XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2023

CANDIDATE NAME	MARK SCHEME		
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **28 August 2023**

Secondary 4 Express **2 hour 15 minutes**

Setter : Ms Vanessa Chia
Vetter : Mrs Wong Li Meng
Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		***Marks Awarded***	
Presentation		***Marks Penalised***	

For Examiner's Use
90

Parent's/Guardian's Signature:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A cuboid has a square base of side $\left(2\sqrt{2}-1\right)$ cm and a volume of $\left(53-29\sqrt{2}\right)$ cm^3.

Without using a calculator, find the height of the cuboid, in cm, in the form $\left(a+b\sqrt{2}\right)$, where a and b are integers. [5]

Let h be the height of the cuboid

$\left(2\sqrt{2}-1\right)^2 \times h = 53-29\sqrt{2}$ [M1]

$$h = \frac{53-29\sqrt{2}}{\left(2\sqrt{2}-1\right)^2}$$

$$= \frac{53-29\sqrt{2}}{\left(2\sqrt{2}\right)^2 - 2\left(2\sqrt{2}\right)(1) + (1)^2}$$

$$= \frac{53-29\sqrt{2}}{8-4\sqrt{2}+1}$$

$$= \frac{53-29\sqrt{2}}{9-4\sqrt{2}} \frac{\times\left(9+4\sqrt{2}\right)}{\times\left(9+4\sqrt{2}\right)}$$

[M1- for $9-4\sqrt{2}$]
[M1- for rationalising denominator, allow ecf]

$$= \frac{\left(53-29\sqrt{2}\right)\left(9+4\sqrt{2}\right)}{(9)^2 - \left(4\sqrt{2}\right)^2}$$

$$= \frac{477+212\sqrt{2}-261\sqrt{2}-116(2)}{49}$$

[M1- for either numerator or denominator]

$$= \frac{245-49\sqrt{2}}{49}$$

$= \left(5-\sqrt{2}\right)$ cm [A1]

2 A curve is such that $\frac{dy}{dx} = ae^{1-x} - 3x^2 + 10$, where a is a constant. The point P (1, 5) lies on the curve. The gradient of the curve at P is 12.

(a) Show that $a = 5$. [1]

$\frac{dy}{dx} = ae^{1-x} - 3x^2 + 10$

When $x = 1$ and $\frac{dy}{dx} = 12$,

$ae^0 - 3(1)^2 + 10 = 12$
$a - 3 + 10 = 12$
$a = 5$ (shown) [A1]

(b) Find the equation of the curve. [4]

$\frac{dy}{dx} = 5e^{1-x} - 3x^2 + 10$

$y = \int 5e^{1-x} - 3x^2 + 10 \, dx$

$= \frac{5e^{1-x}}{-1} - \frac{3x^3}{3} + 10x + c$

$= -5e^{1-x} - x^3 + 10x + c$ [M1- for $-5e^{1-x}$] [M1- for $-x^3 + 10x$]

When $x = 1$ and $y = 5$,

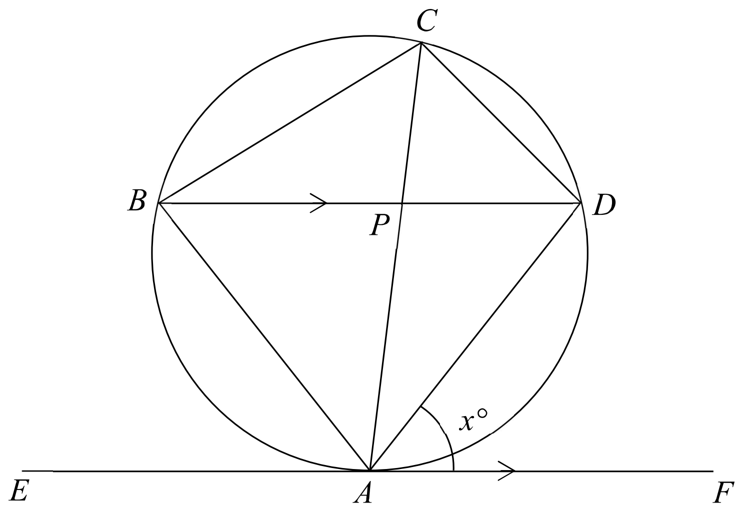
$-5e^0 - (1)^3 + 10(1) + c = 5$ [M1- allow ecf]

$-5 - 1 + 10 + c = 5$

$c = 1$

$\therefore y = -5e^{1-x} - x^3 + 10x + 1$ [A1]

3



A, B, C and D are points on a circle. EF is a tangent to the circle and angle $DAF = x°$.
BD is parallel to EF. AC and BD intersects at P.

Prove that

(a) $AB = AD$, [3]

$\angle ADB = \angle DAF = x°$ (alt. $\angle$s, // lines) [M1- with correct reason]
$\angle ABD = \angle DAF = x°$ (alt. segment thm) [M1- with correct reason]
Since $\angle ADB = \angle ABD$, by (base $\angle$s of isos triangles), $AB = AD$.
[A1- accept "isosceles triangle"]

Deduct 1 m from overall question for incorrectly phrased reasons.

