2024 JC1 H2 Mathematics (9758)

Year-End Examination Revision Package

Name:______________________________ **Class:**__________

Contents

Worked solutions will be uploaded on JPJC Math Google Site on 30/8 (Topical 1-3), 9/9 (Topical 4-6), 18/9 (Revision A-D)

INFORMATION FOR STUDENTS

JC1 H2 Mathematics Year-End Examination:	**27 Sept 2024 (Fri)**

SCHEME OF EXAMINATION PAPERS

For the year-end examination in H2 Mathematics, it will be a 3 hour paper consisting of about 10 to 13 questions of different lengths and marks based on the following Pure Mathematics topics:

Topic 1: Vectors
Topic 2: Graphing Techniques I & II
Topic 3: Equations and Inequalities
Topic 4: Functions
Topic 5: Differentiation Techniques & Applications
Topic 6: Integration Techniques

(Chp 1 to 8 of Lecture Notes)

The total number of marks is 100.

Candidates will be expected to answer all questions.

USE OF GRAPHIC CALCULATOR (GC)

The use of GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

Topic 1: Vectors

1. [CJC/2016/Promo/3]

The position vectors of the points A, B and C with respect to the origin O are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Point C is on AB produced such that $5AB = 3AC$. Given that $\mathbf{a} \bullet \mathbf{c} = 2$, $\mathbf{a}$ is a unit vector and the angle AOB is $\frac{\pi}{3}$, show that $|\mathbf{b}| = \frac{16}{5}$. [5]

2. [HCI/2016/Promo/7]

Relative to the origin O, three distinct fixed points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. It is known that $\mathbf{b}$ is a unit vector, $|\mathbf{a}| = 3$, $|\mathbf{c}| = 2$ and angle $AOC = 60°$.

(i) State the geometrical interpretation of $|\mathbf{b} \bullet \mathbf{c}|$. [1]

It is further given that $\mathbf{a} \times 2\mathbf{b} = \mathbf{b} \times \mathbf{c}$.

(ii) Find the ratio of the area of triangle AOB to the area of triangle BOC. [2]

(iii) Show that $2\mathbf{a} + \mathbf{c} = k\mathbf{b}$ where $k \in \mathbb{R},\ k \neq 0$.

By considering $(2\mathbf{a} + \mathbf{c}) \bullet (2\mathbf{a} + \mathbf{c})$, find the exact values of k. [6]

3. [HCI/2016/P1/3]

Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ with $\mathbf{a}$ not parallel to $\mathbf{b}$. The point P is on AB produced with $AP : AB = 3:1$ and the position vector of point Q is $2\mathbf{a}$.

(a) Find the position vector of the point of intersection of lines OB and PQ, giving your answer in terms of $\mathbf{b}$. [4]

(b) It is given that $\mathbf{a} \times \mathbf{b} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and the point $C(0,\ 3,\ 4)$ does not lie on the plane OAB. Find the coordinates of the foot of the perpendicular from C to the plane OAB. [4]

4. [MJC/2016/PROMO/9]

The points A and B have coordinates $(0,2,0)$ and $(1,0,2)$ respectively, and the plane π_1 has equation $\mathbf{r} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 4$.

(i) Find a vector equation for the line AB. [2]

(ii) Show that the line AB lies in plane π_1. [1]

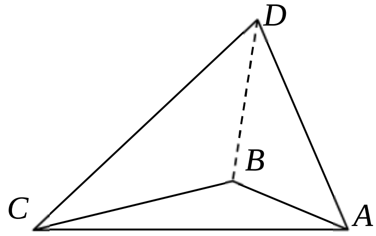
(iii) The plane π_2 contains the line AB and is perpendicular to plane π_1. Show that the equation of plane π_2 is $\mathbf{r} \bullet \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = 2$. [3]

(iv) The point $P(p,0,0)$ is equidistant from the planes π_1 and π_2. Calculate the value of p. [5]

(v) Hence write down an equation of a line that is equidistant from planes π_1 and π_2 which does not intersect both planes π_1 and π_2. [2]

5. [DHS/2016/P1/11]

The diagram below shows a tetrahedron *ABCD*. The equation of the plane *ABD* is $4x+y+2z=16$.



(i) Given that *A* is on the *x*-axis, find the coordinates of *A*. [1]

The equation of the plane *CBD* is $7x-11y-5z=-23$.

(ii) Find a vector equation of the line that passes through *B* and *D*. [2]

(iii) Given that *B* is on the *xy*-plane, find the coordinates of *B*. [2]

The cartesian equation of the line that passes through *A* and *D* is $\dfrac{4-x}{2}=\dfrac{y}{2}=\dfrac{z}{3}$.

(iv) Find the coordinates of *D*. [3]

The coordinates of *C* are (−1, 1, 1).

(v) By considering the area of triangle *ABC*, find the exact volume of the tetrahedron *ABCD*. [5]

[Volume of tetrahedron = $\frac{1}{3}\times$area of base$\times$perpendicular height]

6. [MI/2016/PROMO/13]

The lines l_1 and l_2 have equations $\mathbf{r}=\begin{pmatrix}4\\1\\10\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-6\end{pmatrix}$, $\lambda\in\mathbb{R}$ and $\mathbf{r}=\begin{pmatrix}1\\-2\\4\end{pmatrix}+\mu\begin{pmatrix}4\\3\\4\end{pmatrix}$, $\mu\in\mathbb{R}$ respectively.

(i) Find the acute angle between l_1 and l_2. [2]

(ii) Find a vector equation of the line l_3 which passes through the point *A* with coordinates (1, −2, 4) and is perpendicular to both l_1 and l_2. [2]

(iii) The plane p_1 contains both l_1 and l_2. Find the equation of p_1 in scalar product form. [2]

(iv) Show that the point *B* with coordinates (−5, 10, 1) lies on l_3.

By deducing the position vector of the point of intersection of l_3 and p_1, find the exact perpendicular distance from *B* to p_1. [3]

(v) The plane p_2 has equation $\mathbf{r} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = k$, where *a*, *b*, *c*, and *k* are constants.

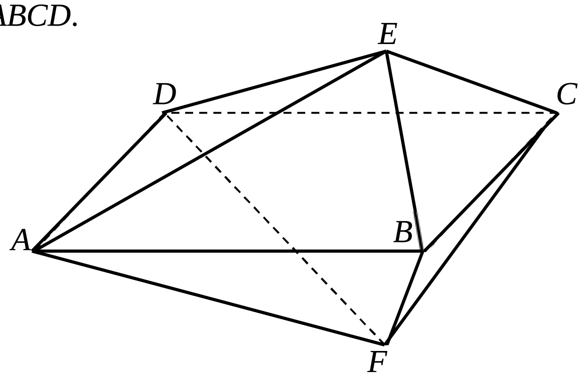
What can be said about the values of *a*, *b*, *c* and *k* when

(a) plane p_2 and line l_2 are perpendicular to each other, [2]

(b) planes p_1 and p_2 do not intersect? [2]

7. [MJC/2011/Promo/12]

In the figure below, the pyramids *ABCDE* and *ABCDF* are symmetrical about the parallelogram *ABCD*.



Referred to an origin *O*, the position vectors of the points *A*, *B* and *E* are $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$,

$7\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $4\mathbf{i} + 7\mathbf{j} - \mathbf{k}$ respectively. The line *l*, which lies on the plane *ABCD* has vector equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}, \ \lambda \in \mathbb{R} .$$

(i) Find a vector equation of the plane *ABCD* in the form **r . n** = *d*. [3]

(ii) Find the acute angle between the lines *AE* and *AB*. [2]

(iii) Find the position vector of the foot of the perpendicular from the point *E* to the plane *ABCD*. Hence find the shortest distance from point *E* to the plane *ABCD*. [4]

(iv) Find a vector equation of the line *AF*. [2]

8. The line *l* has equation $\dfrac{x-3}{-3} = y+3 = \dfrac{z-1}{-2}$ and the plane *p* has equation $3x - y + 2z = 0$.

(i) Show that *l* is perpendicular to *p*. [2]

(ii) Find the coordinates of the point of intersection of *l* and *p*. [3]

(iii) Show that the point *C* with coordinates $(-9,1,-7)$ lies on *l*. Find the coordinates of the point *C'* which is the mirror image of *C* in *p*. [3]

Topic 2: Graphing Techniques I & II

1 [RJC/2014/Promo/3]

The curve C has equation

$$\frac{(x-1)^2}{9}-y^2=1.$$

(i) Sketch the curve C, showing clearly the equations of asymptotes, axial intercepts and coordinates of turning points, if any. [3]

(ii) Given that k is a positive constant and C intersects the curve with equation

$$y^2+\frac{x^2}{k^2}=1$$

at exactly two distinct points, state the range of values of k. [2]

2. [AJC/2016/Promo/6]

A curve C has equation $y=\dfrac{2x^2-5ax+2a^2+1}{x-a}$, $x\neq a$.

(i) Find the equations of the asymptotes, giving your answers in terms of a. [2]

(ii) If curve C has no stationary points, find the range of values of a. [3]

(iii) Sketch curve C for $a>1$, stating clearly the equations of the asymptotes and the y-intercept of the curve only. [3]

3. [IJC/2016/Promo/11]

The curve C has equation

$$y=\frac{x^2+4x+\lambda-5}{x+r},$$

where λ is a non-zero constant, and a vertical asymptote $x=1$.

(i) State the value of r and find the equation of the other asymptote of C. [3]

(ii) Draw a sketch of C for the case when $\lambda<0$. [2]

(iii) By using an algebraic method, find the range of values of λ for which the line $y=2x$ and C have at least one point in common. [3]

It is now given that $\lambda=9$.

(iv) On a separate diagram from part **(ii)**, sketch the graph of C, indicating clearly the coordinates of the stationary points. [2]

Another curve D has equation $\dfrac{(x-4)^2}{4^2}+\dfrac{(y-6)^2}{k^2}=1$.

(v) On the same diagram as part **(iv)**, sketch D for the case when $k=3$. [2]

(vi) Deduce the range of values of k for which C and D intersect more than once. [1]

4. [MJC/2016/Promo/3]

In this question, a, b, c and d are non-zero constants.

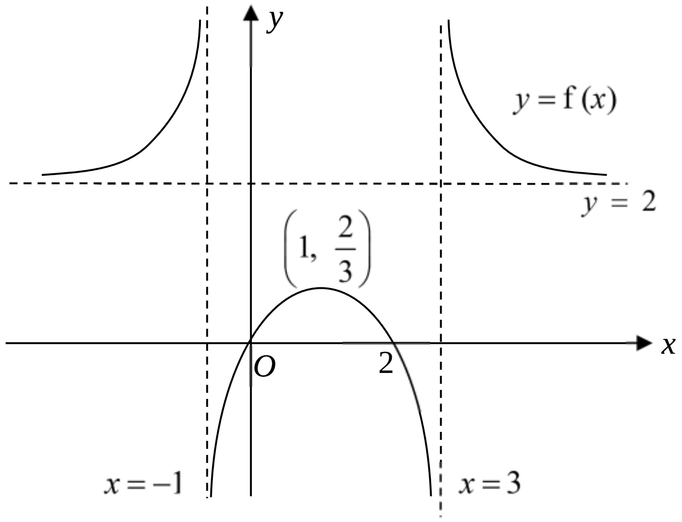
The curve C has equation $y = \frac{ax^2 + bx - 7}{x - c}$ with oblique asymptote $y = dx + 2$ and vertical asymptote intersecting at $(4, 6)$.

