1 Factorise $a^3 + b^3$ and hence express $\dfrac{7^{\frac{3}{2}} + 3^{\frac{3}{2}}}{7^{\frac{1}{2}} + 3^{\frac{1}{2}}}$ in the form $p + q\sqrt{21}$, where p and q are integers. [3]

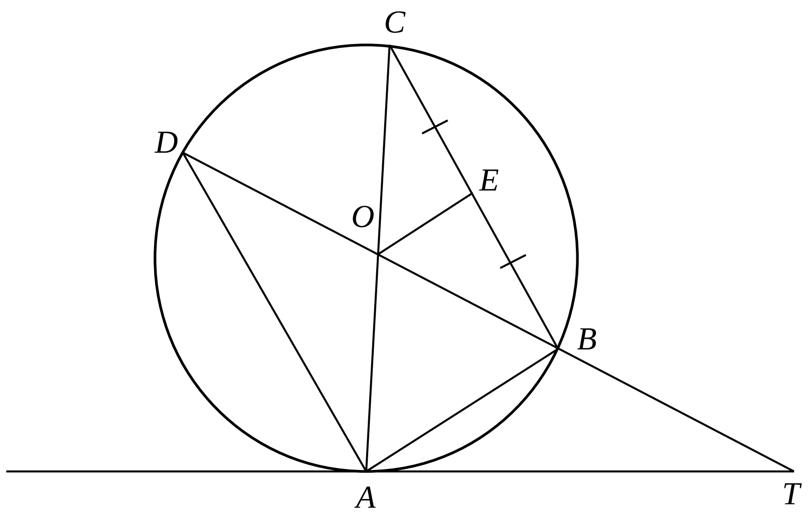
2 The equation of a curve is $y = 4\sin 3x + \frac{1}{2}$.

(a) State the minimum and maximum values of *y*. [2]

(b) Sketch the graph of $y = 4\sin 3x + \frac{1}{2}$ for $0° \le x \le 360°$. [3]

3 The line $\frac{x}{a}+\frac{y}{b}=2$, where *a* and *b* are positive constants, intersects the *x*-axis at *P* and the *y*-axis at *Q*. Given that the gradient of *PQ* is $-\frac{3}{5}$ and that the distance *PQ* = $\sqrt{34}$, find the value of *a* and of *b*. [5]

4 In the diagram, *A*, *B*, *C* and *D* lie on a circle with centre *O*. *E* lie on the line *BC* such that *BE* = *EC*. The tangent to the circle at *A* meets *DB* produced at *T*.



(a) Show that the length of *OE* : *AB* is 1 : 2. [2]

(b) Show that angle *ATB* = $90° - 2 \times \text{angle } TAB$. [3]

5 **(a)** Find the first 3 terms in the expansion, in ascending powers of x, of $\left(3-\frac{2x}{3}\right)^5$. Give the terms in their simplest forms. [3]

(b) Hence find the term independent of x in the expansion $\left(3-\frac{2x}{3}\right)^5\left(x-\frac{5}{x}\right)^2$. [3]

6 In an experiment, 200 grams of a particular radioactive substance is stored in a container.

The amount of substance present after t days is given by $N = 200e^{kt}$, where k is a constant. It is found that the substance is halved after 20 days.

(a) Find the amount of substance remaining after 1 week. [4]

The radioactive substance is considered to be safe to be discarded using the proper procedures when the amount is reduced to 15 grams.

(b) John claimed that it is safe to discard the radioactive substance after 74 days.

Is his claim justified?
Show the calculations and the reasons on which you based your answer. [2]

7 For a particular curve $\frac{d^2y}{dx^2} = 9\cos 3x - 5\sin x$. The curve passes through the point $P\left(\frac{3\pi}{2}, \frac{15\pi}{2}\right)$ and the gradient of the curve at *P* is 8. Find the equation of the curve. [6]

8 The function f is defined by $f(x) = -\dfrac{3x^2}{x^2+1}$, $x > 0$.

(a) Explain, with working, whether f is an increasing or a decreasing function. [4]

(b) A point T moves along the curve $y = f(x)$ in such a way that its y-coordinate is decreasing at a rate of 0.03 units per second. Find the rate of increase of its x-coordinate when $x = 3$. [2]

9 The expression $2x^3 + ax^2 + b$, where *a* and *b* are constants, has a factor of $2x-3$ and leaves a remainder of -14 when divided by $x-2$.