(b) AC bisects angle BCD. [2]

$\angle ACD = \angle DAF = x°$ (alt. segment thm / $\angle$s in same segment)
$\angle ACB = \angle ADB = x°$ ($\angle$s in same segment)
} [M1- one correct reason]
Since $\angle ACD = \angle ACB$, $\therefore$ AC bisects $\angle BCD$.
[A1- both reasons correct + conclusion]

4 **(a)** Express $4\cos^2 x - 6\sin^2 x$ in the form $a\cos 2x + b$. [2]

$4\cos^2 x - 6\sin^2 x$

$= 4\left(\frac{\cos 2x + 1}{2}\right) - 6\left(\frac{1-\cos 2x}{2}\right)$ [M1- for either]

$= 2(\cos 2x + 1) - 3(1 - \cos 2x)$

$= 2\cos 2x + 2 - 3 + 3\cos 2x$

$= 5\cos 2x - 1$ [A1]

Alt mtd

$4\cos^2 x - 6\sin^2 x$

$= 4\cos^2 x - 6(\underline{1-\cos^2 x})$ [M1]

$= 4\cos^2 x - 6 - 6\cos^2 x$

$= 10\cos^2 x - 6$

$= 10\left(\frac{\cos 2x + 1}{2}\right) - 6$

$= 5(\cos 2x + 1) - 6$

$= 5\cos 2x + 5 - 6$

$= 5\cos 2x - 1$ [A1]

(b) The equation of a curve is $y = 2x^2 - kx + 4$. Find the set of values of k for which the line $y = k - 4x$ meets the curve. [4]

$y = 2x^2 - kx + 4$ ---(1)

$y = k - 4x$ ---(2)

(1)=(2),

$2x^2 - kx + 4 = k - 4x$ [M1]

$2x^2 - kx + 4x + 4 - k = 0$

$2x^2 + (4-k)x + (4-k) = 0$

Line meets the curve: $D \geq 0$

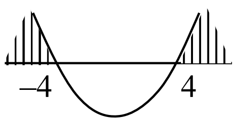
$(4-k)^2 - 4(2)(4-k) \geq 0$ [M1]

$16 - 8k + k^2 - 32 + 8k \geq 0$

$k^2 - 16 \geq 0$

$(k+4)(k-4) \geq 0$ [M1- for factorising]

$K^2 \geq 16$ X



$k \leq -4$ or $k \geq 4$ [A1]

5 A curve has the equation $y = e^{\frac{1}{2}x} + 5e^{-\frac{1}{2}x}$.

(a) Show that the exact value of the y-coordinate of the stationary point of the curve is $2\sqrt{5}$. [4]

$y = e^{\frac{1}{2}x} + 5e^{-\frac{1}{2}x}$

$\frac{dy}{dx} = e^{\frac{1}{2}x}\left(\frac{1}{2}\right) + 5e^{-\frac{1}{2}x}\left(-\frac{1}{2}\right)$ [M1]

$= \frac{1}{2}e^{\frac{1}{2}x} - \frac{5}{2}e^{-\frac{1}{2}x}$

When $\frac{dy}{dx} = 0$, $\frac{1}{2}e^{\frac{1}{2}x} - \frac{5}{2}e^{-\frac{1}{2}x} = 0$ [M1- allow ecf]

$\frac{1}{2}e^{\frac{1}{2}x} = \frac{5}{2}e^{-\frac{1}{2}x}$

$e^{\frac{1}{2}x} = \frac{5}{e^{\frac{1}{2}x}}$

$e^{\frac{1}{2}x} \times e^{\frac{1}{2}x} = 5$

$e^x = 5$ [M1]

Mtd 1

$x = \ln 5$

$y = e^{\frac{1}{2}\ln 5} + 5e^{-\frac{1}{2}\ln 5}$

$= \left(e^{\ln 5}\right)^{\frac{1}{2}} + 5\left(e^{\ln 5}\right)^{-\frac{1}{2}}$

$= 5^{\frac{1}{2}} + 5(5)^{-\frac{1}{2}}$

$= 5^{\frac{1}{2}} + 5^{\frac{1}{2}}$

$= \sqrt{5} + \sqrt{5}$

$= 2\sqrt{5}$ (shown) [A1]

Mtd 2

$y = \left(e^x\right)^{\frac{1}{2}} + 5\left(e^x\right)^{-\frac{1}{2}}$ ---(1)

sub into (1), $y = (5)^{\frac{1}{2}} + 5(5)^{-\frac{1}{2}}$

$= \sqrt{5} + \frac{5}{\sqrt{5}}$

$= \sqrt{5} + \frac{5\sqrt{5}}{5}$

$= \sqrt{5} + \sqrt{5}$

$= 2\sqrt{5}$ (shown) [A1]

