FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS   4052/02

Paper 2

Date: 23 August 2023   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

_____

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	☐ 1			
	☐ 2			**90**
Rounding Off	☐ 1		Parent's/Guardian's Signature	

Setter: Mr James Quek

This question paper consists of **23** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** $p = \dfrac{2+f}{4f-1}$

      (i) Find p when $f = -6$.

Answer p = …………...……………….. [1]

      (ii) Rearrange the formula to make f the subject.

Answer f = …………...……………….. [2]

(b) Write as a single fraction in its simplest form $\dfrac{3}{x-2} - \dfrac{4}{2x+3}$.

Answer …………...………………… [2]

1 **(c)** Solve these simultaneous equations.

$$3x - 2y = 56$$
$$3y + 5x = 1.5$$

You must show your working.

Answer x = ……......……………………..

y = ..………...…..………………… [3]

1   **(d)**   Solve the equation $\dfrac{y+2}{2} - \dfrac{10}{3y+2} = 0$.

Answer y = …………...… or *y* = …………….. [3]

2 **(a)** Jeremy earns $7500 each month.
Jeremy contributes 20% of his first $6000 into his CPF funds.
21% of his CPF funds are distributed to Medi-Save account.
16% of his CPF funds are distributed to Special account.
The remaining of his CPF funds are distributed to Ordinary account.

Calculate the amount of Jeremy's CPF that is distributed to Ordinary account.

Answer $ …………...….…………….. [2]

(b) Jeremy drives a car that consumes 12.5 litres of petrol for every 100 km.
The cost of petrol is $2.50 per litre and there is an additional petrol levy of 20 cents for every litre.
Jeremy drives 440 km to a town in Malaysia.

Calculate the cost of the petrol used for this journey.

Answer $ …………...….…………….. [2]

2   (c)   Jeremy invested $10 000 of his savings for 2 years. The interest from his investment is deposited back to his investment account at the end of each year. The rate of interest for the first year is 2.5%. At the end of the second year, the overall percentage increase of his investment is 8.65%.

Find the rate of interest for the second year.

Answer …………...….…………… % [2]

(d)   Jeremy is in Japan and decides to buy a jacket for ¥50 000 using a credit card.
The cashier offers him 2 options to pay for the jacket;

- Option A: pay in Singapore dollars ($) $468.16 or
- Option B: pay in Japanese yen (¥) ¥50 000.

