \frac{U}{m}}$ and $\phi = -\frac{GM}{r}$ for potential at a distance r away from a **point** mass M

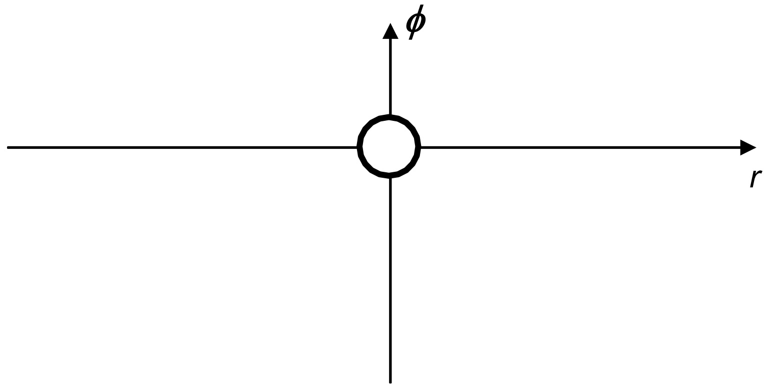
The unit of gravitational potential is J kg^{-1}. It is a **scalar** quantity.

Example 11 **(Significance of negative sign in the equation for Φ)**
Why is the gravitational potential negative in value?

Solution:

INQUIRY 08 :

Sketch the gravitational potential, ϕ against distance, r graph for a 500 kg mass below, by treating it as a point mass:



(h) *solve problems using the equation $\phi = -\frac{GM}{r}$ for the gravitational potential in the field of a point mass.*

Example 12 ***[J80/II/5][Modified]*** **(Use of Formula for Gravitational Potential)**
X and Y are two points at respective distances R and $2R$ from the centre of the Earth, where R is greater than the radius of the Earth. The gravitational potential at X is –800 kJ kg^{-1}. When a 1 kg mass is taken from X to Y, the work done on the mass is

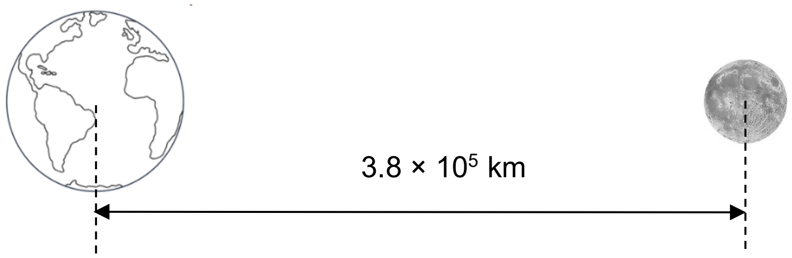
A –400 kJ **B** –200 kJ **C** +200 kJ **D** +400 kJ

Solution: (Ans:)

The potential at X, ϕ_X = –800 kJ kg^{-1}; so the potential at Y, ϕ_Y = –400 kJ kg^{-1} ($\phi \propto -\frac{1}{r}$)

If the potential energy of the 1 kg mass at X is U_X and its potential energy at Y is U_Y, then the work done in taking the 1 kg mass from X to Y,

Example 13 **(Synoptic Question: Scalars)**
Earth is a point mass of 6.0×10^{24} kg and Moon is a point mass of 7.4×10^{22} kg. The distance between them is 3.8×10^5 km. Find the gravitational potential at a point 3.43×10^5 km from the Earth due to both Earth and Moon.



Solution:
The distance of the point from Moon = (3.8 × 10^5) – (3.43 × 10^5) = 0.37 × 10^5 km

Since gravitational potential is a scalar, potential at the point due to Earth and Moon is just the algebraic sum of the potential at the point due to Earth, ϕ_E and the potential at the point due to Moon, ϕ_M.

Potential at the point = $\phi_E + \phi_M$

$$= \left(-\frac{GM_E}{3.43 \times 10^8}\right) + \left(-\frac{GM_M}{0.37 \times 10^8}\right)$$

$$= -(6.67 \times 10^{-11})\left[\frac{6.0 \times 10^{24}}{3.43 \times 10^8} + \frac{7.4 \times 10^{22}}{0.37 \times 10^8}\right]$$