(i) Find the values of a, b, c and d. [3]

(ii) Sketch C. [3]

(iii) Hence find the set of values of k such that the equation $\frac{ax^2 + bx - 7}{x - c} = k$ has no real roots. [2]

5. [JJC/2015/Promo/5]



The graph of $y = \mathrm{f}(x)$ is given above. Sketch, on separate diagrams, the graphs of

(i) $y = \mathrm{f}(2x + 3)$, [3]

(ii) $y = \mathrm{f}'(x)$, [3]

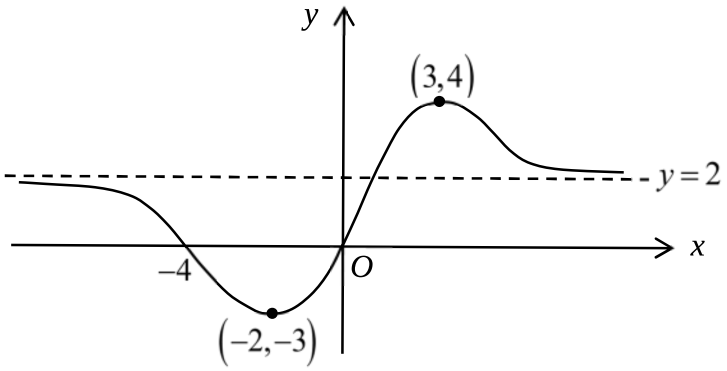
(iii) $y = \frac{1}{\mathrm{f}(x)}$. [3]

Your sketches should show clearly the coordinates of any stationary points, axial intercepts and the equations of any asymptotes.

6. [NJC/2014/Promo/8]

(a) State a sequence of transformations which can transform the graph of $y=\frac{1}{x}$ to the graph of $y=\frac{4}{3-2x}$. [3]

(b) The diagram below shows the graph of $y=\frac{1}{\mathrm{f}(x)}$. The curve has turning points at $(-2,-3)$ and $(3,4)$, and passes through the origin. The line $y=2$ is an asymptote to the curve.



Sketch, on separate clearly labelled diagrams, the graphs of

(i) $y=\mathrm{f}(x)$, [3]

(ii) $y=\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{\mathrm{f}(x)}\right]$, [3]

including the coordinates of any points where the graphs intersect the x- and y-axes, the coordinates of any turning points (where applicable) and the equations of any asymptotes (where applicable).

Topic 3: Equations and Inequalities

1. [AJC/2014/Promo/1]

Without using a calculator, solve the inequality

$$\frac{x}{3x-4} > \frac{1}{x}.$$ [4]

2. [MI/2016/Pre-U 1 Promo 2/Q6]

(i) Sketch the curve C with equation $y = \frac{x-2}{x-1}$, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) By adding an additional line in your sketch, solve the inequality $\frac{x-2}{x-1} \leq 2 - x$. [3]

(iii) Hence solve $\frac{|x|-2}{|x|-1} \leq 2 - |x|$. [3]

3. [MI/2016/Pre-U 2 Promo1/2]

(i) Sketch, on the same diagram, the curves with equations $y = \ln(x-2)$ and $y = \frac{1}{\mathrm{e}^x} + 3$, stating the equations of the asymptotes. [2]

(ii) Hence solve $\frac{1}{\mathrm{e}^x} + 3 \geq \ln(x-2)$. [1]

(iii) Without using a calculator, solve $\frac{1}{\mathrm{e}^{|x|}} + 3 \geq \ln(|x|-2)$. [2]

4. [NJC/2014/MYE/9(a)]

Show algebraically that $x^2 + 2x + 2$ is always positive for all real values of x. [1]

Without using a calculator, solve the inequality $\frac{x^2+2x+2}{6-x-x^2} \leq 0$. [2]

Hence, solve the inequality $\frac{x+2\sqrt{x}+2}{x+\sqrt{x}-6} \geq 0$. [2]

5. [MI/2014/PU1 Promo/3]

(i) Without the use of a calculator, solve the inequality $\frac{3x^2+1}{2x^2+3x-2} > 1$. [4]

(ii) Hence solve the inequality $\frac{3x^2+1}{2x^2+3|x|-2} > 1$. [2]

6. [HCI/2014/MYE/3]

The equation of a curve is given by

$$Ax^2 + By^2 + Cx + Dy + 13 = 0$$

where A, B, C and D are constants.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

The curve has a stationary point at $(1, -1)$ and the tangent to the curve at $(3, -2)$ is parallel to the y-axis.

(ii) Find the values of A, B, C and D. [4]

7. [CJC/2018/Promo/Q2]

The graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = ax^2 + bx + c$, passes through the point $(10, -28)$.

When the graph of $y = \mathrm{f}(x)$ is scaled parallel to the x–axis by factor 2, the resulting graph passes through the point $(-5, -3)$.

It is also observed that the graph of $y = \mathrm{f}(|x|)$ passes through the point $(-5, -3)$.

Find the values of the constants a, b and c. [4]

Topic 4: Functions

1. [MI/2014/MYE PU1/13]

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 - 6x + 14, \qquad x \in \mathbb{R}.$$

(i) Explain why f is not a one-one function. [2]

(ii) If the domain of f is restricted to the subset of $\mathbb{R}$ for which $x \le a$, find the maximum value of a such that f is a one-one function. [1]
Find an expression for f^{-1} for this new domain. [3]

2. [DHS/2014/Promo/2]

The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{1}{x} - 1, \qquad x \ne 0,$$
$$\mathrm{g}: x \mapsto (x+2)^2, \qquad x > -2.$$

(i) Show that $\mathrm{fg}(x) = -\dfrac{(x+1)(x+3)}{(x+2)^2}$. State the domain of fg. [3]

(ii) Hence use an algebraic method to solve the inequality $\mathrm{fg}(x) \ge 0$. [3]

3. [HCI/2014/Promo/12]

The function f is defined by $\mathrm{f}: x \mapsto \cos\left(\dfrac{\pi x}{2}\right), \quad x \in \mathbb{R}, -2 < x \le 0$.

(i) Sketch the graph of $y = \mathrm{f}(x)$, indicating clearly the coordinates of all axial intercepts and end points. [2]

(ii) Show that f^{-1} exists, and find its rule and domain. [4]

The function g is defined by $\mathrm{g}: x \mapsto (2x+1)^{\frac{2}{3}}, \quad x \in \mathbb{R}, -2 < x \le 2$.

(iii) Find the set of values of x such that $\mathrm{g}(x) \ge \mathrm{f}(x)$. [4]

(iv) Explain clearly why gf exists. Hence, find the range of gf. [3]

4. [JJC/2014/Promo/8]

The function f is defined by $\mathrm{f}: x \mapsto \dfrac{x-1}{x}$, $x \in \mathbb{R}$, $x \ne 0$ and $x \ne 1$.

(i) Define f^{-1}. [3]

(ii) Show that the composite function f^2 exists and that $\mathrm{f}^2(x) = \mathrm{f}^{-1}(x)$. [3]

(iii) State $\mathrm{f}^3(x)$. [1]

(iv) Given that $\mathrm{f}^{2014}\mathrm{g}(x) = 1 - \mathrm{e}^{-x}$ for $x > 0$, find $\mathrm{g}(x)$. [3]

5. [CJC/2018/Prelim/1/7]

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 + 4x - 5, \quad \text{for } x \le k, k \in \mathbb{R}.$$

(i) Find the largest exact value of k such that f^{-1} exists. For this value of k, define f^{-1} in a similar form. [4]

Another function g is defined by

$$\mathrm{g}: x \mapsto \begin{cases} 4 - x^2, & \text{for } 0 < x \le 2 \\ 2x - 4, & \text{for } 2 < x \le 4 \end{cases}$$

and that g(x) = g(x + 4) for all real values of x.

(ii) Sketch the graph of $y = \mathrm{g}(x)$ for $-1 < x \le 7$. [3]

(iii) Using the results in part **(i)** and **(ii)**, explain why composite function $\mathrm{f}^{-1}\mathrm{g}$ exists and find the exact value of $\mathrm{f}^{-1}\mathrm{g}(6)$. [4]

6. [ACJC/2014/MYE /8]

The function f is given by

$$\mathrm{f}: x \mapsto \frac{1}{4-(2x+1)^2}, \quad x \in \mathbb{R}, \ x \le -\frac{1}{2}, \ x \ne -\frac{3}{2}.$$

(i) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram, giving the equations of asymptotes, if any. Define f^{-1}in a similar form. [6]

(ii) Explain why the x-coordinate of the point of intersection of the curves in (i) satisfy the equation $4x^3 + 4x^2 - 3x + 1 = 0$ and find the coordinates of this point of intersection. [2]

(iii) The function g is defined by

$$\mathrm{g}: x \mapsto k - e^x, \quad x \in \mathbb{R}, \quad x \le 0, \quad \text{where } k \text{ is an } \textbf{integer}.$$

Find the range of g in terms of k. Determine the largest value of k such that the composite function fg exists and find its range. [3]

Topic 5: Differentiation Techniques & Applications

1. [ACJC/2013/Promo/3(b)] and [CJC/2013/Promo/4]

(a) Differentiate $\ln\sqrt{\frac{(x+1)^3}{x^2-1}}$ with respect to x. [2]

(b) Given that $y = \tan^{-1}\sqrt{x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(c) Given that $\sqrt[x]{y} = \sqrt[y]{x}$, where $x > 0$, $y > 0$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [4]

2. [MJC/2013/Promo/7]

The curve C has equation $y = \frac{x^2}{x-1}$.

Points $A(x_1, y_1)$ and $B(x_2, y_2)$ lie on curve C such that the tangent at A is parallel to tangent at B where $x_2 < x_1$. Given further that the equation of tangent at A is $y = \frac{8}{9}x + \frac{16}{9}$, find the coordinates of B, and hence find the equation of normal at point B. [6]

3. [IJC/2013/Promo/8]

The equation of a curve is $x^2 - 2xy + 2y^2 = 20$.

(i) Find the equations of the tangent and normal to the curve at the point $P(2,4)$. [5]

(ii) The tangent at P meets the y-axis at A and the normal at P meets the x-axis at B. Find the area of triangle APB. [3]

4. [JJC/2013/Promo/10(b)]

The equation of a curve C is $x^2 - 2xy + 2y^2 = k$, where k is a constant.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

Given that C has two points for which the tangents are parallel to the line $y = x$, find the range of values of k. [3]

Given that $k = 4$, find the exact coordinates of each point on the curve C at which the tangent is parallel to the y-axis. [4]

5. [PJC/2018/Promo/10]

The curve C has parametric equations

$$x = 2t^2, \qquad y = 4t.$$

(i) Find the equations of the tangent and normal to C at the point with parameter p. [3]

(ii) The normal at the point $(18, -12)$ cuts C again at the point Q. Find the coordinates of Q. [5]

(iii) Tangents are drawn from the point $(-2, -3)$ to C. Show that the tangents are perpendicular. [3]

6. [TJC/2013/Promo/10]

A curve C is given parametrically by the equations $x = 2\cos^3\theta$, $y = 2\sin^3\theta$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

Show that the normal at the point with parameter θ has equation

$$y\sin\theta = x\cos\theta + 2\left(\sin^4\theta - \cos^4\theta\right). \qquad [4]$$

The normal at the point Q where $\theta = \frac{\pi}{6}$, cuts C again at the point P, where $\theta = p$. Show that $\sin^3 p - \sqrt{3}\cos^3 p + 1 = 0$ and hence find the coordinates of P. [5]

7. [NJC/2013/Promo/6]

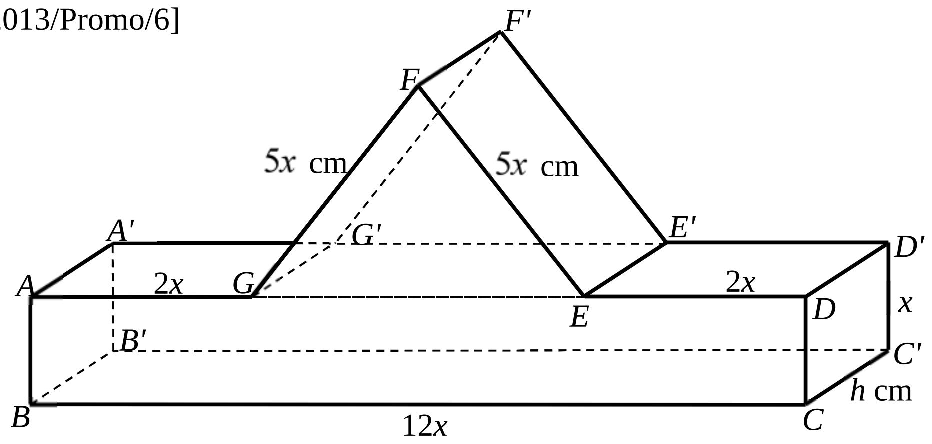


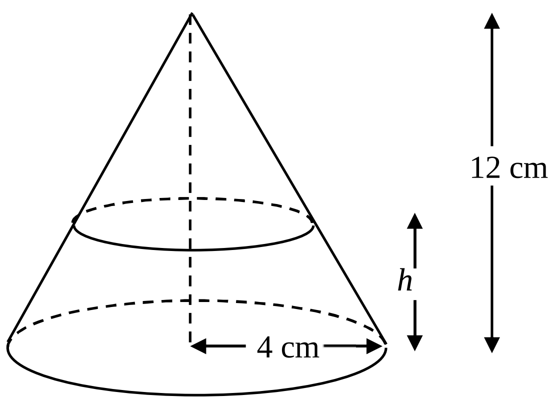
Figure 1

A solid model as shown in Figure 1 is constructed by attaching a triangular prism to a rectangular cuboid. The model has a volume of 28224 cm^3 and a surface area of S cm^2.