(a) Find the value of *a* and of *b*. [4]

(b) Using these values of *a* and *b*, show that the equation $2x^3 + ax^2 + b = 0$ has only one real root. [3]

10 **Without using a calculator,**

(a) show that $\cos 105° = \dfrac{1-\sqrt{3}}{2\sqrt{2}}$. [2]

(b) express $\sec^2 105°$ in the form of $a + b\sqrt{3}$, where *a* and *b* are integers. [5]

11 **(a)** Express each of $-x^2+6x-14$ and $3x^2+12x+13$ in the form $a(x+b)^2+c$, where a, b and c are integers. [4]

(b) Use your answers from part **(a)** to explain why the curves with equations $y=-x^2+6x-14$ and $y=3x^2+12x+13$ will not intersect. [3]

12

Diagram I

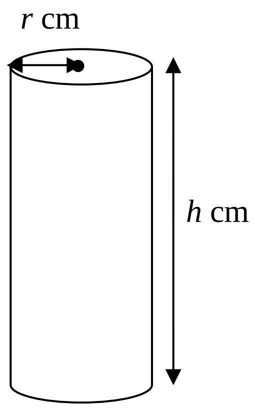


Diagram II

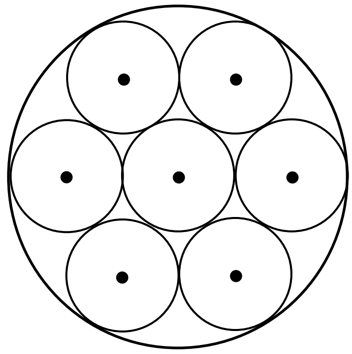


Diagram I shows a cylindrical battery with radius *r* cm and height *h* cm.
Diagram II shows the top view and cross-section of a closed plastic cylindrical container which contains 7 of these batteries.
The batteries are just touching each other and the sides of the container.

(a) Given that the volume of the plastic cylindrical container is 108 cm^3, express *h* in in terms of *r*. [1]

(b) Show that *A*, the total surface area of the plastic cylindrical container is

$$A = \frac{72}{r} + 18\pi r^2$$

[2]

(c) Given that *r* can vary, find the smallest possible total surface area of the cylindrical plastic container. [5]

13 **(a)** Prove the identity $\sec x - \dfrac{\cos x}{1+\sin x} = \tan x$. [4]

(b) Hence solve the equation $\dfrac{1}{2}\left(\sec x - \dfrac{\cos x}{1+\sin x}\right) = \cot x$ for $0° < x < 180°$. [3]

(c) Show that there are no solutions to the equation

$$\sec x - \frac{\cos x}{1+\sin x} = \frac{1}{1-\tan x}.$$

[2]

14 A boy cycling passes a point *A* at the bottom of the slope with a speed of 17 m/s. He then cycled up the slope and passes point *B* at the top of the slope, 5 seconds later, with a speed of 4.5 m/s. Between *A* to *B*, his velocity *v* m/s, is given by $v = 0.5t^2 - pt + q$, where *t* is the time in seconds passing *A*, and *p* and *q* are constants.

(a) Show that $q = 17$ and find the value of *p*. [3]

(b) Find the deceleration of the cyclist when his speed is 6.12 m/s. [4]

(c) Find the distance *AB* in metres. [3]

Answer Key

1	$10-\sqrt{21}$
2(a)	Min value = $-3\frac{1}{2}$ Max value = $4\frac{1}{2}$
3	$a=\frac{5}{2}$ $b=\frac{3}{2}$
5(a)	$243-270x+120x^2+...$
5(b)	570
6(a)	157g
6(b)	It is not safe to discard the radioactive substance after 74 days
7	$y=-\cos 3x+5\sin x+5x+5$
8(a)	$f'(x)=\frac{-6x}{(x^2+1)^2}$ Since $(x^2+1)^2>0$ and $x>0$, $-6x<0$ $\frac{-6x}{(x^2+1)^2}<0$, $f'(x)<0$ Since $f'(x)<0$, $f(x)$ is a decreasing function.
8(b)	$\frac{1}{6}$ units/s
9(a)	$a=5,\ b=-18$
10(b)	$8+4\sqrt{3}$
11(a)	$-x^2+6x-14=-(x-3)^2-5$ $3x^2+12x+13=3(x+2)^2+1$
11(b)	Since *y*-coordinate of min point is > than *y*-coordinate of max point, the two graph will not intersect.
12(a)	$h=\frac{12}{\pi r^2}$
12(c)	126 cm^2
13(b)	$x=54.7°,125.3°$
14(a)	p = 5
14(b)	1.8 m/s^2
14(c)	$43\frac{1}{3}$ m