Regent Secondary School
Additional Mathematics
Sec 4 Express Preliminar Examination 2020
Paper 2
(Setter: Ms Su RY)
Marking Scheme

Qn	Solution	Marks	Total	Marker's Report
1i	$2x^2-4x+3=0$ Sum of roots $(\alpha+\beta)$ $=-\frac{b}{a}$ $=-\frac{-4}{2}$ $=2$ Product of roots $(\alpha\beta)$ $=\frac{c}{a}$ $=\frac{3}{2}$ $\alpha^2+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta$ $=(2)^2-2(\frac{3}{2})$ $=1$ (Shown)	M1 M1 M1 A1	4	
1ii	Sum of new roots $=\alpha^2+3+\beta^2+3$ $=\alpha^2+\beta^2+6$ $=1+6$ $=7$ Product of new roots $=(\alpha^2+3)(\beta^2+3)$ $=\alpha^2\beta^2+3\alpha^2+3\beta^2+9$ $=(\alpha\beta)^2+3(\alpha^2+\beta^2)+9$ $=\left(\frac{3}{2}\right)^2+3(1)+9$ $=\frac{57}{4}$ New quadratic equation	M1 M1		

	$x^2 - 7x + \frac{57}{4} = 0$ $4x^2 - 28x + 57 = 0$	A1	3	
2i	General Term $= \binom{7}{r} (x)^{7-r} \left(\frac{k}{x}\right)^r$ $= \binom{7}{r} k^r (x)^{7-r} (x^{-1})^r$ $= \binom{7}{r} k^r (x)^{7-2r}$ To find term with x^3, $7 - 2r = 3$ $r = 2$ Coefficient of x^3 $= \binom{7}{3} k^2$ $=21k^2$ To find term with x, $7 - 2r = 1$ $r = 3$ Coefficient of x $= \binom{7}{3} k^3$ $=35k^3$ $21k^2 = 35k^3$ $35k^3 - 21k^2 = 0$ $7k^2(5k - 3) = 0$ $k = 0$ (rej) or $k = \frac{3}{5}$	M1 M1 M1 M1 M1 A1	6	
2ii	$7 - 2r = 7$ $r = 0$ Coefficient of x^7 $= \binom{7}{0} k^0$ $= 1$ To find term with x^5, $7 - 2r = 5$ $r = 1$	M1		

<table>
<tr><td></td><td>Coefficient of x^5
$=\binom{7}{1}\left(\frac{3}{5}\right)$
$=\frac{21}{5}$

$\left(1-5x^2\right)\left(x+\frac{k}{x}\right)^7$
$=\left(1-5x^2\right)(x^7+\frac{21}{5}x^5+...)$
$=x^7-21x^7$
$=-20x^7$

Coefficient of $x^7 = -20$</td><td>M1

M1

A1</td><td>4</td><td></td></tr>
<tr><td>2ii</td><td>Alternative Method:

$(1-5x^2)[x^7+7(x^6)\left(\frac{k}{x}\right)+...]$
$=(1-5x^2)[x^7+\frac{21}{5}x^5+...]$

Coefficient of x^7
$= 1-21$
$=-20$</td><td>M1
M1

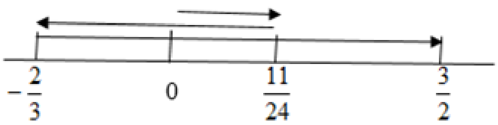
M1
A1</td><td>4</td><td></td></tr>
<tr><td>3i</td><td>$m = m_0 e^{-kt}$
$\ln m = \ln m_0 e^{-kt}$
$\ln m = -kt + \ln m_0$
<table><tr><td>t</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>$\ln m$</td><td>3.88</td><td>3.73</td><td>3.58</td><td>3.43</td><td>3.28</td></tr></table>
See graph at the end.</td><td>B1

C2</td><td>3</td><td></td></tr>
<tr><td>3ii</td><td>$\ln m_0 = 4.03$ (acceptable range 4.02 - 4.03)
$m_0 = e^{4.03}$
$= 56.3$mg (3sf)</td><td>M1

A1</td><td>2</td><td></td></tr>
<tr><td>3iii</td><td>$-k = \frac{3.28-3.88}{10-2}$
$=-0.075$
$k = 0.075$ (± 0.01)</td><td>M1