(i) Express h in terms of x. [2]

(ii) Using differentiation, show that S is minimum when $x = 7$. Justify your answer. [4]

8. [SRJC/2013/Promo/12(a)]

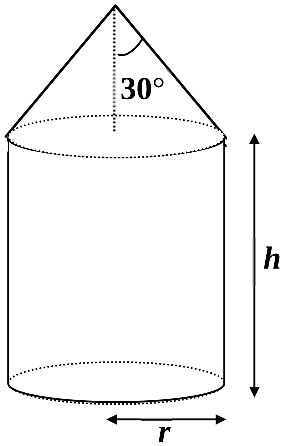


An upright cone, with a closed circular base, is shown in the diagram above. It has a circular base radius of 4 cm and height 12 cm and is initially full of water. Water is leaking from the circular base of the cone at a rate of 2π cm^3 min^{-1}. If h is the depth of water in the cone at time t minutes, show that the volume V, of water remaining, V, in the cone at time t minutes is given by $V = 64\pi - \frac{\pi}{27}(12-h)^3$.

Hence find the rate of change of the depth of water when the depth of water is 6 cm. [4]

9. [TPJC/2013/Promo/3]

The closed container shown below consists of a cylinder of height h cm fixed to a cone. The cylinder and cone have the same base radius r cm and the slant height of the cone is 30° to the vertical. The volume of the container is 70 cm^3.



(i) Show that the surface area, A cm^2, of the closed container is

$$A = 3\pi r^2 - \frac{2\sqrt{3}\pi r^2}{3} + \frac{140}{r}.$$ [3]

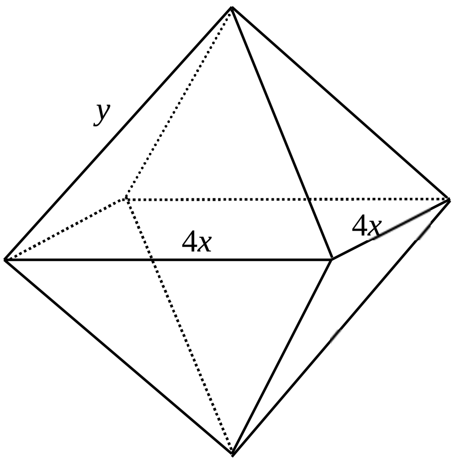
(i) Find the value of r that gives a minimum surface area, leaving your answer to 3 significant figures. [3]

[The volume of a cone of radius r and height h_1 is $\frac{1}{3}\pi r^2 h_1$. The curved surface area of a cone of radius r and slant height l is $\pi r l$.]

10. [PJC/2014/MYE/Q9]

As part of an Art Project, a student designed a lantern in the shape of an octahedron as shown in the diagram. The frame of the lantern consists of twelve straight pieces of wire, joined at their ends to form two identical right pyramids with a square base of sides $4x$ m and slant edges of y m. The wire frame is covered tightly with coloured paper.

The wire cost \$8 per metre and the coloured paper cost \$15 per square metre. The student is required to include the budget for his lantern in this project. Given that the total length of the wire used is 3.2 m, use differentiation to find the maximum amount for his budget, giving your answer to the nearest dollar. [10]



Topic 6: Integration Techniques

1. [CJC/2008/Promo/1]

(a) (i) Express $\dfrac{x^2-3x+7}{(1-x)(4+x^2)}$ in partial fractions. [4]

(ii) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$, where $\mathrm{f}(x)=\dfrac{x^2-3x+7}{(1-x)(4+x^2)}$. [2]

(b) By substituting $u=\sqrt{3x^2+5}$, find the integral $\int 3x\sqrt{3x^2+5}\,\mathrm{d}x$. [3]

2. [CJC/2009/Promo/13]

(a) Find $\displaystyle\int \frac{1}{9-4x^2}\,\mathrm{d}x$. [3]

(b) Using the substitution $\sqrt{x}=\cos\theta$, find $\displaystyle\int \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$. [4]

(c) Evaluate $\int_1^{\mathrm{e}} x(\ln x)^2\,\mathrm{d}x$, giving your answer in exact form. [4]

3. [NJC/2009/Promo/10(a)]

(i) Find the derivative of e^{x^2+2x}. [1]

(ii) Hence, find $\int (x+1)^3\,\mathrm{e}^{x^2+2x}\,\mathrm{d}x$. [3]

4. [ACJC/2010/Promo/9]

Find the following integrals.

(a) $\displaystyle\int \frac{\cos 3x-\operatorname{cosec}^2 3x}{\sin 3x+\cot 3x}\,\mathrm{d}x$ [2]

(b) $\displaystyle\int \frac{1-x}{\sqrt{1-16x^2}}\,\mathrm{d}x$ [4]

(c) $\int (1-x)^{-2}\ln x\,\mathrm{d}x$ [4]

5. [JJC/2010/Promo/12]

(a) Write down constants A and B such that, for all values of x,

$$x+4=A(2x+6)+B.$$ [2]

Hence find $\int \frac{x+4}{x^2+6x+13}\,\mathrm{d}x$. [4]

(b) Using the substitution $x=\frac{1}{u}$, show that $\int_2^4 \frac{1}{x^3}\mathrm{e}^{\frac{1}{x}}\,\mathrm{d}x=\int_{\frac{1}{4}}^{\frac{1}{2}} u\mathrm{e}^u\,\mathrm{d}u$. [2]

Hence evaluate $\int_2^4 \frac{1}{x^3}\mathrm{e}^{\frac{1}{x}}\,\mathrm{d}x$, giving your answer in exact form. [3]

6. [SAJC/2013/Promo/10]

(a) Find the exact value of k such that $\int_k^{\infty} \frac{1}{9+x^2}\mathrm{d}x=\int_{\frac{\sqrt{3}}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{1-4x^2}}\mathrm{d}x$,

where k is a positive constant. [3]

(b) By using the substitution $x=\sin\theta$, show that $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,\mathrm{d}x=\frac{\pi}{12}-\frac{\sqrt{3}}{8}$. [5]

Hence, or otherwise, find the exact value of $\int_0^{\frac{1}{2}} x\sin^{-1}x\,\mathrm{d}x$. [3]

7. [HCI/2014/Promo/7]

(a) Find $\int x\ln\left(x^2-4\right)\mathrm{d}x$. [4]

(b) Use the substitution $x=a\cos u$ to find $\int_0^{\frac{a}{2}} \sqrt{a^2-x^2}\,\mathrm{d}x$ where a is a positive constant. Leave your answer in exact form. [5]

H2 Mathematics (9758) **JC1 2024**
Year-End Revision Paper A **Time : 3h** **Marks :100**

1 **(i)** On the same axes, sketch the graphs of $y=\dfrac{1}{x-2}$ and $y=4x-2$, indicating clearly the equations of the asymptotes and the coordinates of the axial intercepts. [2]

(ii) Hence, or otherwise, solve the inequality $\dfrac{1}{x-2}\le 4x-2$. [3]

2 **(a)** Differentiate the following with respect to *x*.

(i) $\ln\left(9+3e^{3x}\right)$, [1]

(ii) $\sin^4 x^2$, [2]

(iii) $\sec 3x \sin^{-1} 2x$. [3]

(b) A curve has equation $y^3-4y+x^2-9x+10=0$. Find $\dfrac{dy}{dx}$ in terms of *x* and *y*. [2]

3 By using the substitution $u=1+t^3$, find the exact value of $\displaystyle\int_0^2 \frac{t^5}{\left(1+t^3\right)^3}\,dt$ without using a calculator. [5]

4 The curve *C* has equation $y=\dfrac{x^2+kx+4}{x+1}$, where *k* is a constant.

(i) Find the range of values of *k* if *C* has two distinct stationary points. [3]

For the rest of this question, use $k=4$.

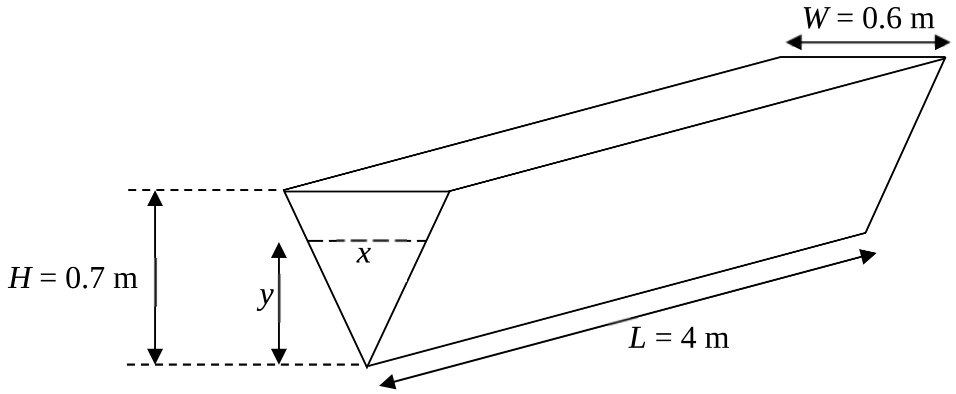
(ii) Sketch the graph of *C*, indicating clearly the equations of any asymptotes and the coordinates of any turning points and axial-intercepts. [3]

(iii) Using your sketch in **(ii)**, find the range of values of *m*, where *m* is a positive real number for which the equation

$$x^2+kx+4-(mx+3)(x+1)=0$$

has no real roots. [2]

5 The diagram shows a V-shaped tank with dimensions $L = 4$ m, $W = 0.6$ m and $H = 0.7$ m. The tank is initially empty. Water is pumped into the tank at a rate of 0.0025 m^3/s. At any instant from the start of water flowing into the tank, the water in the tank has a depth of y m and a surface width of x m.



