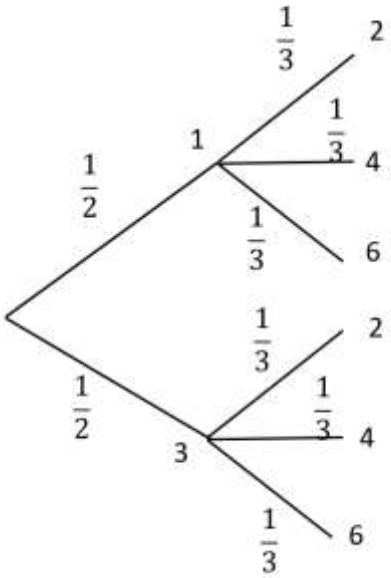
Serangoon Garden Secondary School
2021 Sec 4E/5NA Prelims P2 Suggested Mark Scheme

Qn	Solution	Mark Scheme
1(a)	$\frac{4ab^2}{3}\times\frac{b}{2a^{-2}}$ $=\frac{2a^3b^3}{3}$	**M1 (for "×")** **B1**
1(b)	$9^{(2x+1)}=3^{-x}$ $3^{2(2x+1)}=3^{-x}$ $4x+2=-x$ $x=-\frac{2}{5}$	**M1** **(base 3)** **A1**
1(c)	$x=\frac{2}{a}+\frac{1}{b}$ $x=\frac{2b+a}{ab}$ $abx=2b+a$ $b(ax-2)=a$ $b=\frac{a}{ax-2}$ or $\frac{1}{b}=x-\frac{2}{a}$ $\frac{1}{b}=\frac{ax-2}{a}$ $b=\frac{a}{ax-2}$	**M1 (remove denominator)** **A1**
1(d)	$\frac{4x^2-16}{3x^2-5x-2}$ $=\frac{4(x+2)(x-2)}{(3x+1)(x-2)}$ $=\frac{4(x+2)}{3x+1}$	**M1 for** $4(x+2)(x-2)$ **M1 for** $(3x+1)(x-2)$ **A1**
1(e)	$x-2y=10$ $2x-y=8$ $x=2,\ y=-4$	**M1** **Correctly eliminate 1 variable.** **A1, A1**
	Total for Q1	**12 m**

2(a)	$\frac{55}{x}$	**B1**
2(b)	$\frac{55}{x-10}$	**B1**
2(c)(i)	$\frac{55}{x-10}-\frac{55}{x}=1\frac{15}{60}$ $60(55x-55(x-10))=75x(x-10)$ $75x^2-750x-33000=0$ $x^2-10x-440=0$ (shown)	**M1** **Accept mix number or improper fraction** **M1 remove denominator** **A1**
2(c)(ii)	$x^2-10x-440=0$ $x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(-440)}}{2(1)}$ $x=\frac{10\pm\sqrt{1860}}{2}$ $x=26.563858$ or $x=-16.563858$ $x\approx 26.6$ (3sf) or $x\approx -16.6$ (3sf)	**B1** **B1,B1**
2(c)(iii)	$\frac{55}{26.563858}+\frac{55}{26.563858-10}$ $=5.390964$ $=5$ hours, 23.45789 mins ≈ 5 hours, 23 mins	**M1** **Sub 26.563858 into correct expression** **A1**
	Total for Q2	**10 m**

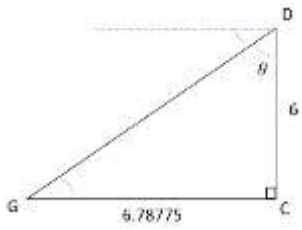
3(a)(i)		**B2** **Minus 1 mark for each error.**
3(a)(ii)	P(sum more than 5) = $3\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)=\frac{1}{2}$	**M1** **For** $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$ **A1**
3(b)	Let x be the initial no. of green balls $2\left(\frac{3}{x+3}\right)\left(\frac{x}{x+2}\right)=\frac{15}{28}$ $28(6x)=15(x+3)(x+2)$ $15x^2-93x+90=0$ $x=\frac{-(-93)\pm\sqrt{(-93)^2-4(15)(90)}}{2(15)}$ $x=\frac{93\pm\sqrt{3249}}{30}$ $x=5$ or $x=1.2$ (rej) Possible no. of initial green balls is 5.	**M1** $\left(\frac{3}{x+3}\right)\left(\frac{x}{x+2}\right)$ **M1** **remove denominator correctly** **M1** **A1**
	Total for Q3	**8 m**