A1</td><td>2</td><td></td></tr>
</table>

3iv	Half original mass $=\frac{e^{4.03}}{2}$ $= 28.13046...$ $\ln(28.13046...) = 3.33685...$ Time taken = 9.2 hr	 M1 A1	√ based 3ii 2	
4i	$y = 2x^2 + kx + 6 - k$ $b^2 - 4ac < 0$ $k^2 - 4(2)(6-k) < 0$ $k^2 - 48 + 8k < 0$ $(k+12)(k-4) < 0$ $-12 < k < 4$	 M1 M1 A1	 3	
4ii	When k = 2, $y = 2x^2 + 2x + 4$ $y = mx - 4$ $2x^2 + 2x + 4 = mx - 4$ $2x^2 + (2-m)x + 8 = 0$ $b^2 - 4ac = 0$ $(2-m)^2 - 4(2)(8) = 0$ $m^2 - 4m - 60 = 0$ $(m-10)(m+6) = 0$ $m = 10$ or $m = -6$	 M1 M1 M1 A1	 4	
5i	$AF = 2\left(4\cos\theta\right) = 8\cos\theta$ $AB = 2\left(3\sin\theta\right) = 6\sin\theta$ $Perimeter = 3+3+4+4+6\sin\theta + 8\cos\theta$ $= 14 + 6\sin\theta + 8\cos\theta$	M1 M1 A1	 3	
5ii	$Perimeter = 14 + 6\sin\theta + 8\cos\theta$ $6\sin\theta + 8\cos\theta = R\sin\left(\theta + \alpha\right)$ $R = \sqrt{6^2 + 8^2} = 10$ $\tan\alpha = \frac{8}{6}$ $\alpha = 53.13°$ $P = 14 + 10\sin\left(\theta + 53.13°\right)$	 M1 M1 A1	 3	
5iii	$23 = 14 + 10\sin\left(\theta + 53.13°\right)$ $10\sin\left(\theta + 53.13°\right) = 9$ $\sin\left(\theta + 53.13°\right) = 0.9$ $Basic\ angle = 64.158°$ $\theta + 53.13° = 64.158°, 115.842°$ $\theta = 11.028°, 62.712°$ $= 11.0°, 62.7°$	 M1 M1 A1	 3	

6i	$a=\frac{dv}{dt}=4t-5$ Decelerating means $a=4t-5<0$ $t<\frac{5}{4}$ seconds	M1 M1 A1	3	
6ii	When instantaneously at rest, $v=2t^2-5t+2=0$ $(2t-1)(t-2)=0$ $t=\frac{1}{2}$, $t=2$	M1 A1	2	
6iii	$s=\int vdt=\int(2t^2-5t+2)dt$ $=\frac{2t^3}{3}-\frac{5t^2}{2}+2t+c$ When t =0, s =0 , so c =0 Hence, $s=\frac{2t^3}{3}-\frac{5t^2}{2}+2t$	M1 A1	2	
6iv	When $t=0$, $s=0$ When, $t=\frac{1}{2}$. $s=\frac{2}{3}(\frac{1}{2})^3-\frac{5}{2}(\frac{1}{2})^2+2(\frac{1}{2})=\frac{11}{24}$ When $t=2$, $s=\frac{2}{3}(2)^3-\frac{5}{2}(2)^2+2(2)=-\frac{2}{3}$ When3, $s=\frac{2}{3}(3)^3-\frac{5}{2}(3)^2+2(3)=\frac{3}{2}$  Total distance travelled $d=\frac{11}{24}+(\frac{11}{24}+\frac{2}{3})+(\frac{3}{2}+\frac{2}{3})$ $=3\frac{3}{4}$ m	M1 M1 M1 M1 A1	5	

7i	$f(x)=2x^3+ax^2+bx+6$ Since $(x+2)$ s a factor, $f(-2)=2(-2)^3+a(-2)^2+b(-2)+6=0$ $-16+4a-2b+6=0$ $4a-2b=10$ --------------- (1) $f(3)=2(3)^3+a(3)^2+b(3)+6=15$ $54+9a+3b+6=15$ $9a+3b=-45$ $a=-2$ and $b=-9$	M1 M1 A1, A1	4	
7ii	$f(x)=2x^3-2x^2-9x+6=0$ $(x+2)(2x^2+kx+3)=0$ $kx^2+4x^2=-2x^2$ $k=-6$ $(x+2)(2x^2-6x+3)=0$ $x=-2$ or $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$ $x=-2$ or $x=\frac{3\pm\sqrt{3}}{2}$	M1 M1 A1, A1	√ based on 7i 4	
7iii	$16y^3-8y^2-18y+6=0$ $2(2y)^3-2(2y)^2-9(2y)+6=0$ Consider $2y=x$ $2y=-2$ or $2y=\frac{3\pm\sqrt{3}}{2}$ $y=-1$ or $y=\frac{3\pm\sqrt{3}}{4}$	M1 M1 A1, A1	4	

