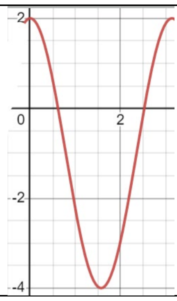
2023 Sec 4NA Preliminary Examinations Paper 1

1	$\frac{18x+7}{(x-1)(4x+1)}=\frac{A}{x-1}+\frac{B}{4x+1}$ $18x+7=A(4x+1)+B(x-1)$ $Let\ x=1$ $18+7=5A$ $A=5$ $Let\ x=-\frac{1}{4}$ $18(-\frac{1}{4})+7=B(-\frac{1}{4}-1)$ $\frac{5}{2}=-\frac{5}{4}B$ $B=-2$ $\frac{18x+7}{(x-1)(4x+1)}=\frac{5}{x-1}-\frac{2}{4x+1}$	**2**	$3x^2+x+5=1-2x$ $3x^2+3x+4=0$ $D=(3)^2-4(3)(4)=-39$ Since $D<0$, the line will not intersect the curve.
		3(i)	$(5x+3)(x-2)>6$ $5x^2-10x+3x-6>6$ $5x^2-7x-12>0$ $(x+1)(5x-12)>0$ $x<-1$ (reject) or $x>\frac{12}{5}$
		(ii)	$x=3$
4(i)	$x+3\overline{)\ 2x^3+5x^2-4x-3}$ quotient $2x^2-x-1$ $-\left(2x^3+6x^2\right)$ $-x^2-4x$ $-\left(-x^2-3x\right)$ $-x-3$ $-\left(-x^2-3x\right)$ 0	**5**	$x^3-8=0$ $(x-2)(x^2+2x+4)=0$ $x-2=0\ \ or\ \ x^2+2x+4=0$ $x=2\ \ or\ \ D=2^2-4(1)(4)=-12<0$ Since $x^2+2x+4=0$ has no solutions, $x=2$ is the only real root of the equation $x^3-8=0$
4 (ii)	$x+3$ is a factor of f(x).	**6(i)**	$-2x^2-4x+3$ $=-2(x^2+2x-\frac{3}{2})$ $=-2(x^2+2x+(-1)^2-(-1)^2-\frac{3}{2})$ $=-2[(x+1)^2-\frac{5}{2}]=-2(x+1)^2+5$
4 (iii)	$(x+3)(2x^2-x-1)=0$ $(x+3)(2x+1)(x-1)=0$ $x=-3$ or $-\frac{1}{2}$ or 1	**6(ii)**	Maximum value is 5.

7	$V=\frac{4x^3}{3}-2x+3$, $\frac{dV}{dx}=4x^2-2$ At $x=5, \frac{dV}{dx}=4(5)^2-2=98$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $52=98\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{26}{49}\quad=0.531$ cm/s (to 3 sf) Rate of change of $x=0.531$ cm/s	**8**	$\frac{d^2y}{dx^2}=12x-2$ $\frac{dy}{dx}=\int 12x-2\,dx=\ 6x^2-2x+c$ $At\,(2,15)$, $\frac{dy}{dx}=27$ $6(2)^2-2(2)+c=27\longrightarrow c=7$ $\frac{dy}{dx}=6x^2-2x+7$ $y=\int 6x^2-2x+7\,dx=2x^3-x^2+7x+c$ *When* $x=2, y=15$ $\therefore 15=2(2)^3-2^2+7(2)+c$ $c=-11$ $y=2x^3-x^2+7x-11$
9a (i)	$\sin^{-1}(\frac{\sqrt{3}}{2})=60°$	**9b (ii)**	
9a (ii)	$\cos^{-1}(-\sin 45°)=135°$		
9b (i)	Period = 180°, Amplitude = 3		
10 (i)	$AC^2=(\sqrt{7}+2)^2+(\sqrt{7}-2)^2$ $=\ 7+4\sqrt{7}+4+7-4\sqrt{7}+4$ $=22$ $AC=\sqrt{22}\,cm$ *reject* $AC=-\sqrt{22}$	**10 (ii)**	$\tan A=\frac{\sqrt{7}-2}{\sqrt{7}+2}$ $=\frac{(\sqrt{7}-2)^2}{(\sqrt{7})^2-2^2}$ $=\frac{7-4\sqrt{7}+4}{3}\quad=\frac{11-4\sqrt{7}}{3}$
11 (i)	$LHS=\frac{\sin 2x}{1+\cos 2x}$ $=\frac{2\sin x\cos x}{1+2\cos^2 x-1}$ $=\frac{2\sin x\cos x}{2\cos^2 x}$ $=\frac{\sin x}{\cos x}$ $=\tan x=RHS$	**11 (ii)**	$\frac{\sin 2x}{1+\cos 2x}=2$ $\tan x=2$ $BA=1.1071$ $x=1.1071,\ \ \pi+1.1071$ $=1.11,\ \ 4.25$

12 (a)	$y=\frac{x^4}{\sqrt{1+2x}}$ $\frac{dy}{dx}=\frac{4x^3\sqrt{1+2x}-x^4(\frac{1}{2})(1+2x)^{-\frac{1}{2}}(2)}{1+2x}$ $=\frac{4x^3\sqrt{1+2x}-x^4(1+2x)^{-\frac{1}{2}}}{1+2x}$ $=\frac{x^3[4(1+2x)-x]}{(1+2x)^{\frac{3}{2}}}$ $=\frac{x^3(4+7x)}{(1+2x)^{\frac{3}{2}}}$	**b (i)**	$y=x^2(x-1)^5$ $\frac{dy}{dx}=2x(x-1)^5+x^2(5)(x-1)^4$ $=x(x-1)^4[2(x-1)+5x]$ $=x(x-1)^4(7x-2)$
		b (ii)	Since y is decreasing, $\frac{dy}{dx}<0$ $x(x-1)^4(7x-2)<0$ $\text{Since } (x-1)^4\geq 0 \textit{ for all } x,$ $x(7x-2)<0$ $0<x<\frac{2}{7}$
13	$y=x^2+2x+8$ $\frac{dy}{dx}=2x+2$ At x = 1: $\frac{dy}{dx}=4, y=11$ Equation of tangent: $\frac{y-11}{x-1}=4$ $y=4x+7$	At x = 4: $\frac{dy}{dx}=2(4)+2=10,\ y=32$ Gradient of normal is $-\frac{1}{10}$ Equation of normal is $\frac{y-32}{x-4}=-\frac{1}{10}$ $y-32=-\frac{1}{10}x+\frac{4}{10}$ $y=-\frac{1}{10}x+32\frac{2}{5}$	
	Solve the 2 equations simultaneously: $4x+7=-\frac{1}{10}x+32\frac{2}{5}$ $4\frac{1}{10}x=25\frac{2}{5}$ $x=6\frac{8}{41}\quad;\quad y=31\frac{32}{41}$ $P(6\frac{8}{41},31\frac{32}{41})$		