When paying in Japanese yen, the credit card company will convert the amount to Singapore dollars and there is a currency conversion fee of 1.5%.
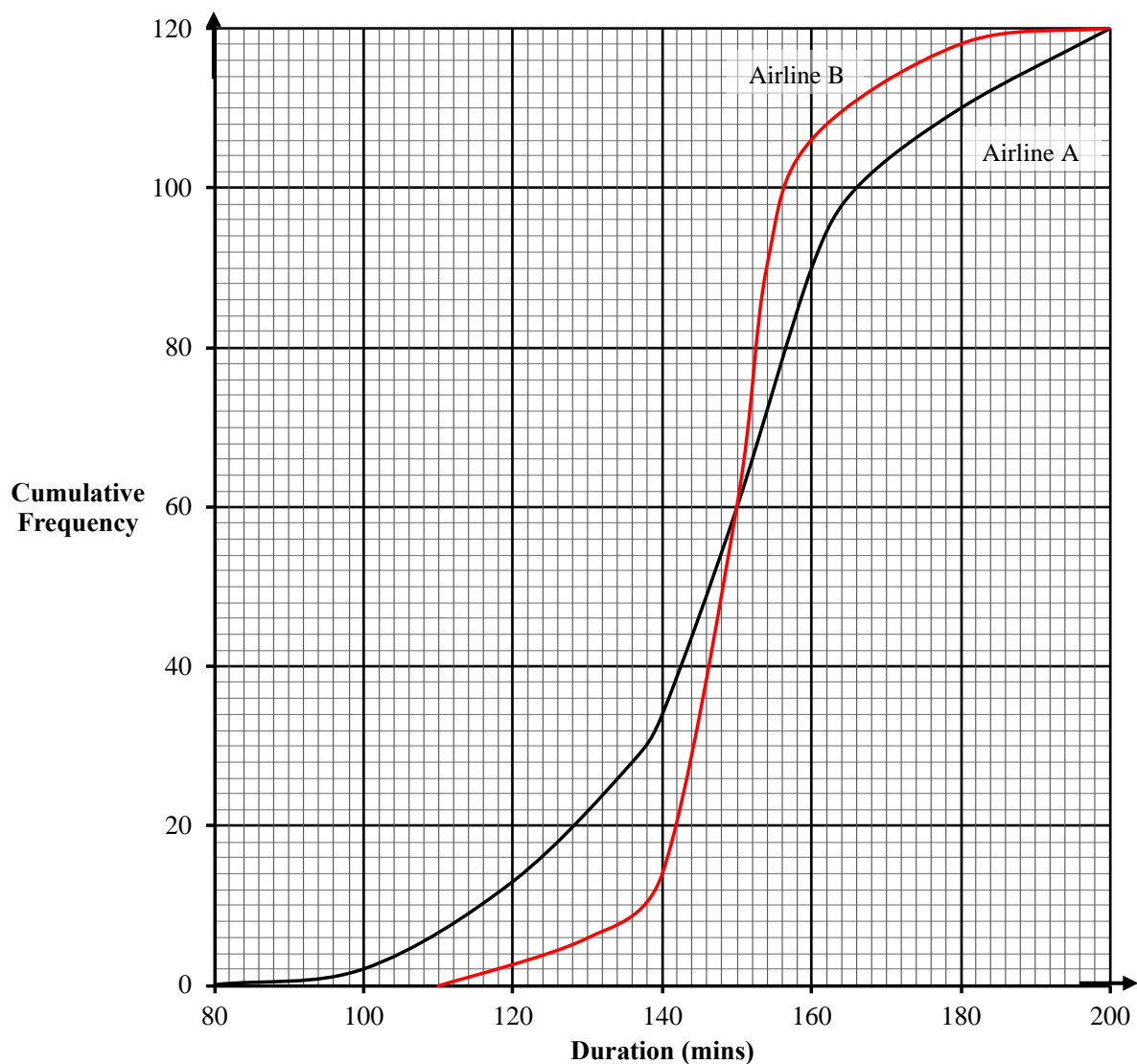The exchange rate between Singapore dollars and Japanese yen is $1 = ¥108.

Which option should Jeremy choose? Show your working clearly.

Option …………… because ………………………………………………………..

……………………………………………………………………………………[3]

3   The cumulative frequency graph for the duration of 120 movies on Airline A and Airline B is shown below.

(a)   Use the graph to find
(i)   the median duration of movies for Airline B,

Answer …………..….…………… mins [1]

(ii)   the interquartile range of duration of movies for Airline A.

Answer …………..….…………… mins [2]

3 **(b)** Ali flies with Airline A and he watched 2 different movies with no break between them. What is the maximum possible duration of the 2 movies?

Answer …………..….hours.…………… mins [1]

(c) Ali prefers to watch movies with a duration between 140 minutes and 160 minutes. Which airlines would be a better choice for Ali?
Justify your answer using appropriate figures.

Airline …………… because ……………………………………………………

…………………………………………………………………………………… [2]

(d) Meals on Airline A are served 2 hours into the flight. Ali starts to watch a movie as the plane takes off. What is the probability that the movie is still playing when his meal is served? Leave your answer as a fraction in its simplest form.

Answer …………...…………….....… [1]

(e) There are 50 Action movies, 10 Romance movies and 60 Comedy movies.
Ali chose 3 different movies at random to watch on the plane.
Find, as a fraction in its simplest form, the probability that he chose at least 2 Comedy movies.