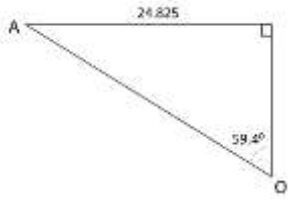
4(a)	See Graph Paper	**B1** **All points plotted correctly** **B1** **All points joined** **B1** **Smooth Curve**
4(b)	By drawing the line $y = 2$, the equation only has 1 solution because the line cuts the curve only at 1 point.	**M1** **Draw** $y = 2$ **A1** **"Line cuts curve at only 1 point"**
4(c)	Mathematically calculated gradient = 1.25. Based on points selected, sub points into gradient formula and calculate.	**M1** **Draws acceptable tangent at** $(2, -0.5)$ **A1 (mark base on points chosen) Cannot give marks if negative gradient.**
4(d)(i)	See Graph Paper	**B1** **Cuts *y*-axis at 1** **Or correct table of values** **B1**
4(d)(ii)	$x = -2.3, -0.6$ and 2.9 But mark based on the graph drawn.	**B1**
4(d)(iii)	$\frac{1}{2}x + 1 = \frac{1}{4}x(x^2 - 5)$ $2x + 4 = x^3 - 5x$ $x^3 - 7x - 4 = 0$ (shown)	**M1** **Line = curve** **A1**
4(e)	Mathematically calculated answer = $(-2,\ 0.5)$ But mark based on drawing a parallel tangent o the curve.	**B1**
	Total for Q4	**13 m**

5(a)	$P = 7M = \begin{pmatrix} 420 & 280 & 210 \\ 280 & 350 & 140 \end{pmatrix}$	**B1**
5(b)	$T = PN = \begin{pmatrix} 420 & 280 & 210 \\ 280 & 350 & 140 \end{pmatrix}\begin{pmatrix} 1.00 \\ 1.50 \\ 2.50 \end{pmatrix}$ $= \begin{pmatrix} 1365 \\ 1155 \end{pmatrix}$	**M1** for $\begin{pmatrix} 1.00 \\ 1.50 \\ 2.50 \end{pmatrix}$ **A1**
5(c)	T represents the total cost to make caramel and strawberry cakes for the whole week respectively.	**B1**
5(d)(i)	Cakes sold = $\frac{6}{7}\begin{pmatrix} 420 & 280 & 210 \\ 280 & 350 & 140 \end{pmatrix}$ $= \begin{pmatrix} 360 & 240 & 180 \\ 240 & 300 & 120 \end{pmatrix}$	**M1 for** $\frac{6}{7}$ **A1**
5(d)(ii)	Selling price = $\begin{pmatrix} 1.40 \\ 2.10 \\ 3.50 \end{pmatrix}$ Sales = $\begin{pmatrix} 360 & 240 & 180 \\ 240 & 300 & 120 \end{pmatrix}\begin{pmatrix} 1.40 \\ 2.10 \\ 3.50 \end{pmatrix}$ $= \begin{pmatrix} 1638 \\ 1386 \end{pmatrix}$ Profit = $\begin{pmatrix} 1638 \\ 1386 \end{pmatrix} - \begin{pmatrix} 1365 \\ 1155 \end{pmatrix} = \begin{pmatrix} 273 \\ 231 \end{pmatrix}$ Total Profit = 273 + 231 = \$504	**M1** **A1**
	Total for Q5	**8 m**

6(a)(i)	49	**B1**
6(a)(ii)	$6n+1$	**B1**
6(a)(iii)	Let $6n+1=256$ $n=42.5$ Since n is not an integer, 256 is not part of the sequence.	**M1** **A1 "Not an integer"**
6(b)	4^{th} term = 25 8^{th} term = 49 A B $25cm^2 : 49cm^2$ $5cm : 7cm$ $125cm^3 : 343cm^3$ Volume of Cone B = $\frac{343}{125}(437.5)=1200.5cm^3$	**M1 for square root** **M1 for cube** **A1**
	Total for Q6	**7 m**
7(a)	Area $=\frac{134.8}{360}\pi(13)^2-\frac{1}{2}(13)^2\sin(134.8)$ $=138.8447\approx 139cm^2$	**M1 Sector** **M1 Triangle** **M1 attempts to do (sector – triangle)** **A1**
7(b)	$\sin(67.4)=\frac{AM}{13}$ $AM=12.0017$ $\cos(67.4)=\frac{OM}{13}$ $OM=4.9958$ k = Gradient$_{OA}=\frac{OM}{AM}=\frac{4.9958}{12.0017}$ $=0.41625\approx 0.4$ (1dp)	**M1** **M1** **M1 (OM ÷ AM)** **A1**
	Total for Q7	**8 m**
8(a)(i)	Angle ABD = 90° (angle in semi circle) Angle $ADB=180-90-60=30^\circ$ (angle sum of triangle)	**M1** **A1**
8(a)(ii)	Angle $EOF=180-40-40$	**M1** **A1**