(i) Find the rate of change of the water depth when $y = 0.4$ m, leaving your answer to 4 decimal places. [4]

(ii) Find, to the nearest second, the time taken to completely fill up the tank from the instant when $y = 0.4$ m. [2]

6 The equation of a curve $y = \mathrm{f}(x)$ is in the form of a cubic polynomial. The curve has a maximum point at $(-1, 0)$ and passes through the point with coordinates $(4, 25)$. The tangent to the curve at $x = 3$ is parallel to the line with equation $y = 24x + 5$. Find $\mathrm{f}(x)$. [5]

7 Find

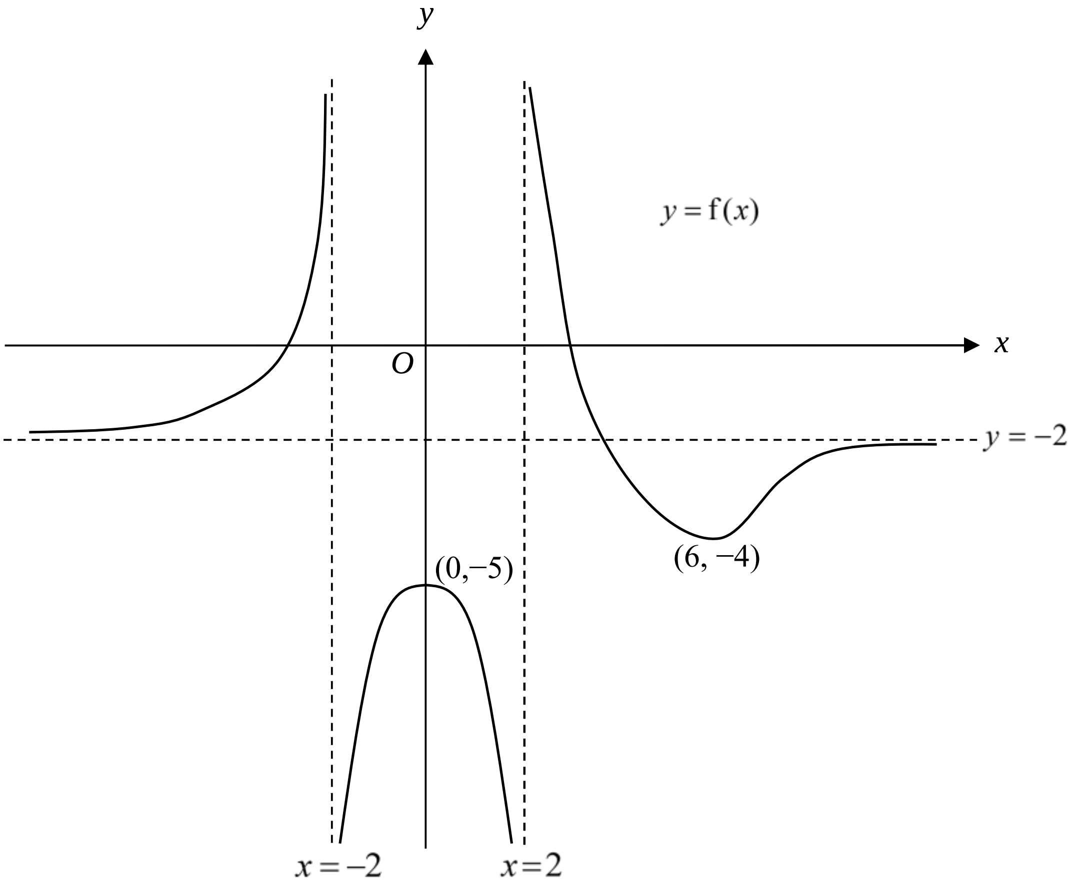
(a) $\displaystyle\int \frac{10\mathrm{e}^x}{5 - 2\mathrm{e}^x}\,\mathrm{d}x$, [2]

(b) $\displaystyle\int \frac{x}{\sqrt{1+8x^2}}\,\mathrm{d}x$, [2]

(c) $\displaystyle\int x(\ln x)^2\,\mathrm{d}x$. [5]

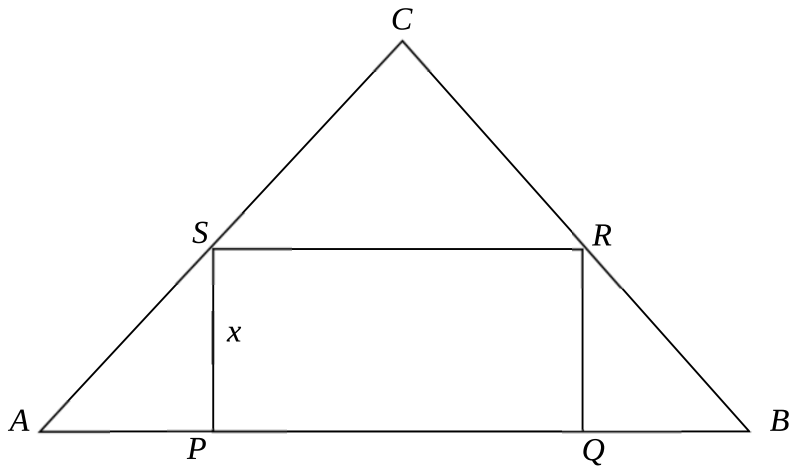
8 **(a)** By expressing the equation of the curve $y = \dfrac{12x+11}{2x+1}$ in the form $y = A + \dfrac{B}{2x+1}$, where A and B are constants, describe a sequence of three transformations which maps the graph of $y = \dfrac{1}{2x-3}$ onto the graph of $y = \dfrac{12x+11}{2x+1}$. [4]

(b) The diagram shows the graph of $y = \mathrm{f}(x)$. The curve has a maximum point at $(0, -5)$ and a minimum point at $(6, -4)$. The equations of the asymptotes of the curve are $x = -2$, $x = 2$ and $y = -2$.



Sketch the graph of $y=\mathrm{f}\left(-x+2\right)+4$, indicating clearly the equations of the asymptotes and the coordinates of the axial intercepts and turning points. [3]

9



In the isosceles triangle *ABC*, $AC = BC$, $AB = 20$ cm and $\angle BAC = 30^\circ$. A rectangle *PQRS* is inscribed in *ABC* with points *P* and *Q* on *AB*, point *R* on *BC* and point *S* on *AC* as shown in the diagram.

Taking *PS* to be *x* cm, show that the area of *PQRS* may be expressed as $20x - 2\sqrt{3}x^2$. [2]

Hence, as *x* varies, find the exact values of *PS* and *PQ* such that the area of *PQRS* is a maximum, and find the corresponding area of *PQRS*. [4]

10 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point C lies on *OA* such that $OC : CA = 2 : 1$. The point D lies on *OB* produced such that $OD : BD = 4 : 1$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of **a** and **b**. [2]

(ii) The lines *BC* and *AD* meet at the point *E*. Show that *E* has position vector $4\mathbf{b} - 2\mathbf{a}$. [4]

(iii) Show that the area of triangle *CDE* can be written as $k|\mathbf{a} \times \mathbf{b}|$, where *k* is a constant to be found. [4]

11 A curve *C* has parametric equations

$$x = 3t^2, \qquad y = 6t^3, \qquad \text{where } t \text{ is a real parameter.}$$

(i) Sketch *C*. [1]

(ii) Find the exact coordinates of the point *P* on *C* where the tangent is parallel to the line $y = 4 - 2x$. [3]

(iii) Show that the equation of the tangent to *C* at the point $Q\left(\frac{4}{3}, \frac{16}{9}\right)$ is $9y = 18x - 8$. [2]

(iv) The tangent at *Q* cuts the *x*-axis at the point *R*. Find the area of triangle *PQR*. [2]

(v) The tangent at *Q* cuts *C* again at the point *S*. Find the coordinates of *S*. [3]

12 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{2x}{x^2 - 1}, x \in \mathbb{R}, x > 1$.

(i) Sketch the graph of f and show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state its domain. [4]

(iii) Write down the equation of the line in which the graph of f must be reflected in order to obtain the graph of f^{-1} and hence find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

13 The planes p_1 and p_2 have equations $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + s\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \\ 12 \end{pmatrix}$ and $x + 3y = 1$ respectively, where s and t are parameters.

(i) Find the line of intersection of p_1 and p_2. [3]

(ii) Find the acute angle between p_1 and p_2. [2]

The point A has position vector $5\mathbf{i} - 4\mathbf{j} + 15\mathbf{k}$ and the point B has position vector $\mathbf{i} - 2\mathbf{k}$.

(iii) Find the foot of perpendicular from A to p_2. [3]

(iv) Find the length of projection of AB onto p_2. [3]

1 A curve C has equation $y=\frac{a}{x}+bx^2+c$, where a, b and c are constants. It is given that C passes through the point with coordinates (1, 2) and intersects the x-axis at $x=-1$. The gradient of C is -1 at the point where $x=0.5$. Find the values of a, b and c and state the equation of C.

[5]

2 **(i)** Sketch, on the same diagram, the graphs of $y=|2x+3|$ and $y=-2x^2-8x-3$. (You are not required to label any axial intercepts and stationary points.) [2]

(ii) Solve exactly the inequality $|2x+3|>-2x^2-8x-3$. [3]

3 The curve C defined by $y=\frac{ax+1}{bx-2}$ passes through $(3, 13)$ and has vertical asymptote $x=2$.

(i) Find the values of a and b. [2]

(ii) Describe a sequence of three transformations that transform the graph of $y=\frac{1}{x}$ onto the graph of C. [4]

4 **(a)** Differentiate the following expressions with respect to x.

(i) $\frac{\ln x}{2+3x}$ [2]

(ii) $\sin^{-1}\left(x^3+2x\right)$ [2]

(b) The curve C has equation $2xy-y^2=(1+y)^2$.

Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y and show that there are no tangents to C which are parallel to the x-axis. [4]

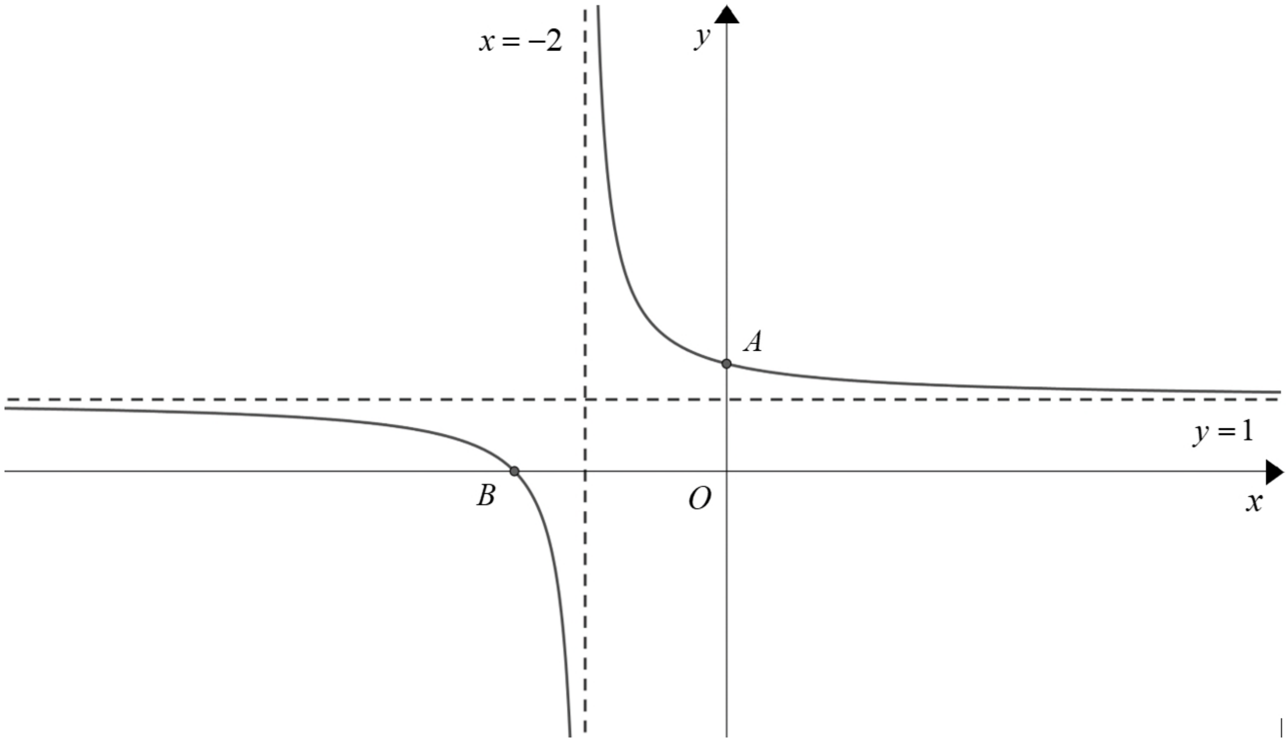
5 **(a)** Show that $\frac{4x^2-6x+1}{\left(x^2+1\right)(x-2)}$ can be expressed as $\frac{Ax+B}{x^2+1}+\frac{C}{x-2}$, where A, B and C are constants to be determined. Hence find $\int \frac{4x^2-6x+1}{\left(x^2+1\right)(x-2)}\,\mathrm{d}x$. [3]

(b) Differentiate $\mathrm{e}^{\sqrt{x}}$ with respect to x. Hence, or otherwise, find $\int \frac{\mathrm{e}^{\sqrt{x}}}{\sqrt{x}\left(\mathrm{e}^{\sqrt{x}}+1\right)^4}\,\mathrm{d}x$. [3]

(c) Using the substitution $u=x\cos x$, find

$$\int (\cos x - x\sin x)(x\cos x)\,\mathrm{d}x.$$ [3]

6 The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve cuts the axes at $A(0,1.5)$ and $B(-3,0)$. The asymptotes of the curve are $x=-2$ and $y=1$.