Answer ………...……………...… [2]
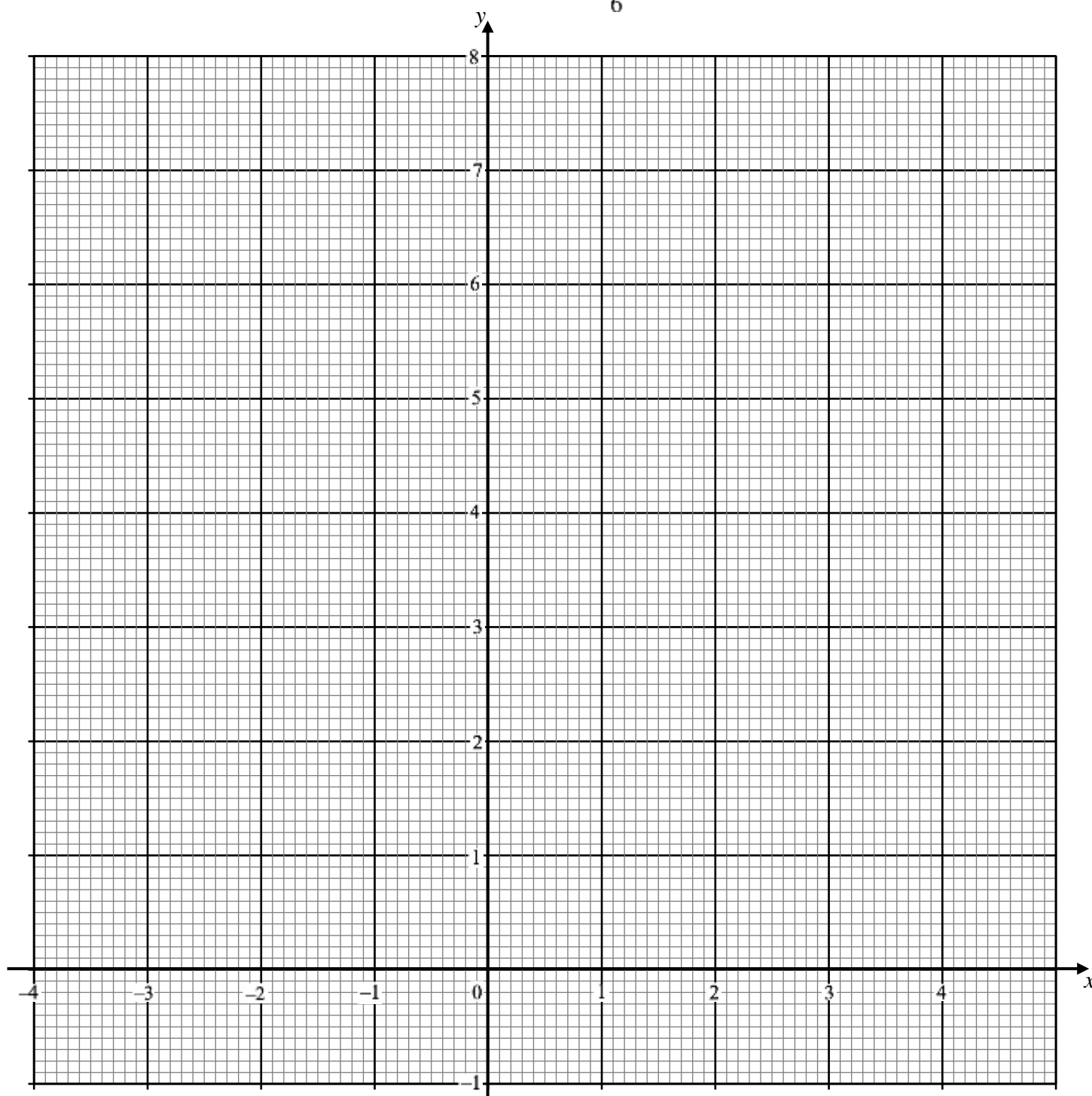
Name: _____ (   )   Class: _____

