KUO CHUAN PRESBYTERIAN SECONDARY SCHOOL
2023 Preliminary Examination
Secondary 4 Express / 5 Normal Academic

NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/02

Paper 2

24 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.
Setter: Ms Rebecca Kang

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use		
Paper 2		/ 90 m

Areas to Note		
	Accuracy	
	Presentation	

	Pencil	
	Units	

This document consists of **19** printed pages and 1 blank page.

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\mathrm{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae of $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 In 2005, Andrew bought a painting. In 2008, the value of the painting is estimated to be \$3878.58. The value $V(t)$, in dollars, of the painting t years after being purchased is given by the function $V(t) = 2400\,e^{kt}$, where k is a constant.

(a) What was the value of the painting after 10 years? [4]

(b) If Andrew intended to sell the painting after its value had at least doubled, which is the earliest year that he can sell it? [2]

2 **(a)** Prove that $3\cos^2 x + \sin^2 x = 2 + \cos 2x$. [3]

Hence, using your result from part (a),

(b) **(i)** find the exact value of $3\cos^2 22.5° + \sin^2 22.5°$, [2]

(ii) solve the equation $6\cos^2 x + 2\sin^2 x = 5$ for $0° \le x \le 360°$. [4]

3 It is given that $f(x) = x^3 + ax^2 + bx + 5$, where a and b are constants, has a factor of $x - 1$ and leaves a remainder of -1 when divided by $x - 2$.

(a) Find the values of a and b. [3]

(b) Using these values of a and b, factorise $f(x)$ completely into three linear factors, using surds where necessary. [3]

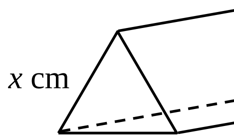
4

A miniature model in the shape of a triangular prism is shown above. A piece of wire, 36 cm in length, runs through all the edges of this triangular prism.

The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is y cm.

(i) Show that the total surface area A cm^2, of the prism is given by

$$A = \frac{\sqrt{3} - 12}{2}x^2 + 36x\,.$$ [4]



(ii) Given that x can vary, find the value of x which gives a stationary value of A. [3]

(iii) Justify if A is a maximum or a minimum for this value of x and find this stationary value of A. [3]

5 **(a)** Without using a calculator, find the values of the integers *a*, *b* and *c* for which the solution to the equation $5x - \sqrt{3} = 4 + \sqrt{3}x$ is $\frac{a + b\sqrt{3}}{c}$. [4]

(b) It is given that $sinsin\, A = \frac{3}{5}$ where A is acute, $tantan\, B = \frac{5}{12}$ and that A and B are in different quadrants. Evaluate, without using a calculator, the values of

(i) $\sin\left(\frac{\pi}{2} - B\right)$, [2]

(ii) $\tan 2A$. [2]

6 **(a)** Solve the equation $6^z(6^z - 1) = 12$. [4]

(b) A curve has the equation $y = xe^{3x}$.

(i) Find $\frac{dy}{dx}$. [2]

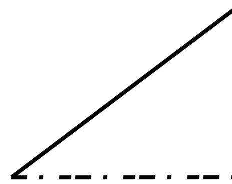
(ii) Find the value of k for which $\frac{d^2y}{dx^2} = ke^{3x}(2 + 3x)$. [3]

7

Q
q

40 m
18 m
R
P

S



The diagram shows a play area for babies that is surrounded by partitions at *PQ*, *QR* and *PS*, where *PQ* = 18 m, *PS* = 40 m, angle *SPQ* = angle *QRS* = 90^{o} and the acute angle *PSR* = q can vary.

(i) Show that *L* m, the length of all the partitions can be expressed as $L = 58 + 40 sinsin\,\theta\, - 18 coscos\,\theta$. [3]

(ii) Express L in the form $58 + R\sin(\theta - \alpha)$, where $R > 0$ and $0 \le a \le 90$. [4]

(iii) Given that the exact length of all the partitions is 74 m, find the value of q . [2]

8 The equation of a curve is $y = x\sqrt{x+1}$.

(a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\sqrt{x+1}) = \frac{3x+2}{2\sqrt{x+1}}$. [4]

(b) A particle moves along the curve such that the *x*-coordinate is moving at a constant rate of 0.2 units per second. Find the rate of change of the *y*-coordinate at the point where $x = 5$. [2]

(c) Using your result from **part (a),** evaluate $\int_{-1}^{3} \frac{x}{2\sqrt{x+1}}\,dx$. [4]

9 The equation of a circle with centre C is $x^2 + y^2 - 8x - 2y - 152 = 0$.

(i) Find the coordinates of C and the radius of the circle. [4]

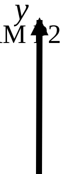
(ii) Show that the point $P(9, 13)$ lies on the circle. [1]

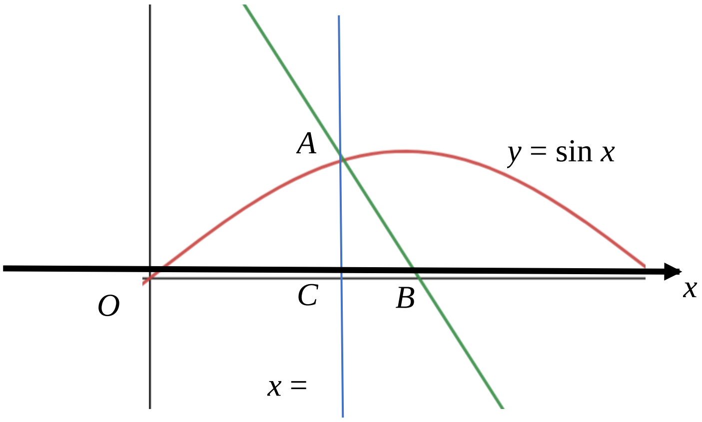
(iii) A line, *l* is parallel to the tangent to the circle at *P*. *Q* is a point on line *l* such that *P* is the midpoint of the line *QC*.

(a) Find the equation of the line *l*. [5]

(b) Explain if the line *l* will intersect the circle. [1]

10



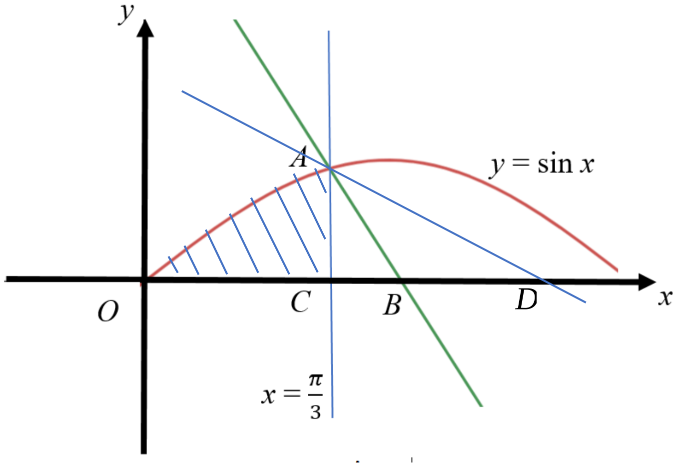


The diagram shows part of the curve of $y = \sin x$.

A is a point on the curve that lies on the line $x = \frac{p}{3}$. The line AC is perpendicular to the x-axis.

(i) If the normal of the curve at point A meets the x-axis at B, find the exact coordinates of A and B. [6]

(ii)



D is a point on the x-axis such that the shaded area OAC and triangle ACD are equal.
Find the distance BD in the form $\frac{m\sqrt{3}}{n}$. [6]

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