Sketch, on separate diagrams, the graphs of

(i) $y = -\mathrm{f}(1+x)$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(iii) $y = \mathrm{f}'(x)$, [2]

indicating clearly the asymptotes, axial intercepts and the points corresponding to A and B where possible.

7 **(i)** Sketch the graph of $(x-2)^2 + 4(y+2)^2 = 16$, stating the coordinates of all the vertices.[3]

(ii) On the same diagram, sketch the graph of $y = \dfrac{x^2 - 2x + 6}{x+1}$, stating the equations of any asymptotes, the coordinates of turning points and the points of intersection with the axes. [3]

(iii) Find the range of values of m, where $m > 0$, such that $(x-2)^2 + 4m^2\left(\dfrac{x^2-2x+6}{x+1}+2\right)^2 = 16$ has no real solutions. [2]

8 Referred to the origin O, the points A and B have position vectors **a** and **b** respectively such that $|\mathbf{a}| = 5$ and $|\mathbf{b}| = 3$.

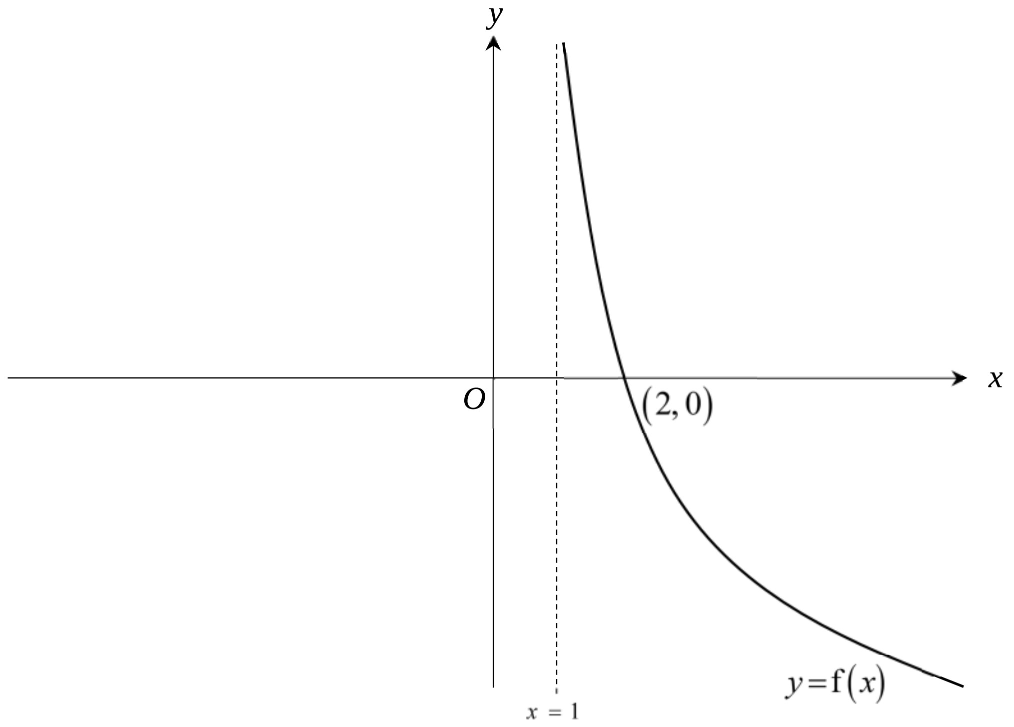
(i) It is given that $|2\mathbf{b} - \mathbf{a}| = 7$. Using a suitable scalar product, show that $\mathbf{a} \bullet \mathbf{b} = 3$. [2]

(ii) Deduce the length of projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$. [1]

The point P lies on AB such that $AP : PB = 1 : k$, where k is a positive constant.

(iii) Write down the position vector of P in terms of k, **a** and **b** and show that the area of triangle OAP is $\dfrac{3\sqrt{6}}{k+1}$. [5]

9 **(a)** The diagram below shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, $x > 1$. The curve $y = \mathrm{f}(x)$ has an asymptote $x = 1$ and passes through $(2, 0)$. Sketch on this same diagram the graph of $y = \mathrm{f}^{-1}(x)$, showing clearly the geometrical relationship between the two graphs. [2]



(b) Functions g and h are defined by

$$\mathrm{g} : x \mapsto \frac{1}{1-x^2}, \qquad x \in \mathbb{R}, x > 1,$$
$$\mathrm{h} : x \mapsto 1 - 2x, \qquad x \in \mathbb{R}.$$

(i) Explain why the composite function gh does not exist. [2]

(ii) Find $\mathrm{hg}(x)$. [1]

(iii) Find the range of $\mathrm{hg}(x)$. [2]

(iv) By using the result in part **(ii)**, or otherwise find $(\mathrm{hg})^{-1}(4)$. [3]

10 A curve C has parametric equations

$$x = e^{2t}, \; y = t^2 .$$

(i) Find the value of t at the point where the tangent has gradient $\dfrac{1}{2e}$. [3]

The tangent at the point $P(e^{2p}, p^2)$ on C meets the y-axis at the point D.

(ii) Find the coordinates of D in terms of p. [2]

(iii) The point M is the midpoint of PD. Find a Cartesian equation of the curve traced by M as p varies. [3]

11 The line l has equation $1-x=\dfrac{z-\alpha}{2}, y=1$, where α is a constant.

(i) Given that the point of intersection between the line l and the yz-plane is $(0,1,5)$, show that $\alpha=3$. [2]

Referred to the origin, the points A and B have position vectors $-\mathbf{j}+3\mathbf{k}$ and $12\mathbf{i}+5\mathbf{j}-\mathbf{k}$ respectively.

(ii) Find the cartesian equation of the plane π_1 which contains the point A and the line l. [3]

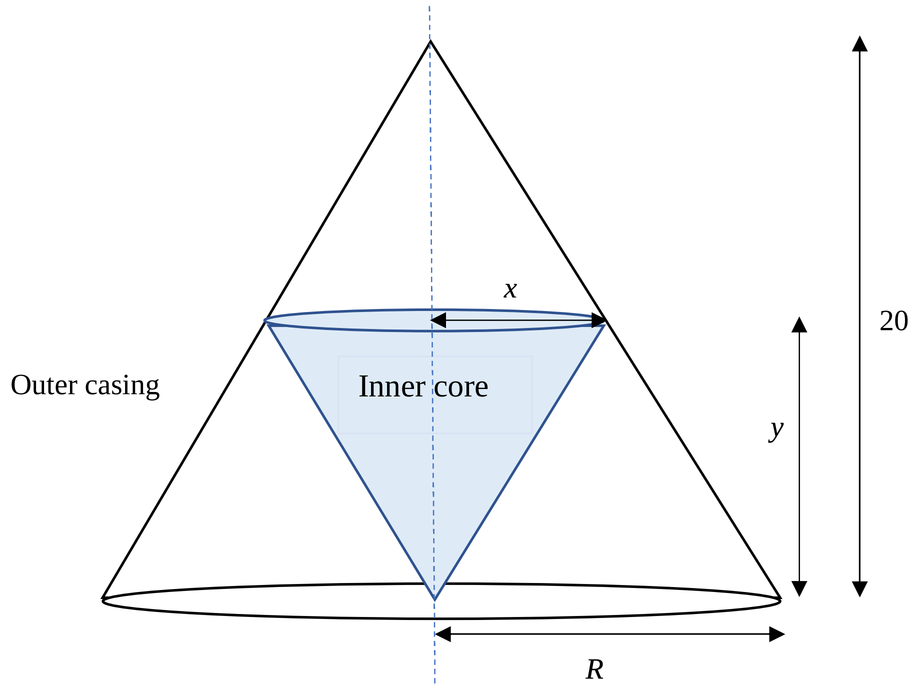
(iii) Find the acute angle between the line AB and π_1. [3]

The plane π_2 has cartesian equation $x+2z=5$.

(iv) Find the cartesian equations of the planes such that the perpendicular distance from each plane to π_2 is $2\sqrt{5}$. [4]

12 In a popular fantasy online game Ginseng Impact, there is a powerful in-game item known as the "Ring of Knowledge" which the players can buy or "craft" for their game characters. "Crafting" is a game mechanic where the player gets to make a particular item by gathering the required materials and deciding on some of its visual details.

Nathan, an avid player, decides to craft the "Ring of Knowledge", which is an enchanted jewel set on ring that is worn on the finger. The enchanted jewel itself consists of two components: a conical outer casing, and a conical inner core, as shown in the diagram.



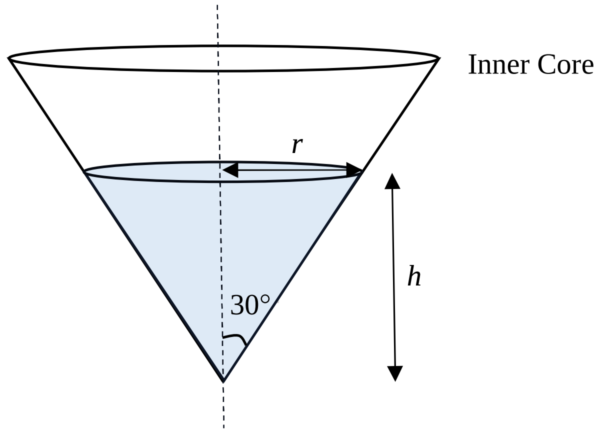
The outer casing has fixed height of 20 units, and fixed base radius of R units. Nathan is able to decide the base radius x, and the height y, of the inner core during his crafting. The inner core acts as a store of destructive power that can be unleashed on enemies, hence Nathan wants the volume of the inner core, V to be as large as possible.

[The volume of a cone is $\frac{1}{3} \times \text{base area} \times \text{height}$.]

(a) **(i)** Show that $V = \frac{20\pi}{3}x^2 - \frac{20\pi}{3R}x^3$. [2]

(ii) Find the largest possible volume of the inner core that Nathan can craft, giving your answer in terms of R. [6]

Focusing on the inner core, Nathan decides to make the values of x and y such that the slant edge of the core makes an angle of $30°$ with the centre axis, as shown in the diagram.



When the Ring is in use, the core continually charges up and the accumulated energy slowly fills up the space within the core, starting from the bottom. The height and base radius of this accumulated energy at time t are given as h and r respectively.

If the rate of energy charge is $8 \text{ unit}^3 \text{ per second}$,

(b) (i) show that $r = \dfrac{h}{\sqrt{3}}$, [1]

(ii) find the rate of change of h, when it reaches the height of 5 units. [3]

- END –

1 **(a)** Without using a calculator, solve the inequality $\dfrac{4}{x+2} \geq 3-x.$ [3]

Hence solve the inequality $\dfrac{4}{\ln x+3} \geq 2-\ln x.$ [2]

(b) Solve the inequality $\dfrac{|2x+1|}{x} \leq 2x+1.$ [4]

2 Payprice Supermarket held a sale at the "Fresh Food Corner". Ms Ng, Mdm Lim, Mr Yeo and Mrs Heng were at Payprice for this sale. As the items were not pre-packed, customers were charged by the weight of the seafood they have selected. The weight of their seafood purchase was summarized in the following table.

	Ms Ng	Mdm Lim	Mr Yeo	Mrs Heng
Cuttlefish (kg)	0.75	0.60	1.30	1.10
Japanese Sea Bass (kg)	1.20	1.45	0.90	1.80
Tiger Prawns (kg)	0.20	0.45	0.50	0.30

Ms Ng, Mdm Lim and Mr Yeo paid \$28.51, \$35.79 and \$33.40 respectively for their purchase. Mrs Heng found that she only had \$50 with her. Determine whether she has enough money to pay for her purchase. [4]

3 Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for each of the following, simplifying your answers.