8i	$\tan A + \cot A = 2\operatorname{cosec} 2A$ *LHS* $\tan A + \cot A$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \frac{1}{\frac{1}{2}(\sin 2A)}$ $= \frac{2}{\sin 2A}$ $= 2\operatorname{cosec} 2A$	M1 M1 M1 A1	4	
8ii	$\tan A + \cot A = 5$ $2\operatorname{cosec} 2A = 5$ $\operatorname{cosec} 2A = \frac{5}{2}$ $\frac{1}{\sin 2A} = \frac{5}{2}$ $\sin 2A = \frac{2}{5}$ Basic angle = $\sin^{-1}(\frac{2}{5})$ = 23.57818° 2*A* is in 1st of 2nd quadrant 2*A* $= 23.57818°, 180° - 23.57818°, 360° + 23.57818°, 180° - 23.57818° + 360^\circ$ $A = 11.8°, 78.2°, 191.8°, 258.2°$	M1 M1 M1 B2	5	
9i	$\frac{d}{dx}(\cos^3 x - 3\cos x)$ $= 3\cos^2 x(-\sin x) + 3\sin x$ $= -3\sin x(1 - \sin^2 x) + 3\sin x$ $= 3\sin^3 x$ (Shown)	M1 A1	2	
9ii	$\int_{0.5}^{1} (3\sin^3 x - 2\sin x)dx$ $= \int_{0.5}^{1} 3\sin^3 x dx - \int_{0.5}^{1} 2\sin x dx$ $= \left[\cos^3 x - 3\cos x\right]_{0.5}^{1} + 2[\cos x]_{0.5}^{1}$ $= \left[\cos^3 1 - 3\cos 1 - \cos^3 0.5 + 3\cos 0.5\right] + 2[\cos 1 - \cos 0.5]$ = -0.181 (3sf)	M1, M1 M1 A1	4	

10i	When $y=0$, $x=2\sqrt{x}$ $x^2=4x$ $x(x-4)=0$ $x=0$ or $x=4$ $\therefore A(4,0)$ $\frac{dy}{dx}$ $=1-2(\frac{1}{2}x^{-\frac{1}{2}})$ $=1-\frac{1}{\sqrt{x}}$ When $x=4$, $\frac{dy}{dx}=\frac{1}{2}$ Gradient of AB = -2 Equation of AB $y-0=-2(x-4)$ $y=-2x+8$	M1 M1 M1 A1	4	
10ii	$1-\frac{1}{\sqrt{x}}=0$ $x=1$ $y=-2(1)+8$ $y=6$ $B(1,6)$	M1 M1 A1	√ based on 10i 3	
10iii	$\left\lvert\int_1^4 x-2\sqrt{x}\ \ dx\right\rvert+\frac{1}{2}\times3\times6$ $=\left\lvert\left[\frac{x^2}{2}-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4\right\rvert+9$ $=\left\lvert-2\frac{2}{3}-(-\frac{5}{6})\right\rvert+9$ $=10\frac{5}{6}$ units^2	M1 M1 A1	√ based on 10i and 10ii 3	
11i	Centre $=\left(\frac{2+8}{2},\frac{3+11}{2}\right)$ $=(5,7)$ Radius	B1		

	$=\sqrt{(2-5)^2+(3-7)^2}$ = 5 units	M1 A1	 3	
11ii	$(x-5)^2+(y-7)^2=25$	B1	1	
11iii	Consider $x=0$, $(0-5)^2+(y-7)^2=25$ $y^2-14y=49=0$ Consider b^2-4ac $=14^2-4(49)=0$ Since $b^2-4ac=0$, y axis is tangent to the circle.	M1 A1	 2	
	or $(y-7)^2=0$ $y=7$ Since there is only 1 point of intersection, y-axis is tangent to the circle.	M1 A1	 2	
11iv	Gradient OQ $=\frac{7-11}{5-8}$ $=\frac{4}{3}$ Gradient of tangent at Q $=-\frac{3}{4}$ Equation of tangent at Q $y-11=-\frac{3}{4}(x-8)$ = $y=-\frac{3}{4}x+17$	M1 M1 A1	 3	