4   The table below shows some values of x and corresponding values of y for
$y = -\dfrac{x^3}{6} + 2x + 4$.

(a)   Complete the table of values, giving your answer correct to 1 decimal place.

x	-4	-3	-2	-1	0	1	2	3	4
y	6.7	2.5	1.3	2.2	4		6.7	5.5	1.3

[1]

On the grid below, draw the graph of $y = -\dfrac{x^3}{6} + 2x + 4$ for $-4 \leq x \leq 4$.   [3]

4   (b)   The equation $-\dfrac{x^3}{6}+2x+4=k$ has three solutions.

Use your graph to find the range of values for k.

Answer …………...….………………… [1]

(c)   The equation $x^3-15x+3=0$ can be solved by finding the points of intersection of the straight line $y=ax+b$ and the curve $y=-\dfrac{x^3}{6}+2x+4$.

(i)   Find the values of a and the value of b.

Answer a = …………...….………………...

b = …………...….……………… [2]

(ii) By drawing the line $y=ax+b$, solve the equation $x^3-15x+3=0$.

Answer x = …………... or …………... or …………... [3]

5 **(a)** The position vector of the point P is $\begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

The position vector of the point Q is $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$.

(i) Find the vector that represent the translation from P to Q.
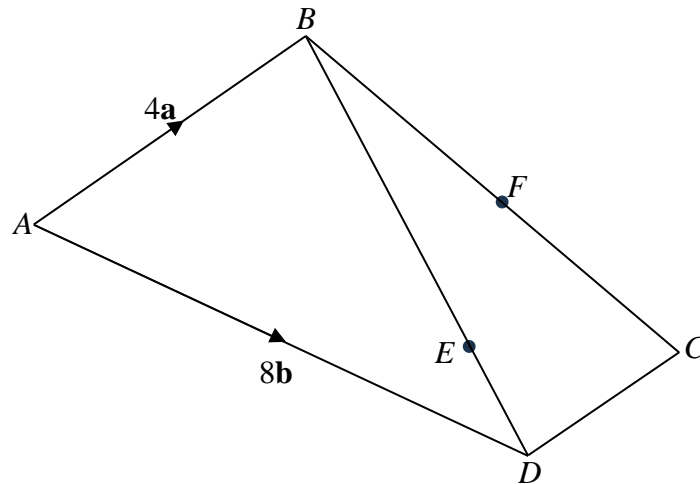
Answer $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(ii) Find the magnitude of $\overrightarrow{PQ}$.

Answer ……………… units[1]

(iii) R is the point on the line PQ with the coordinates $(2, k)$.
Find the position vector of R.

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [2]

5 (b)

In the diagram, $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{AD} = 8\mathbf{b}$ and $\overrightarrow{EF} = 2(\mathbf{a} - \mathbf{b})$.
E is the point on BD such that $BE : BD = 3 : 4$.
F is the midpoint of BC.

(i) Express $\overrightarrow{BE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible.

Answer …………...….………………… [2]

(ii) Express $\overrightarrow{BC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, as simply as possible.

Answer …………...….………………… [2]

(iii) What type of quadrilateral is *ABCD*?
       Justify your answer using vectors.

ABCD is a ……………………… because …………………………………………..