(i) $y=\sin^{-1}\left(\mathrm{e}^{x}\right)$ [2]

(ii) $y=\ln\left(\dfrac{1+\ln x}{x^{x}}\right)$ [3]

4 The function f is defined by

$$\mathrm{f} : x \mapsto (x-2)^2-4, \quad x \in \mathbb{R}, \quad x \geq 0.$$

(i) Sketch the graph of f and explain why the function f^{-1} does not exist. [2]

(ii) If the domain of f is restricted to $x \geq k$, where k is a real number, state the smallest value of k for which f^{-1} exists. [1]

(iii) With the domain in **(ii)**, find f^{-1} in a similar form, stating its domain. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto 3x-5, \quad x \in \mathbb{R}, \quad x \geq 3.$$

(iv) Find an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$, and find also the range of $\mathrm{f}^{-1}\mathrm{g}$. [3]

5 **(a)** Find $\int \frac{e^{2\tan^{-1}x}}{1+x^2}\,dx$. [2]

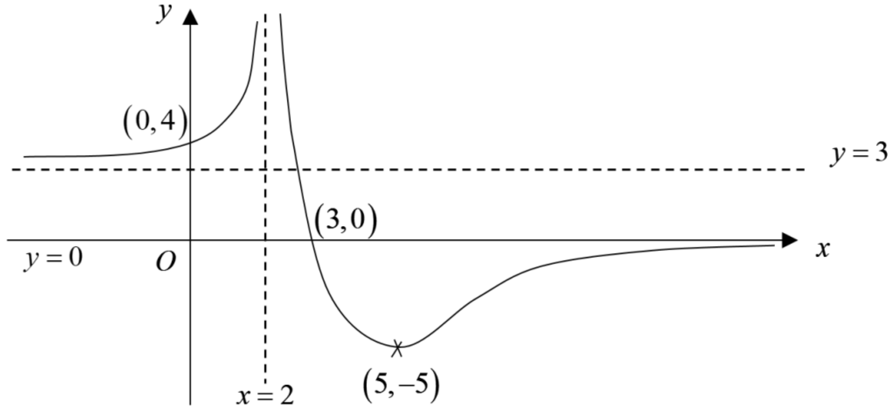
(b) Find $\int \frac{2x+1}{\sqrt{(2x^2+2x-5)}}\,dx$. [2]

(c) **(i)** Using the substitution $x = a\sin\theta$, show that $\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{(a^2-x^2)}\,dx$ can be rewritten as

$\int_{\alpha}^{\beta} a^2\cos^2\theta\,d\theta$, where α and β are constants to be determined in exact form. [3]

(ii) Hence, find the exact value of $\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{(a^2-x^2)}\,dx$. [4]

6 The diagram below shows the graph of $y = f(x)$. The graph cuts the x- and y-axes at the points $(3,0)$ and $(0,4)$ respectively. It has a turning point at $(5,-5)$. The equations of the asymptotes are $x=2$, $y = 0$ and $y=3$.



On separate diagrams, sketch the graph of

(i) $y = f(2x+1)$, [3]

(ii) $y = 2f(x) - 3$,

[3]

(iii) $y = f(|x|)$, [2]

(iv) $y = \frac{1}{f(x)}$, given that $\frac{1}{f(2)} = 0$. [4]

7 A curve C has parametric equations

$$x = (1+t)^2, \quad y = t(1+t)^2, \quad t \geq -1.$$

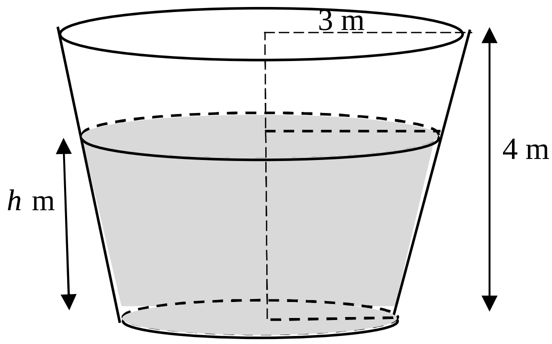
(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. Hence find the equation of the tangent to C at the point $x = 0$. [4]

(ii) Show that the normal to C at the point $x = 0$ intersects C again. Find the coordinates of this point of intersection. [3]

(iii) Sketch C, stating the coordinates of any points of intersection with the axes. [2]

(iv) Find a cartesian equation of C. [2]

8 **(a)**



Water is poured at a rate of 9 m^3 per minute into an open container in the form of a frustum of a right circular cone as shown in the diagram above. The container has bottom radius 2 m, top radius 3 m and height 4 m. After t minutes, the radius of the water surface is r m and the depth of water is h m, where $0 \le h < 4$.

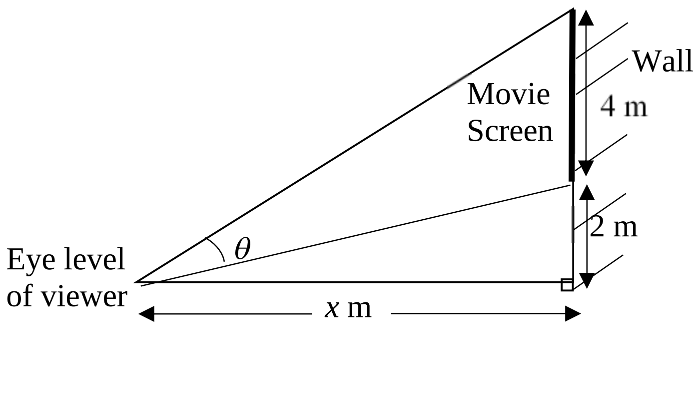
(i) Show that $r = \dfrac{h}{4} + 2$. [2]

(ii) Find, in exact form, the rate of increase of the depth of water when $h = 1$. [4]

[The volume of a frustum of a right circular cone of bottom radius r_1, top radius r_2 and height h is given by $V = \dfrac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$.]

(b) The vertical viewing angle is the angle between two lines from the eye level of the viewer; one from the eye level to the top of the screen, and the other from the eye level to the bottom of the screen. Research has shown that viewer comfort will be compromised if this angle exceeds $\dfrac{\pi}{5}$ radians.

The diagram below shows a movie screen which is 4 m high with its lowest point mounted at 2 m above the eye level of a viewer. The viewer positions himself at a variable horizontal distance x m from the vertical wall, resulting in a vertical viewing angle θ. Determine, by differentiation, if the viewer comfort will be compromised based on what the research has found. [7]



9 The lines l_1 and l_2 have equations

$$\frac{x-1}{3}=\frac{y-2}{a},\ z=1 \qquad \text{and} \qquad \mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix},\ \lambda\in\mathbb{R}$$

respectively, where a is a constant.

(i) Given that l_1 and l_2 intersect at the point N, find the coordinates of N and the value of a. [4]

(ii) Show that the position vector of F, the foot of the perpendicular from the point $P(2,1,1)$ to the line l_2 is $\frac{4}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}$. [3]

(iii) Hence, find the exact distance from P to l_2. [2]

(iv) The plane π contains the point P and the line l_2. Find the equation of π in Cartesian form. [3]

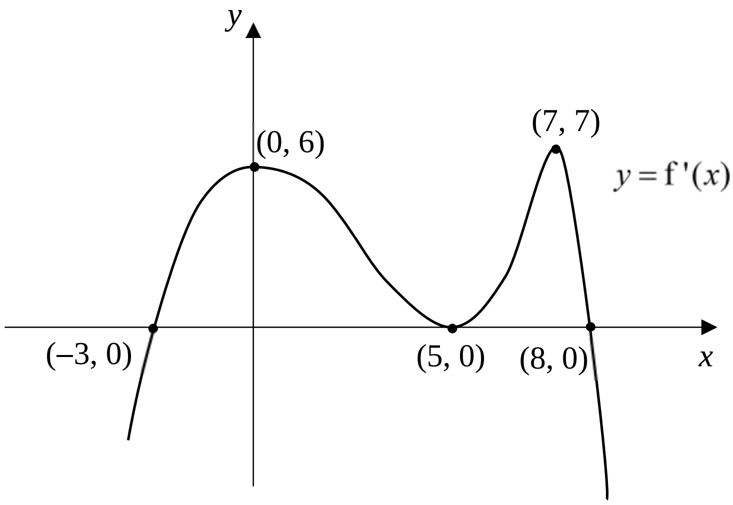
10 Relative to the origin O, two points A and B have position vectors given by $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ respectively. The point P divides AB in the ratio 3 : 1.

(i) Find the position vector of P. [2]

(ii) The vector $\mathbf{c}$ is a unit vector in the direction of $\overrightarrow{OP}$. Write $\mathbf{c}$ as a column vector, and give the geometrical meaning of $|\mathbf{a}\bullet\mathbf{c}|$. [2]

(iii) Find angle OAB to the nearest degree. [3]

11 The diagram below shows the curve $y=\mathrm{f}'(x)$. It has turning points at (0, 6), (5, 0) and (7, 7) and intersects the x-axis at (–3, 0) and (8, 0).



(i) State the x-coordinates of the stationary points for the curve $y=\mathrm{f}(x)$. Hence determine the nature of these stationary points. [3]

(ii) State the range of values of x such that the curve $y=\mathrm{f}(x)$ is concave downwards. [2]

(iii) A student claims that if the curve $y = \mathrm{f}(x)$ passes through the point (–3, 7), then $\mathrm{f}(x)>7$ for $-3<x<5$. Explain briefly whether the student is right to make such a claim. [1]

H2 Mathematics (9758) **JC1 2024**
Year-End Revision Paper D **Time : 3h** **Marks : 100**

1 Differentiate with respect to x:

(a) $e^{\frac{4}{\sqrt{x}}}$, [2]

(b) $\ln\left(\dfrac{\sin x}{2-\sin x}\right)^2$. [3]

2 If $x^3+5x^2+8x+4=\left(x+2\right)^2\left(x+1\right)$, find the range of values of x for which

$$\frac{18}{1-x}\geq x^2+6x+14$$ [3]

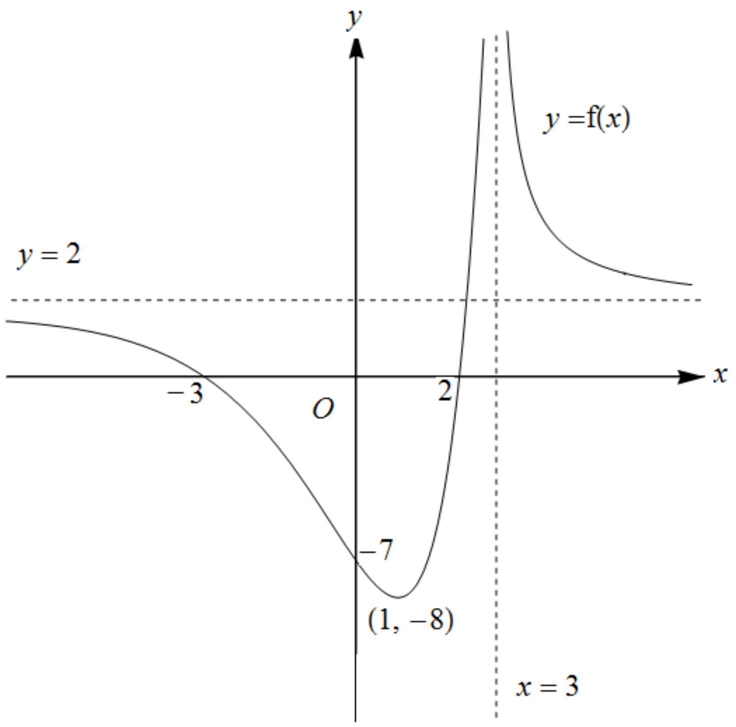
Hence find the range of values of x for which $\dfrac{18}{1-|x|}\geq |x|^2+6|x|+14$. [3]

3 A farmer has 100 acres of land on which tomatoes, peas and carrots can be planted. The labour cost, materials cost and gross income for one acre of these crops are summarised as follows.