	$= 100^\circ$ (Base angles of isosceles triangle)	
8(a)(iii)	Angle ECF $= 100 \div 2$ $= 50^\circ$ (angle at centre = 2 angle at circumference)	**B1**
8(b)	$(EF)^2 = (10)^2 + (10)^2 - 2(10)^2(10)^2\cos(100)$ $EF = 15.32088 \approx 15.3$ cm	**M1** **A1**
	Total for Q8	**7 m**
9(a)	$AC = \sqrt{20^2 + 20^2} = 28.28427$ $\Rightarrow AX = 14.142135$ $\tan(60) = \dfrac{EX}{14.142135}$ $EX = 24.49489 \approx 24.5$ cm (shown)	**M1 (uses Pythagoras)** **M1** **A1**
9(b)	Let L be the slant height of pyramid $L = \sqrt{24.49489^2 + 10^2} = 26.457506$ $SA = 20^2 + (4)\left(\dfrac{1}{2}(20)(26.457506)\right)$ $= 1458.300 \approx 1460$ cm^3	**M1** **M1 (attempts to add square + 4 triangles)** **A1**
9(c)	Volume of Pyramid $= \dfrac{1}{3}(20^2)(24.49489) = 3265.985$ Volume of lower part $= (4)(10)(40) = 1600$ Volume of upper part $= 3265.985 - 1600 = 1665.985$ Height of upper part $= \dfrac{1665.985}{(40)(40)} = 1.04124$ Total height = 4 + 1.04124 = 5.04 cm	**M1** **M1** **M1** **M1** **A1**
	Total for Q9	**11 m**

10(a)	$\tan(\angle BCD) = \frac{6}{7}$ $\angle BCD = 40.60129 \approx 40.6^\circ \ (1\text{dp})$	**M1** **A1**
10(b)(i)	Let *x* be be the shortest distance from C to AB $\frac{\sin(\angle BAC)}{7} = \frac{\sin(40)}{5}$ $\angle BAC = 64.14527^\circ$ $\angle ABC = 180 - 64.14527 - 40 = 75.85473^\circ$ $\frac{AC}{\sin(75.85473)} = \frac{5}{\sin(40)}$ $AC = 7.54276$ $\frac{1}{2}(7)(7.54276)\sin(40) = \frac{1}{2}(5)(x)$ $x = 6.78775 \approx 6.79 \text{ m}$	**Error carry forward for entire 10(b)** **M1 (sine rule usage)** **M1 (for AC)** **A1**
10b(ii)	Let G be the position of the guest when he is nearest to C. $\tan(\angle DGC) = \frac{6}{6.78775}$ $\angle DGC = 41.4749^\circ$  smallest θ required $= 180 - 90 - 41.4749$ $= 48.5251^\circ$ The spotlight needs to bend the most (41.4749°) when the guest is nearest to C and θ needs to be 48.5251° Since θ can be as small as 45°, **the spot light can reach the guest at all times**.	**M1 (find depression)** **M1 (find θ)** **A1 (compares 45° to the required 48.5°.)**
	Total for Q10	**8 m**

11(a)	Angle AOC (reflex) = 241.2 (angle at center = twice angle at circumference) Angle AOC =  $360 - 241.2 = 118.8$ $\sin(59.4) = \frac{24.825}{AO}$ $AO = 28.841 \approx 28.8 \text{ m} \quad (3sf)$	**M1** **M1 (trigo ratio)** **A1**
11(b)	$\text{Arc Length} = \frac{118.8}{360}(2\pi(28.841))$ $= 59.8004 \approx 59.8 \text{ m} \quad (3sf)$	**M1** **A1**
11(c)	<table><tr><th>Speed (km/h)</th><th>Speed (M/min)</th></tr><tr><td>4.5 km / h</td><td>$\frac{4.5(1000)}{60} = 75 \text{ m / min}$</td></tr><tr><td>5.0 km / h</td><td>$\frac{5(1000)}{60} = 83.33 \text{ m / min}$</td></tr><tr><td>4.0 km / h</td><td>$\frac{4(1000)}{60} = 66.67 \text{ m / min}$</td></tr><tr><td>3.5 km / h</td><td>$\frac{3.5(1000)}{60} = 58.33 \text{ m / min}$</td></tr></table> Since distance of walkway is 59.8 m, the speed of the shoppers must be able to cover at least 59.8 m per min. John should avoid 5 pm to 7 pm on both weekdays and weekends. Or Min Speed required is 59.8 m / min. $= \frac{59.8}{1000}(60) = 3.588 \text{ km / h}$	**M2** **Converts km/h to m / min correctly** **A1** **or** **M1 (59.8m/min)** **M1 converts correctly**

	Should avoid speed slower than 3.588 km / h. John should avoid 5 pm to 7 pm on both weekdays and weekends.	**A1**
	Total for Q11	**8 m**