Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
Paper 1

4049/01

25 August 2021   Wednesday   2 hours 15 min

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2021 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen. You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the spaces provided below each question.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

For Examiner's Use

Total Marks

90

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Setter: Mrs Yim Meng Choo
Vetter: Mr Tan Chee Wee

This question paper consists of **17** printed pages and **1** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \square + \binom{n}{r}a^{n-r}b^r + \square + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)............(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all questions in the space provided.

1 Solve the following simultaneous equations.

$$2^{x+6y} = \frac{1}{32} \quad \text{and} \quad \left(9^x\right)\left(729^y\right) = 243.$$

[4]

2 **(i)** Express $y = -2x^2 + 6x - 1$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [2]

(ii) Hence, determine with explanation, whether the curve $y = -2x^2 + 6x - 1$ lies entirely below the x-axis. [2]

3 If the line $x - ky = 10$ is a tangent to the curve $x^2 + y^2 = 20$, find the possible values of k. [5]

4 Given that $\tan^2\theta = p$, where $90^o \leq \theta \leq 180^o$, express in terms of p,

   (i) $\cos\theta$, [2]

   (ii) $\operatorname{cosec} 2\theta$. [3]

5 **(i)** *PQR* is an equilateral triangle whose side is $\left(3\sqrt{3}-1\right)$ cm. Find the exact value of the area of the equilateral triangle *PQR*, in the form $a\sqrt{3}+b$ where *a* and *b* are rational numbers. [4]

(ii) A right prism with the equilateral triangle *PQR* as the cross-sectional base is to be made such that the volume of the prism is $8\left(3\sqrt{3}-1\right)$ cm³. Find the height of the prism, giving your answer in surd form. [3]

6 A man buys a precious gem. The value, *V* dollars, of the gem after *t* years is given by
$V = N(0.97)^{kt}$, where *N* and *k* are constants. At the beginning, the value of the gem is $12000.

   (i) Find the value of *N*. [1]

   (ii) The value of the gem after 5 years is $10000. Find the value of *k*. [4]

   (iii) After 15 years, a gem dealer offers to pay the man $5000 for the gem. Based on the given equation, would you advise him to sell it? Justify your answer. [2]

7 A curve has equation $y = \dfrac{3\tan^2 x}{e^x}$.

   (i) Find the gradient of the curve when $x = \dfrac{\pi}{4}$, leaving your answer in the exact form.   [5]

   (ii) Given that x is increasing at a constant rate of 0.12 units per second, find the rate of change of y when $x = \dfrac{\pi}{4}$.   [2]
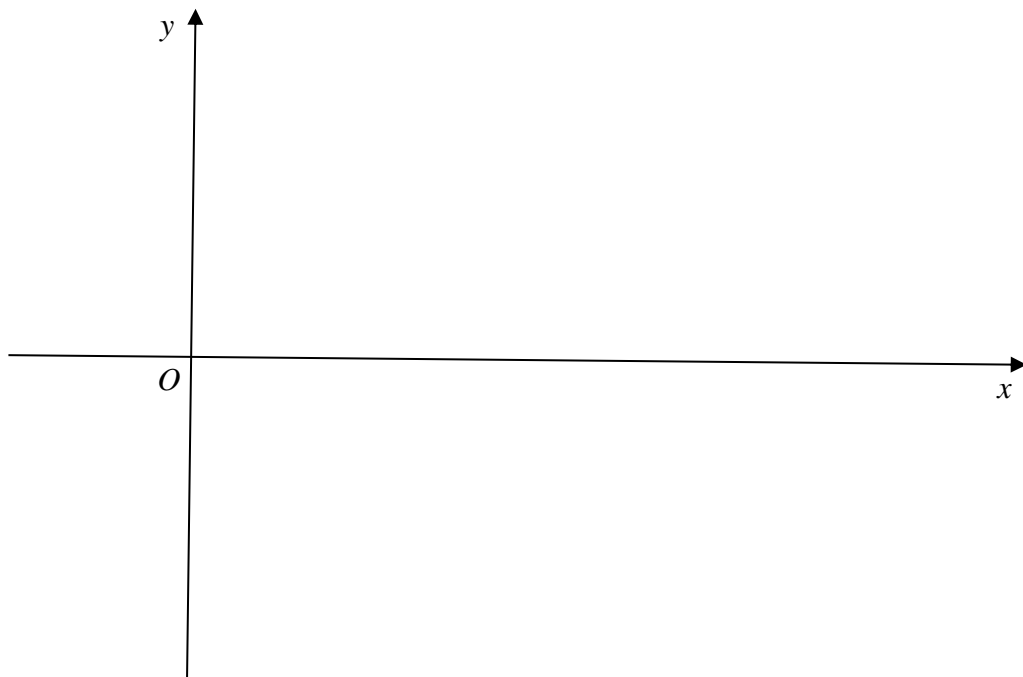
8 The function $f(x) = 3\cos 2x + 1$ is defined for $x \geq 0$.