=

4.3 Relationship summary

		at a point of distance *r* measured from a point mass *M*	two point masses *M* and *m*, distance *r* apart	Definition
vectors	$\propto \frac{1}{r^2}$	magnitude of gravitational field strength $g = \frac{GM}{r^2}$	gravitational force $F = \frac{GMm}{r^2}$	$\vec{g} \equiv \frac{\vec{F}}{m}$
scalars	$\propto \frac{1}{r}$	gravitational potential $\phi = -\frac{GM}{r}$	gravitational potential energy $U = -\frac{GMm}{r}$	$\phi \equiv \frac{U}{m}$

	field strength is negative of potential gradient	force is negative of potential energy gradient	
relationship	$g = -\frac{d\phi}{dr}$ • The negative sign on the derivative shows that if the potential ϕ increases with increasing *r*, the field strength points toward smaller *r*. • The field strength *g* is the negative of the slope of the potential curve. Plots of potential functions are valuable aids to visualizing the change of the field strength in a given region of space.	$F = -\frac{dU}{dr}$ • The negative sign on the derivative shows that if the potential energy *U* increases with increasing *r*, the force will tend to move it toward smaller *r* to decrease the potential energy. • The force *F* on *m* is the negative of the slope of the potential energy curve. Plots of potential energy functions are valuable aids to visualizing the change of the force in a given region of space.	

(i) analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes.

5.1 Satellite in Circular Orbits

In Newton's cannonball thought experiment, he visualizes a cannon on top of a very high mountain. If a gravitational force acts on the cannonball and air resistance is negligible, it will follow a different path depending on its initial velocity. If the speed is low, it will simply fall back on Earth (like path A and B). If the speed is the orbital speed at that altitude, it will go on circling around the Earth along a fixed circular orbit (just like path C or the Moon). Many man-made satellites move in circular orbits around the Earth. The first man-made satellite, the "Sputnik 1", was launched by Soviet Union in 1957. Since then, hundreds of satellites have been launched into orbit around the Earth.

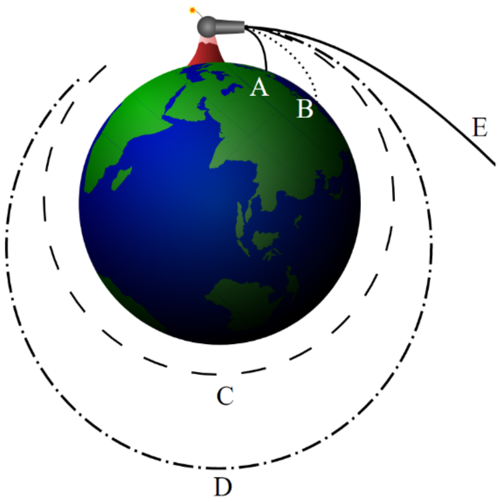


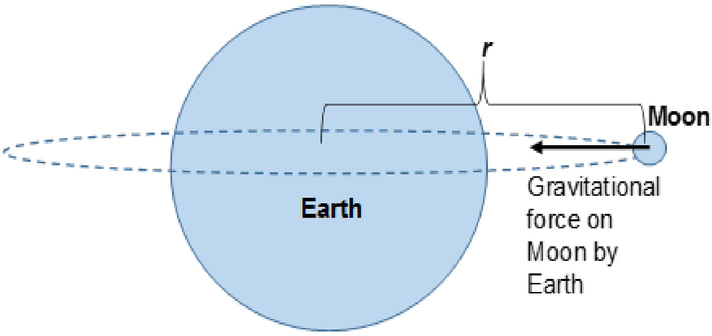
Fig. 6
Newton's cannonball

If the speed is higher than the orbital velocity, but not high enough to leave Earth altogether (lower than the escape velocity), it will continue revolving around Earth along an elliptical orbit (like path D). If the speed is very high, it will leave Earth in a parabolic (at exactly escape velocity) or hyperbolic trajectory (like path E).

5.2 Centripetal Acceleration caused by Gravitational Force

Another example of a circular motion is the motion of our Moon (of mass m), assumed to be circulating about our Earth (of mass M) in a circular orbit of radius r.

> **Note:**
> The assumption effectively simplifies the case to a Gravitational One-Body Problem, in which one object is orbiting about a point under the influence of gravity.



The **gravitational force** acting on the Moon by the Earth must **provide the centripetal force**, which causes the centripetal acceleration $a = r\omega^2$.