	Labour (hr)	Materials Cost ($)	Gross Income ($)
Tomatoes	30	50	500
Peas	45	30	225
Carrots	50	40	325

The farmer has 4000 hours of labour and $4250 of capital to invest. Find the distribution of land area for each crop and the total gross income for the farmer. [6]

4 **(a)**



The graph of $y=\mathrm{f}\left(x\right)$ is shown above. It has a turning point at $\left(1,-8\right)$ and cuts the axes at $\left(-3,0\right)$, $\left(2,0\right)$ and $\left(0,-7\right)$. The lines $x=3$ and $y = 2$ are asymptotes. Sketch, on separate diagrams, the graphs of

(i) $y=\mathrm{f}\left(|x|\right)$, [2]

(ii) $y=-\mathrm{f}\left(x\right)-8$, [3]

(iii) $y=\dfrac{1}{\mathrm{f}\left(x\right)}$, [3]

giving the coordinates of any turning points, axial intercepts and the equations of any asymptotes.

(b) The curve with equation $y=\ln\left(x-1\right)$ undergoes in succession the following transformations.

A: A translation of 3 units in the negative *x*-direction.

B: A reflection in the y-axis.

C: A scaling parallel to the y-axis with scale factor 2.

Determine the equation of the resulting curve after the three transformations are effected. [2]

5 **(a)** The curve C_1 has equation $y=\dfrac{x-1}{x-2}$. The curve C_2 has equation

$$(x-2)^2+(y-1)^2=2.$$

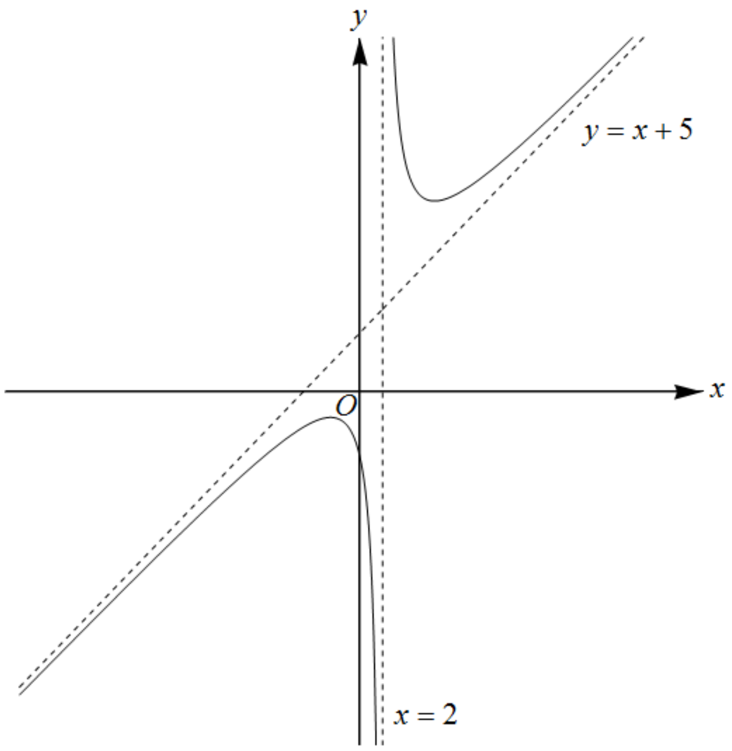
(i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

(ii) Hence, deduce the number of roots of the equation

$$(x-2)^2+\left(\frac{1}{x-2}\right)^2=2.$$ [2]

(iii) Another curve C_3 has the equation $(x-2)^2+(y-1)^2=h$. State the range of values of $h\in\mathbb{R}$ such that C_1 and C_3 intersect at 4 points. [1]

(b) A sketch of the curve $y=\dfrac{Ax^2+Bx+11}{x+C}$, where A, B and C are constants, is shown below. The lines $x=2$ and $y=x+5$ are asymptotes to the curve. Find the values of A, B and C. [3]



6 **(a)** State the constants A and B such that, for all values of x,

$$11-4x = A(4-2x)+B.$$

Hence, find $\displaystyle\int \frac{11-4x}{\sqrt{x(4-x)}}\,\mathrm{d}x$. [4]

(b) Use the substitution $x=\sin\theta$ to find $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x$. [4]

(c) Evaluate $\displaystyle\int_1^2 x\mathrm{e}^{2x}\,\mathrm{d}x$, giving your answer in exact form. [4]

7 **(a)** The function f is defined by

$$\mathrm{f}: x \mapsto \left|\mathrm{e}^{x^2}-2\right|, \quad x\in\mathbb{R}.$$

If the domain of f is restricted to $k \le x \le 0$,

(i) state the least value of k for which the function f^{-1} exists, [1]

(ii) find $\mathrm{f}^{-1}(x)$, stating the domain of f^{-1}. [3]

(b) The functions g and h are defined by

$$\mathrm{g}: x \mapsto \begin{cases} \sin^{-1}x & \text{for } -1 < x \le 1, \\ \pi\left(1-\dfrac{x}{2}\right) & \text{for } 1 < x \le 3, \end{cases}$$

$$\mathrm{h}: x \mapsto 2x, \text{ for } -\frac{\sqrt{3}}{4} \le x \le 1.$$

(i) Show that the composite function gh exists and find its range, giving all values in terms of π. [3]

It is given that $\mathrm{g}(x+4)=\mathrm{g}(x)$ for all real values of x.

(ii) Evaluate $\mathrm{g}(17)$. [2]

(iii) Sketch the graph of $y=\mathrm{g}(x)$ for $-1 \le x \le 5.5$. [3]

8 Relative to an origin O, the points A and B have position vectors $4\mathbf{i}+2\mathbf{j}$ and $3\mathbf{i}+\mathbf{k}$ respectively. The point C is such that $OACB$ is a parallelogram and the point E is such that $\overrightarrow{BE}=\frac{1}{4}\overrightarrow{BC}$. The point M is the midpoint of AB.

(i) Find the vector equations of the lines AB and OE. [4]

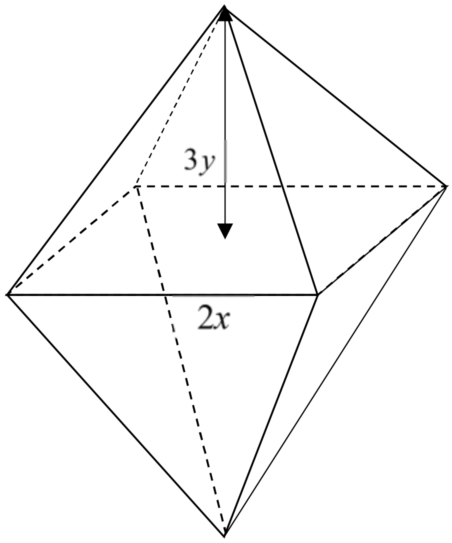
(ii) Find the position vector of point F, the point of intersection of AB and OE.

Hence deduce $\dfrac{OF}{OE}$. [4]

(iii) Find angle OMA, giving your answer correct to the nearest 0.1°. [3]

(iv) Find the exact length of the projection of OM onto AB. [2]

9



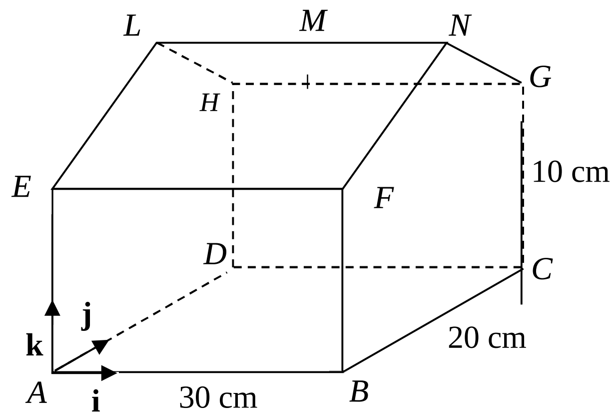
A lighting consultant intends to construct a giant light structure for Christmas that is octahedral in shape. The edges of this structure are made of 12 straight pieces of steel rods of negligible thickness that are welded at their ends to form 2 congruent right pyramids. Each pyramid has a height of $3y$ metres and a square base of side $2x$ metres as shown in the diagram above. Given that the total length of steel rods used is 480 metres,

(i) show that $3y = \sqrt{3600 - 120x - x^2}$. [3]

Upon the completion of the giant light structure, the lighting consultant intends to wrap it tightly with a transparent plastic sheet of negligible thickness.

(ii) Deduce that the total surface area of the giant light structure is given by $8x\sqrt{3600-120x}$ square metres. Hence find the exact value of x that maximises the total surface area as x varies. [6]

10



The diagram above shows a toy house with horizontal rectangular base $ABCD$ where $AB = 30$ cm and $BC = 20$ cm. There are two vertical rectangular walls, $ABFE$ and $DCGH$, where $AE = BF = CG = DH = 10$ cm. There are two vertical walls, $AELHD$ and $BFNGC$, where $LE = LH = NF = NG$ and L and N are 15 cm from the base respectively.

The point A is taken as the origin and vectors **i**, **j**, **k** each of length 1 cm, are taken along AB, AD and AE respectively.

(i) There is a slanted rectangular false ceiling $PQGH$ where P and Q are lying on AE and BF respectively with $PE = QF = 1$ cm. Show that a vector normal to the plane $PQGH$ is given by $\mathbf{j} - 20\mathbf{k}$. Hence find the cartesian equation of plane $PQGH$. [3]

(ii) Find, in surd form, the cosine of the acute angle between $ABCD$ and $PQGH$. [3]

A wire fastened at M, the midpoint of LN, is connected to the point V on the false ceiling. Find the coordinates of V for the length of the wire used to be minimum. [3]

11 An *astroid* is a curve with parametric equations

$$x = a\cos^3 t, \quad y = a\sin^3 t.$$

(i) Sketch an astroid with $a = 2$ and $0 \le t \le \frac{\pi}{2}$, indicating clearly the coordinates of all axial intercepts. [2]

For the rest of the question, take $a = 1$.

(ii) Show that the equation of the tangent to the curve at the point P where $t = p$ is $y = (-\tan p)x + \sin p$ [3]

(iii) The tangent to the curve at P intersects the x- and y-axes at the points Q and R respectively. Find the cartesian equation for the path traced by the midpoint of QR. [3]

Answers:

Topic 1: Vectors

2. (ii) Area of ΔAOB : Area of ΔBOC = 1 : 2; $k = \pm 2\sqrt{13}$

3. (a) $\frac{3}{2}\mathbf{b}$ (b) $\left(\frac{1}{3}, \frac{7}{3}, \frac{13}{3}\right)$

4. (i) Line AB: $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$ or $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\lambda \in \mathbb{R}$

   (iv) $p = 0.5$, (v) Line: $\mathbf{r} = \begin{pmatrix} 0.5 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\mu \in \mathbb{R}$

5. (i) $A(4, 0, 0)$, (ii) $l_{BD} : \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \lambda \in \mathbb{R}$ (iii) $B(3, 4, 0)$, (iv) $D(2, 2, 3)$ (v) $\frac{17}{2}$ units^3

6. (i) $54.4°$, (ii) $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$, $\alpha \in \mathbb{R}$ (iii) $\mathbf{r} \bullet \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} = 14$ (iv) $\sqrt{189}$

7. (i) $\mathbf{r} \bullet \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 11$ (ii) $36.5°$ (iii) $\begin{pmatrix} 8 \\ 3 \\ 1 \end{pmatrix}$, 6 (iv) $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$, $\mu \in \mathbb{R}$

8. (ii) (0, -2, -1) (iii) $(9, -5, 5)$

Topic 2: Graphing techniques

1. (ii) $2 < k < 4$
2. (i) $y = 2x - 3a$ and $x = a$. (ii) $a < -1$ **or** $a > 1$
3. (i) $r = -1$, $y = x + 5$ (iii) $-4 \le \lambda < 0$ *or* $\lambda > 0$ (or $\lambda \ge -4,\ \lambda \ne 0$) (vi) $k > 6$
4. (i) $a = 1,\ b = -2,\ c = 4,\ d = 1$ (iii) $\{k \in \mathbb{R} : 4 < k < 8\}$