………………………………..………………………………………………………….. [3]

6 **(a)**
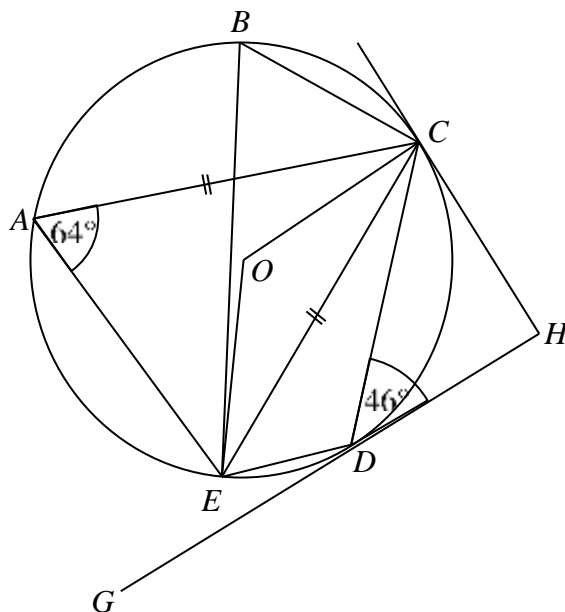
A, B, C, D and E are points on the circumference of a circle with centre O.
GH and CH are tangent to the circle at D and C respectively. Triangle ACE is an
isosceles triangle. Angle $EAC = 64°$ and angle $CDH = 46°$.

(i) Find, stating your reasons clearly,

(a) angle EBC,

Answer …………...………………° [1]

(b) reflex angle EOC,

Answer …………...………………° [1]

(c) angle EDG.

Answer …………...………………° [2]

(ii) Explain why a semicircle with CD as diameter, does not past through H.

…………………………………..…………………………………………………..........

…………………………………..…………………………………………………........ [2]

6 (b)
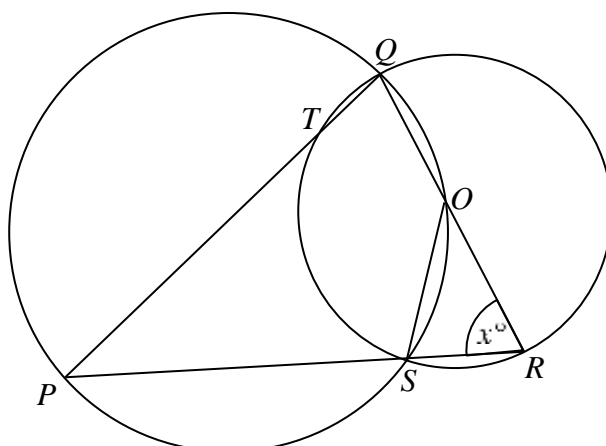
Circle *RSTQ* with centre *O*, intersects circle *OSPQ* at *Q* and *S*. *PTQ, PSR* and *QOR* are straight lines and angle *QRS = x°*.
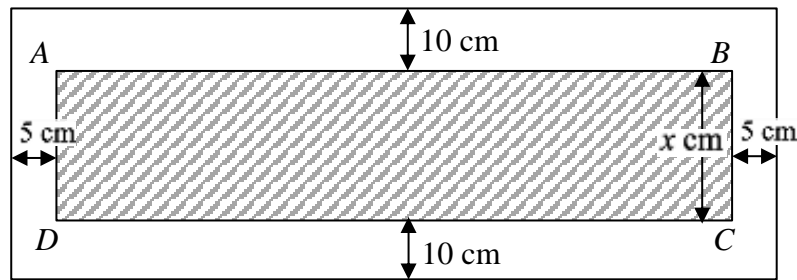
(i) Show that triangles *ORS* and *PRQ* are similar.

…………………………………..……………………………………………………...

…………………………………..……………………………………………………...

…………………………………..……………………………………………………….. [2]

(ii) Given *x°* = 60°, find the ratio of the area of triangle *ORS* and quadrilateral *PQOS*.

Answer ……...…… : ………… [2]

7   A rectangular Chinese painting, *ABCD*, is placed inside a rectangular frame.

The length, *AB*, of the painting is four times its width, *x* cm. The dimensions of the frame are shown in the diagram above.

(a)   The total area of the painting and the frame is 13550 cm².
Form an equation in *x* and show that it simplifies to $2x^2 + 45x - 6675 = 0$.