Mathematically, applying Newton's 2nd Law for an object with constant mass,

$$\sum F = ma \rightarrow \frac{GMm}{r^2} = m(r\omega^2) \qquad \text{- - - - - (3)}$$

From equation (3),

$$\rightarrow \quad \omega^2 = \frac{GM}{r^3} \rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3} \qquad \text{since } \omega = \frac{2\pi}{T}$$

This leads to $$r^3 = \frac{GM}{4\pi^2}T^2 \rightarrow r^3 \propto T^2 \qquad \text{since } \frac{GM}{4\pi^2} \text{ is constant}$$

This is also known as **Kepler's 3rd law**.

Example 14 ***[J90/II/2]*** **(Use of Kepler's 3rd law I)**

A planet of mass m is orbiting the Sun of mass M in a circular path of radius r.

(a) Write down an expression, in terms of G, m, M and r, for the force exerted by the Sun on the planet.

(b) Use this expression to find the angular velocity of the planet in its orbit.

(c) Deduce the time taken to complete one orbit about the Sun.

(d) The Earth is 1.50×10^{11} m from the centre of the Sun and takes exactly one year to complete one orbit. The planet Jupiter takes 11.9 years to complete an orbit about the Sun. Calculate the radius of Jupiter's orbit.

Solution:

(a) $F = \dfrac{GMm}{r^2}$

(b) $\dfrac{GMm}{r^2} = mr\omega^2 \rightarrow \omega^2 = \dfrac{GM}{r^3} \rightarrow \omega = \sqrt{\dfrac{GM}{r^3}}$

(c) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{r^3}{GM}}$

(d) Since $r^3 = \dfrac{GM}{4\pi^2}T^2$

for Earth, - - - - - (4)

for Jupiter, - - - - - (5)

(5)/(4) : $\dfrac{r^3}{(1.50 \times 10^{11})^3} = \dfrac{(11.9)^2}{1^2}$

$\rightarrow$ $r =$

Example 15 ***[J77/II/7; 94/I/8; J98/I/8]*** **(Use of Kepler's 3rd law II)**

A communications satellite which takes 24 hours to orbit the Earth is replaced by a new satellite which has twice the mass of the old one. The new satellite also has an orbit time of 24 hours. What is the ratio of

$$\frac{\text{radius of orbit of new satellite}}{\text{radius of orbit of old satellite}}?$$

A $\dfrac{1}{2}$ **B** $\dfrac{1}{1}$ **C** $\dfrac{\sqrt{2}}{1}$ **D** $\dfrac{2}{1}$

Solution: (Ans:)

(j) show an understanding of geostationary orbits and their application.

5.3 Geostationary orbits

Geostationary orbits are circular orbits made by satellites about the Earth's axis such that these satellites **would appear stationary when observed from a point on surface of the Earth**, this characteristic enables it provide permanent coverage[2] of a given wide area.

(1) They must move ______ (so that it moves in the same sense of self rotation of the Earth about its own axis);

(2) The satellite's ______ must be equal to ______ (so that it is the same as the Earth's rotational period about its own axis);

(3) Geostationary orbit is an ______ (i.e. an orbit in the plane of the Equator).

This is because the gravitational force exerted must be directed towards the centre of the Earth so its circular orbit must have its centre at the centre of the Earth. If the orbit is not over the Equator it will sometimes be over the northern hemisphere and sometimes the southern hemisphere and so cannot be geostationary.

<u>Geostationary satellites are often used for telecommunications</u>

Most commercial communications satellites, broadcast satellites and Satellite-Based Augmentation System (SBAS) satellites operate in geostationary orbits. Satellites in geostationary orbits always remain at the same location, in constant line-of-sight to ground stations on Earth's surface, allowing their dish antenna to be easily directed (fixed) to communicate.

<u>Other type of satellites and their applications</u>

Other than geostationary satellites, there are polar orbiting satellites and Global Positioning System (GPS) satellites. Polar orbits are often used for Earth-mapping, Earth observation, capturing the earth as time passes from one point, reconnaissance satellites, as well as for some weather satellites.

~ THE END ~

[2] Note that a minimum of three geostationary satellites to cover the entire Earth, except the areas near the poles.

Self-Reading: Guided Exam Question

[99/III/2] **(Long Structured Question)**
A satellite P of mass 2400 kg is placed in a geostationary orbit at a distance of 4.23×10^7 m from the centre of the Earth.

(a) Explain what is meant by the term *geostationary orbit*.