(i) State the amplitude and period of f. [2]

(ii) Sketch on the same diagram below, the graphs of $f(x) = 3\cos 2x + 1$ and $g(x) = \sin\left(\dfrac{x}{2}\right)$ for $0° \leq x \leq 360°$. [4]

(iii) Hence determine the value of k for which the equation $3\cos(2x) + 1 = \sin\left(\dfrac{x}{2}\right) + k$ has 3 solutions for $0° \leq x \leq 360°$. [1]

10 **(a)** **(i)** Find the first three terms in the expansion, in ascending powers of x, of $\left(2-\dfrac{x}{3}\right)^7$. [2]

(ii) Hence find the value of p, where p is an integer, such that the coefficient of x^2 in

the expansion of $(p+x)^2\left(2-\dfrac{x}{3}\right)^7$ is $-\dfrac{32}{3}p^2$. [3]

(b) Explain why there is no independent term in the expansion of $\left(x^2-\dfrac{1}{2x}\right)^{17}$. [4]

11 [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and the surface area is $4\pi r^2$.]

Mr Lim wants to make a solid cylinder with a hemisphere on top as shown in the diagram below. The cylinder has radius $4x$ cm and height h cm.

The volume of the entire solid is 896π cm^3.
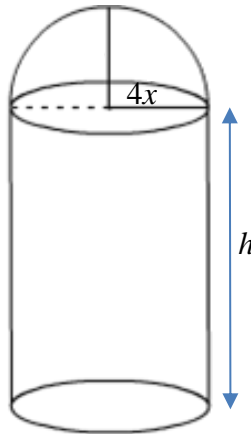
(i) Show that $h = \dfrac{56}{x^2} - \dfrac{8}{3}x$. [2]

(ii) Show that the total surface area of the solid is given by $A = \dfrac{80}{3}\pi x^2 + \dfrac{448\pi}{x}$ cm^2. [3]

**(iii)** Find the value of x for which A is stationary. [2]

(iv) The solid is to be painted completely. The cost of painting is $0.70 per cm^2.
Using the value of x in **(iii)**, calculate the cost of painting the solid and determine whether Mr Lim would be pleased with the cost. [3]

12 The diagram below shows a quadrilateral *ABCD* in which coordinates of *A*, *C* and *D* are $(0, 8)$, $(2k, k-7)$ and $(6, -4)$ respectively.

The equation of *BC* is $19y = -7x + 197$ and the line $y = 8$ bisects angle *BAD*.
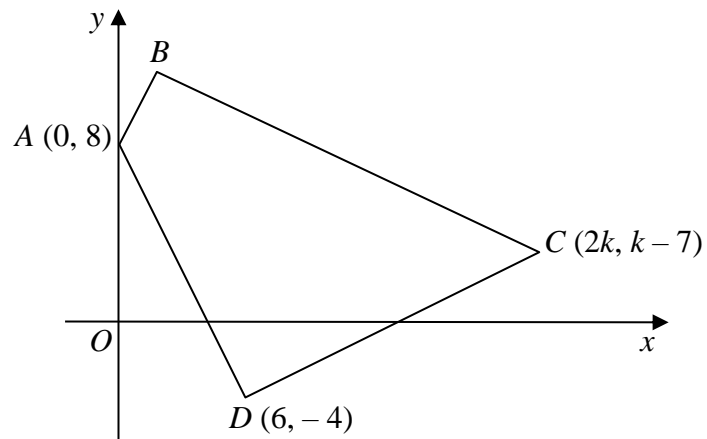
(i) Show that *k* = 10. [2]

(ii) Explain with working why angle *ADC* is $90°$. [2]

(iii) Find the equation of *AB*. [2]

(iv) Calculate the area of quadrilateral *ABCD*. [4]

13 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v m/s is given by $v = kt^2 + 12t - 16$ where k is a constant. When $t = 1$, the acceleration of the particle is 8 m/s^2.

(i) Show that $k = -2$. [2]

(ii) Find the value of t when the velocity of the particle is equal to its initial velocity. [2]

(iii) Find the values of t when the particle is instantaneously at rest. [2]

(iv) Find the distance travelled by the particle during the first 5 seconds. [4]

END OF PAPER

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