[3]

(b)   Solve the equation $2x^2 + 45x - 6675 = 0$.
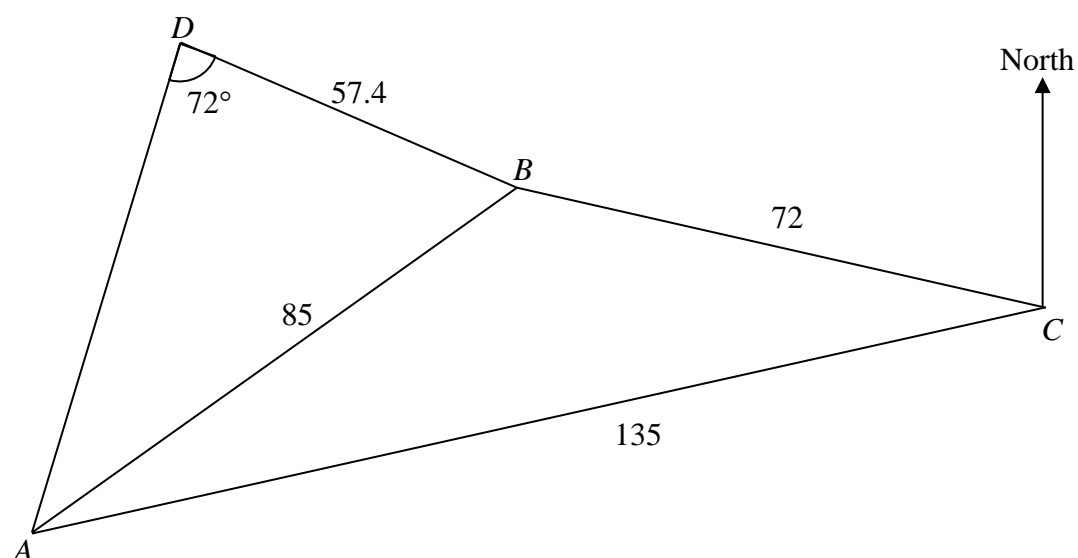Give your solutions correct to two decimal places.

Answer x = …….…… or *x =* ………… [3]

7 **(c)** The painting is painted on premium rice paper. The cost of premium rice paper is $63 per square metres. Calculate the cost of the premium rice paper that was used in the painting.

Answer $ …………………… [3]

8

ABC and *ADB* are two triangular plots of land.
AB = 85 m, *BC* = 72 m, *AC* = 135 and *BD* = 57.4 m.
The bearing of *B* from *C* is 290° and angle *ADB* = 72°.

(a)   Find the bearing of *A* from *B*.

Answer …………………….° [4]

8 (b) The plot of land *ADB* is to be filled with top soil. Calculate the area to be filled with top soil.

Answer …………….……m^2 [4]

(c) A 40 m antenna mast is installed vertically at point *D*. Find the largest angle of depression from the top of the mast to a point along the path *AB*.

Answer ……………………° [3]

Name: _____ (   )   Class: _____

9   Three friends, Tan, Mei and Jaya are planning for an 8-day holiday trip to Japan in December 2023. They plan to travel from Osaka to Tokyo and then back to Osaka. Below is the itinerary.