(b) Calculate
(i) the angular velocity of the satellite,
(ii) the speed of the satellite,
(iii) the acceleration of the satellite,
(iv) the force of attraction between the Earth and the satellite,
(v) the mass of the Earth,
(vi) the kinetic energy of the satellite,
(vii) the gravitational potential energy of the satellite,
(viii) the total energy of the satellite.

(c) Explain why a geostationary satellite
(i) must be placed vertically above the equator,
(ii) must move from West to East.

(d) Why is a satellite in a geostationary orbit often used for telecommunications?

Solution:

(a) **Geostationary orbits** are orbits of satellites orbiting around the Earth such that these satellites **would appear stationary when observed from a fixed location from Earth**.

(b)(i) Since the orbit is geostationary, the period of the satellite = 24 hrs

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(24)(60)(60)} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

(ii) $v = r\omega = (4.23 \times 10^7)(7.27 \times 10^{-5}) = 3075 \approx 3080 \text{ m s}^{-1}$

(iii) $a = r\omega^2 = (4.23 \times 10^7)(7.27 \times 10^{-5})^2 = 0.224 \text{ m s}^{-2}$

(iv) The force of attraction between the Earth and the satellite is the **resultant force that provides the centripetal force** for the satellite to orbit the Earth.
By Newton's 2nd law,

$$F_g = ma = mr\omega^2 = (2400)(0.224) = 537.6 \approx 538 \text{ N}$$

(v) $F_g = \dfrac{GMm}{r^2}$

$$537.6 = \frac{(6.67 \times 10^{-11})(2400)M}{(4.23 \times 10^7)^2} \rightarrow M = 6.0 \times 10^{24} \text{ kg}$$

(vi) $KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(2400)(3075)^2 = 1.13 \times 10^{10} \text{ J}$

OR

$$KE = \frac{GMm}{2r} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(2400)}{2(4.23 \times 10^7)} = 1.13 \times 10^{10} \text{ J}$$

(vii) $GPE = -\dfrac{GMm}{r} = -\dfrac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(2400)}{4.23 \times 10^7} = -2.27 \times 10^{10} \text{ J}$

(viii) $TE = GPE + KE = 1.13 \times 10^{10} + (-2.27 \times 10^{10}) = -1.14 \times 10^{10}$ J

OR

$$TE = -\frac{GMm}{2r} = -\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(2400)}{2(4.23 \times 10^{7})} = -1.14 \times 10^{10} \text{ J}$$

(c)(i) The **gravitational force by the Earth** provides the centripetal force for the satellite and is directed towards the centre of the Earth. Since the gravitational force is directed **towards the centre** of its orbit, the **centre of the orbit must be the centre of the Earth** and the **axis of rotation of the satellite is the same as the Earth**. This means that all geostationary orbits must be **vertically above** the Earth's equator**.**

(ii) In order to appear stationary from a fixed location from the Earth's surface, the period of the satellites in geostationary orbits must be the same as that of the Earth, i.e. **24 hours**. These satellites would also have to orbit about the Earth's axis of rotation and must **rotate from west to east** as the Earth rotates from west to east.

(d) A satellite in a geostationary orbit is often used for telecommunications because it will always remain at the same location, in constant electronic line-of-sight[3] to the data receiving stations on the Earth's surface, which facilitates telecommunications.

[3] The minimum distance between an emitter and the receiver of electromagnetic radiation. It is the path traversed by electromagnetic radiation without being affected by reflection or refraction en route.

ANNEXES

Extra Reading for further understanding and students interested in H3
– not explicitly required in syllabus

(A) Energies of a satellite

INQUIRY 09:

Consider a satellite of mass m in orbit around the Earth of mass M at distance r from its centre. Express the following physical quantities of the satellite in terms of G, M, m and r.