Topic 3 : Equations and Inequalities

1. $x < 0$ or $x > \frac{4}{3}$.
2. (ii) $x \le 0$ or $1 < x \le 2$ (iii) $x = 0$ or $-2 \le x < -1$ or $1 < x \le 2$
3. (ii) $2 < x \le 22.1$ (iii) $-22.1 \le x < -2$ or $2 < x \le 22.1$
4. $x < -3$ *or* $x > 2$; $x > 4$
5. (i) $x < -2$ or $x > \frac{1}{2}$ (ii) $x < -\frac{1}{2}$ or $x > \frac{1}{2}$
6. (i) $2Ax + 2By\frac{\mathrm{d}y}{\mathrm{d}x} + C + D\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2Ax + C}{2By + D}$ (ii) $A = 1, B = 4, C = -2, D = 16$
7. $a = -0.4,\ b = 1,\ c = 2$

Topic 4 : Functions

1. (ii) $f^{-1}(x)=3-\sqrt{x-5}$, $x\geq 5$ 2. (i) $D_{fg}=D_g=(-2,\infty)$ (ii) $-2<x\leq -1$

3. (ii) $f^{-1}(x)=\frac{2}{\pi}\cos^{-1}(x)$; $(-1,1]$ (iii) $\{x\in\mathbb{R}:-2<x\leq -0.673 \text{ or } x=0\}$ (iv) $R_{gf}=[0, 2.08]$

4. (i) $f^{-1}(x)=\frac{1}{1-x}$, $x\in\mathbb{R}\setminus\{0,1\}$ (iii) $f^3(x)=x$ (iv) $g(x)=e^x$

5. (i) $k=-2$; $f^{-1}: x\mapsto -2-\sqrt{x+9}$, $x\geq -9$ (iii) $f^{-1}g(6)=-5$

6. (i) f^{-1}: $x\mapsto -\frac{1}{2}-\frac{1}{2}\sqrt{4-\frac{1}{x}}$, $D_{f^{-1}}=R_f=(-\infty,0)\cup\left[\frac{1}{4},\infty\right)$ (ii) $(-1.58,-1.58)$

(iii) $R_g=[k-1,\ k)$; Largest integer $k=-2$; $R_{fg}=(-\frac{1}{5},-\frac{1}{21}]$

Topic 5: Differentiation

1. (a) $\frac{x-3}{2(x+1)(x-1)}$ (b) $\frac{1}{2\sqrt{x}(1+x)}$ (c) $\frac{1+\ln(x)}{1+\ln(y)}$

2. $\left(-2,-\frac{4}{3}\right)$; $y=-\frac{9}{8}x-\frac{43}{12}$

3. (i) $y=\frac{1}{3}x+\frac{10}{3}y$; $y=-3x+10$ (ii) $\frac{40}{9}$

4. $\frac{y-x}{2y-x}$; $k>0$; $(-2\sqrt{2},-\sqrt{2})$ and $(2\sqrt{2},\sqrt{2})$

5. (i) $py=x+2p^2$; $y=-px+2p^3+4p$ (ii) $\left(\frac{242}{9},\frac{44}{3}\right)$

6. $(0.795,-0.622)$

7. (i) $h=\frac{1176}{x^2}$

8. $-\frac{1}{2}$

9. (ii) 2.29

10. \$29

Topic 6: Integration Techniques

1. (a) (i) $\frac{1}{1-x}+\frac{3}{4+x^2}$ (ii) $-\ln|1-x|+\frac{3}{2}\tan^{-1}(\frac{x}{2})+c$ (b) $\frac{1}{3}(3x^2+5)^{\frac{3}{2}}+c$

2. (a) $\frac{1}{12}\ln\left|\frac{3+2x}{3-2x}\right|+c$ (b) $-\sqrt{x(1-x)}-\cos^{-1}\sqrt{x}+c$ (c) $\frac{1}{4}(e^2-1)$

3. (i) $2(x+1)e^{x^2+2x}$ (ii) $\frac{1}{2}(x^2+1)^2e^{x^2+2x}-\frac{1}{2}e^{x^2+2x}+c$

4. (a) $\frac{1}{3}\ln|\sin 3x+\cot 3x|+c$ (b) $\frac{1}{4}\sin^{-1}(4x)+\frac{1}{16}\sqrt{1-16x^2}+c$

(c) $\frac{\ln x}{1-x}+\ln|1-x|-\ln|x|+c$

5. (a) $\frac{1}{2}\ln|x^2+6x+13|+\frac{1}{2}\tan^{-1}(\frac{x+3}{2})+c$ (b) $\frac{3}{4}e^{\frac{1}{4}}-\frac{1}{2}e^{\frac{1}{2}}$

6. (a) 3 (b) $\frac{\sqrt{3}}{16}-\frac{\pi}{48}$

7. (a) $\frac{x^2}{2}\ln(x^2-4)-\frac{1}{2}x^2-2\ln(x^2-4)+c$ (b) $a^2(\frac{\sqrt{3}}{8}+\frac{\pi}{12})$

Paper A

Q1(ii) $0.349\le x<2$ or $x\ge 2.15$ Q2(a)(i) $\frac{3e^{3x}}{3+e^{3x}}$ (a)(ii) $8x\sin^3 x^2\cos x^2$

Q2(a)(iii) $\sec 3x\left(\frac{2}{\sqrt{1-4x^2}}+3\tan 3x\sin^{-1}2x\right)$ (b) $\frac{dy}{dx}=\frac{9-2x}{3y^2-4}$

Q3 $\frac{32}{243}$ Q4(i) $k<5$ (iii) $0<m\le 1$ Q5(i) 0.0018 m/s (4 d.p) (ii) 226 s

Q6 $f(x)=2x^3-3x^2-12x-7$ Q7(a) $-5\ln\left|5-2e^x\right|+c$ (b) $\frac{1}{8}\left(1+8x^2\right)^{\frac{1}{2}}+c$

Q7(c) $(\ln x)^2\left(\frac{x^2}{2}\right)-(\ln x)\left(\frac{x^2}{2}\right)+\frac{x^2}{4}+c$ Q8(a) $y=6+\frac{5}{2x+1}$

Q9 $PS=\frac{5}{3}\sqrt{3}$ cm, $PQ=20-2\sqrt{3}\left(\frac{5}{3}\sqrt{3}\right)=10$ cm Area of rectangle $PQRS=\frac{50}{3}\sqrt{3}$ cm^2.

Q10(i) $\overrightarrow{OC}=\frac{2}{3}\mathbf{a}$, $\overrightarrow{OD}=\frac{4}{3}\mathbf{b}$ (iii) $k=\frac{4}{9}$ Q11(ii) $P\left(\frac{4}{3},-\frac{16}{9}\right)$ (iv) $\frac{128}{81}$ (v) $S\left(\frac{1}{3},-\frac{2}{9}\right)$

Q12(ii) $f^{-1}(x)=\frac{1+\sqrt{1+x^2}}{x}$ $D_{f^{-1}}=(0,\infty)$ (iii) $y=x$; $x=\sqrt{3}$

Q13(i) $\mathbf{r}=\frac{1}{7}\begin{pmatrix}19\\-4\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\7\end{pmatrix}$, where $\lambda\in\mathbb{R}$. (ii) $\theta=38.2^\circ$ (nearest to 0.1°)

Q13(iii) $(5.8,-1.6,15)$ (iv) 17.7 units (to 3 s.f.)

Paper B

Q1. $y=\frac{1}{x}+3x^2-2$ Q2 (ii) $x>\frac{-5+\sqrt{13}}{2}$ or $x<-3$ Q3 (i) $a=4$; $b=1$

Q4a(i) $\frac{2+3x-3x\ln x}{x(2+3x)^2}$ a(ii) $\frac{3x^2+2}{\sqrt{1-(x^3+2x)^2}}$ (b) $\frac{dy}{dx}=\frac{y}{2y-x+1}$

Q5 (a) $A=3, B=0, C=1$; $\frac{3}{2}\ln\left(x^2+1\right)+\ln|x-2|+C$ (b) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$; $-\frac{2}{3}\left(e^{\sqrt{x}}+1\right)^{-3}+C$

(c) $\frac{1}{2}x^2\cos^2 x+c$

Q6(iii) $m>\frac{1}{2}$ Q8(ii) 1 (iii) $\overrightarrow{OP}=\frac{k\mathbf{a}+\mathbf{b}}{k+1}$ Q9b(ii) $1-\frac{2}{1-x^2}$ b(iii) $R_{hg}=(1,\infty)$ b(iv) $\sqrt{\frac{5}{3}}$

Q10(i) $t=\frac{1}{2}$ (ii) $(0,-p+p^2)$ (iii) $y=-\frac{1}{4}\ln(2x)+\frac{1}{4}(\ln(2x))^2$

Q11(ii) $-2x+y-z=-4$ (iii) $\theta=24.1^\circ$ (1dp) (iv) $x+2z=15$ or $x+2z=-5$

Q12 a(ii) $\frac{80\pi}{81}R^2$ unit^3 b(ii) 0.306 or $\frac{24}{25\pi}$ unit/s

Paper C

1. (a) $-2<x\le -1$ or $x\ge 2$; $e^{-3}<x\le e^{-2}$ or $x\ge e$ (b) $-1\le x<0$ or $x\ge 1$
2. Yes
3. (i) $\frac{e^x}{\sqrt{1-e^{2x}}}$ (ii) $\frac{1}{x(1+\ln x)}-1-\ln x$
4. (ii) 2 (iii) $f^{-1}: x\mapsto 2+\sqrt{x+4}$, $x\ge -4$ (iv) $f^{-1}g(x) = 2+\sqrt{3x-1}$, $\left[2+\sqrt{8},\ \infty\right)$
5. (a) $\frac{1}{2}e^{2\tan^{-1}x}+c$, (b) $\sqrt{2x^2+2x-5}+c$, (c) $a^2\left(\frac{\pi}{8}-\frac{1}{4}\right)$

7 (i) $\frac{1}{2}(1+3t)$, $y=-x$ (ii) (4, 4) (iv) $x^3=(x+y)^2$

8. (ii) $\frac{16}{9\pi}$ m min^{-1} (b) Viewer comfort is not compromised.
9. (i) (0, 3, 1), $a=-3$ (iii) $\frac{2}{3}\sqrt{6}$ (iv) $x+y=3$
10. (i) $\begin{pmatrix}4\\-1\\6\end{pmatrix}$ (iii) 98°
11. (ii) $0<x<5$ or $x>7$

Paper D

1. (a) $-2x^{-\frac{3}{2}}e^{\left(\frac{4}{\sqrt{x}}\right)}$, (b) $\frac{4\cos x}{(\sin x)(2-\sin x)}$
2. $x=-2$ or $-1\le x<1$, $-1<x<1$
3. Tomatoes: 45 acres, peas: 20 acres, carrots: 35 acres; \$38375
4. (b) $y=2\ln(-x+2)$
5. (iii) $h>2$ (b) $A=1, B=3, C=-2$
6. (a) A = 2, B = 3; $4\left(4x-x^2\right)^{\frac{1}{2}}+3\sin^{-1}\frac{x-2}{2}+c$ (b) $\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x+c$

   (c) $\frac{3e^4}{4}-\frac{e^2}{4}$

7 (a)(i) $-\sqrt{\ln 2}$, (ii) $f^{-1}(x)=-\sqrt{\ln(2-x)}$, $D_{f^{-1}}=[0,1]$,

(b)(i) $\left[-\frac{\pi}{3},\frac{\pi}{2}\right]$, (ii) $\frac{\pi}{2}$

8. (i) $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, \quad \lambda \in \mathbb{R}$, $\mathbf{r} = \mu \begin{pmatrix} 8 \\ 1 \\ 2 \end{pmatrix}, \quad \mu \in \mathbb{R}$ (ii) $\overrightarrow{OF} = \frac{2}{5}\begin{pmatrix} 8 \\ 1 \\ 2 \end{pmatrix}, \quad \frac{OF}{OE} = \frac{4}{5}$

(iii) 123.7^0 (iv) $\frac{5\sqrt{6}}{6}$

9. (ii) 20

10. (i) $y - 20z = -180$ (ii) $\frac{20}{\sqrt{401}}$; $V = \left(15, \frac{4120}{401}, \frac{3815}{401}\right)$

11. (iii) $x^2 + y^2 = \left(\frac{1}{2}\right)^2$