Day	1	2	3	4	5	6	7	8
Location	Singapore to Osaka	Osaka to Tokyo	Tokyo	Tokyo	Tokyo	Tokyo	Tokyo to Osaka	Osaka to Singapore
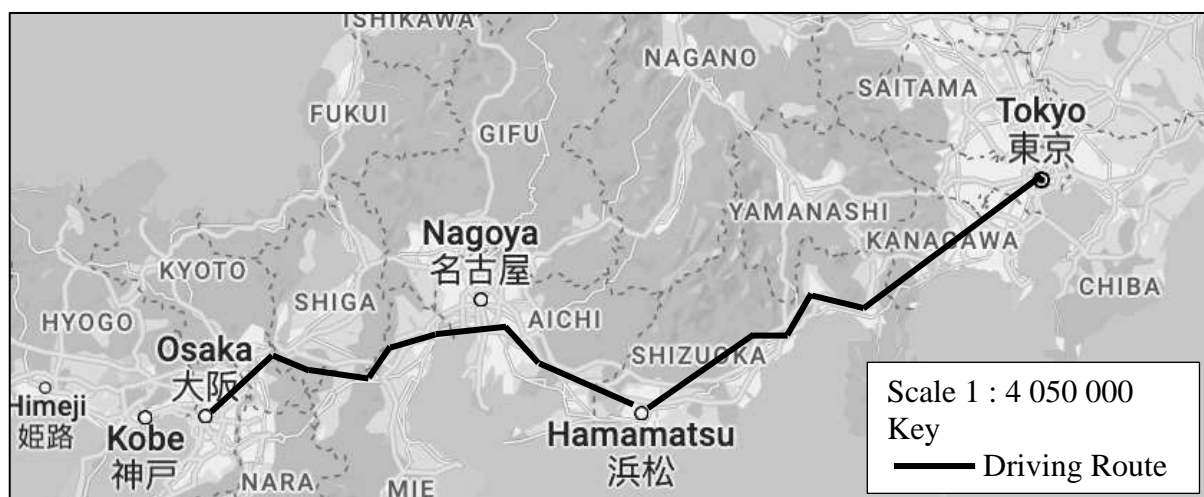
   (a)   Estimate, correct to the nearest km, the driving distance between Osaka and Tokyo.

Answer …………………….km [2]

9 **(b)** There are 2 rest stops between Osaka and Tokyo. Jaya plans to drive for 2 hours to the first rest stop at an average speed of 90 km/h.
Mei will drive to the second rest stop 200 km away from the first rest stop at average speed of 100 km/h.
Lastly, Tan will drive to Tokyo at average speed of 110 km/h.
Tan, Mei and Jaya will take a 30-minutes break at each rest stop.

Calculate the total amount of time taken to reach Tokyo. Give your answer to the nearest hour.

Answer …………………….hours [2]

9 (c)

	Small Car	Medium Car	Large Car	SUV	Car is filled fully with petrol for collection
Amount of fuel used (litres/100 km)	5.5	6.3	7.8	8.2	
Fuel Tank size	40 *l*	45 *l*	45 *l*	50 *l*	
Rental in Japanese Yen (¥) for 6 Days	58 000	60 000	65 000	90 000	Pick up/return point: Osaka
Other Information	Fit 1 luggage	Fit 1 luggage	Fit 2 luggage	Fit 3 luggage	Return with full tank of petrol
Fuel Prices	¥170 per litre				
Tokyo Hotel Parking Fees	• ¥6000 per day upon check-in • Free on check-out day				

Approximate toll fare (in ¥)			
	Fukuoka	**Hiroshima**	**Osaka**
Tokyo	25 000	18 000	13 500
Osaka	13 000	7 000	
Hiroshima	7 000		
Fukuoka			

Bullet Train Ticket Prices (in thousand ¥) Osaka Tokyo (One Way)		
Cabin	**Price per Pax***	*There is a 45% increment in price after 1st Oct 2023
Econ	16	
Econ with luggage	18	
1st Class	25	
1st Class with luggage	27	Duration: 2 hr 30 min

Tan, Mei and Jaya plan to rent a car that can fit 3 luggage to drive to Tokyo and back to Osaka.
Mei collated some information (shown above) that she found on the Internet and suggested to her other 2 friends that taking the bullet train is a cheaper option to drive.

Do you agree with Mei?
Justify your answer and show your working clearly.

Name: _____ (   )   Class: _____

…………………………………….....……………………………………………………...

…………………………………….....……………………………………………………...

…………………………………….....………… …………………………………….. [6]

~ End of Paper ~