Note:
- 2013/P2/Q3
- 2011/P3/Q6
- 2009/P1/Q17

- **Gravitational potential energy**, *GPE* of the satellite

$$GPE = -\frac{GMm}{r}$$

- **Kinetic energy**, KE of the satellite

$$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{1}{2}\frac{GMm}{r}$$

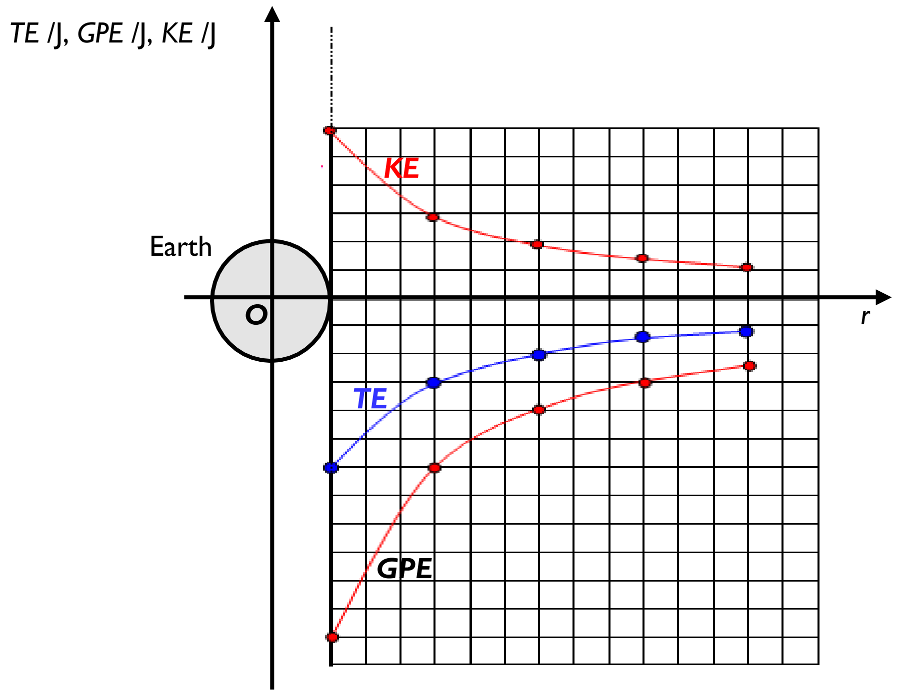
$$\left(\frac{GMm}{r^2} = \frac{mv^2}{r} \rightarrow \frac{GMm}{r} = mv^2\right)$$

- **Total energy** of the satellite, *TE*

$$TE = GPE + KE = -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r} = \left(-\frac{1}{2}\right)\frac{GMm}{r}$$

INQUIRY 10:

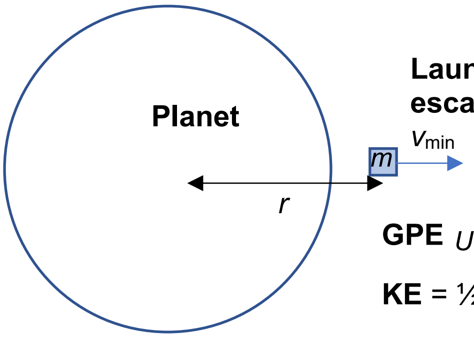
Sketch and label the graphs of *GPE*, *KE* and *TE* with respect to the distance, *r*, from the centre of Earth, *O*, of a satellite.



(B) Escape Velocity

Note:
- 2019/P1/Q12
- 2017/P2/Q2
- 2013/P3/Q1

Escape velocity (v_{min}) from a point on the **surface of a planet** of mass M is the minimum velocity required to project a mass m to infinity, i.e. escape velocity is the minimum velocity for the mass to be projected so as to totally "escape" the planet's gravitational field.



Launched at escape velocity, v_{min}

GPE $U = -\dfrac{GMm}{r}$

KE = ½ mv_{min}^2

∞ **Infinity**

GPE = 0

KE = 0

By conservation of energy,

GPE gain = KE loss

$$0-\left(-\frac{GMm}{r}\right)=\frac{1}{2}mv_{min}^2-0 \qquad \text{where } r = \text{radius of planet}$$

$$v_{min}=\sqrt{\frac{2GM}{r}}=\sqrt{2gr}$$

where g = gravitational field strength on Earth's surface = 9.81 N kg^{-1}

For Earth, $v_{min} = \sqrt{2gR_E} = \sqrt{2(9.81)(6.4\times10^6)}$ = 11 km s^{-1}!

The value of escape velocity on the Moon is ~ 2400 m s^{-1}. The Moon's surface temperature cycles between −155 °C and 100 °C. As temperature increases, the speed of molecules become more than the escape velocity (refer to the topic of "*1st Law of Thermodynamics*"). Thus, the Moon's atmosphere is negligible in comparison with the gaseous envelopes surrounding Earth.

(C) Equipotential lines in a gravitational field

Gravitational potential in a gravitational field may be represented by lines of **equal potential** indicated by **- - - -** (known as equipotential lines).

In a gravitational field, the equipotential lines are perpendicular to the field lines.

No work is done by an external force when a mass is moved between points which are lying on the same equipotential line.