(b) Determine the nature of this stationary point. [2]

$\frac{d^2y}{dx^2} = \frac{1}{2}e^{\frac{1}{2}x}\left(\frac{1}{2}\right) - \frac{5}{2}e^{-\frac{1}{2}x}\left(-\frac{1}{2}\right)$ [M1]

$= \frac{1}{4}e^{\frac{1}{2}x} + \frac{5}{4}e^{-\frac{1}{2}x}$

When $e^x = 5$, $\frac{d^2y}{dx^2} = \frac{1}{4}(5)^{\frac{1}{2}} + \frac{5}{4}(5)^{-\frac{1}{2}}$

$= 1.11803$ or $\frac{1}{2}\sqrt{5}$

Since $\frac{d^2y}{dx^2} > 0$, it is a minimum point. [A1- with $\frac{d^2y}{dx^2} = 1.18$ or $\frac{d^2y}{dx^2} > 0$]

Alt mtd: First Derivative Test

$\frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x} - \frac{5}{2}e^{-\frac{1}{2}x}$

x	1.5	ln5	1.7
$\frac{dy}{dx}$	– ve	0	+ ve
	\	–	/

[M1]

∴ it is a minimum point. [A1]

6 The binomial expansion of $(1 + ax)^n$, where $n > 0$, in ascending powers of x is

$$1 - 6x + 36a^2x^2 + bx^3 + \dots .$$

Find the value of n, a and b. [6]

$(1 + ax)^n$

$= (1)^n + \binom{n}{1}(1)^{n-1}(ax)^1 + \binom{n}{2}(1)^{n-2}(ax)^2 + \binom{n}{3}(1)^{n-3}(ax)^3 + \dots$ [M1]

$= 1 + nax + \frac{n(n-1)}{2!}a^2x^2 + \frac{n(n-1)(n-2)}{3!}a^3x^3 + \dots$

$= 1 + nax + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{6}a^3x^3 + \dots$

By comparing with $1 - 6x + 36a^2x^2 + bx^3 + \dots$,

$\frac{n(n-1)}{2}a^2x^2 = 36a^2x^2$ [M1- with $\binom{n}{2}$ expanded correctly]

$\frac{n(n-1)}{2} = 36$

$n(n - 1) = 72$

$n^2 - n - 72 = 0$

$(n - 9)(n + 8) = 0$

$n = 9$ or $n = -8$ (rej) } [A1]

$9ax = -6x$

$a = \frac{-6}{9}$

$= -\frac{2}{3}$ [A1]

$bx^3 = \frac{9(8)(7)}{6}\left(-\frac{2}{3}\right)^3 x^3$ [M1- allow ecf]

$b = -\frac{224}{9}$ [A1]

7 **(a)** The equation of a curve is $y = -x^2 + 10x - 17$.

(i) Express $-x^2 + 10x - 17$ in the form $a - (x + b)^2$, where a and b are constants. [2]

$-x^2 + 10x - 17$

$= -(x^2 - 10x + 17)$

$= -\left(x^2 - 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2 + 17\right)$

$= -((x-5)^2 - 8)$

$= -(x-5)^2 + 8$

$= 8 - \underbrace{(x-5)^2}_{[M1]}$ [A1]

Alt Mtd

$a - (x + b)^2$

$= a - (x^2 + 2bx + b^2)$

$= a - x^2 - 2bx - b^2$

$= -x^2 - 2bx + (a - b^2)$

By comparing,

$-2b = 10$

$b = -5$

$a - (-5)^2 = -17$

$a - 25 = -17$

$a = 8$

$\therefore 8 - \underbrace{(x-5)^2}_{[M1]}$ [A1]

(ii) The straight line L meets the curve at one point only. Given that L is not a tangent to the curve, what can be deduced about L? [1]

L is a vertical line. [B1]

*No marks if students wrote specific vertical line equation, e.g. $x = 1$.

(b) Express $\dfrac{3x^2 - 14x - 20}{(x-3)^2(2x+1)}$ in partial fractions. [4]

Let $\dfrac{3x^2 - 14x - 20}{(x-3)^2(2x+1)} = \dfrac{A}{x-3} + \dfrac{B}{(x-3)^2} + \dfrac{C}{2x+1}$ [M1]

$3x^2 - 14x - 20 = A(x-3)(2x+1) + B(2x+1) + C(x-3)^2$

When $x = 3$,

$3(3)^2 - 14(3) - 20 = 0 + B(2(3) + 1) + 0$

$7B = -35$

$B = -5$

When $x = -\frac{1}{2}$,

$3\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) - 20 = 0 + 0 + C\left(-\frac{1}{2} - 3\right)^2$

$\frac{49}{4}C = -\frac{49}{4}$

$C = -1$

When $x = 0$,

$-20 = A(-3)(1) - 5 - (-3)^2$

$-20 = -3A - 14$

$3A = 6$

$A = 2$

[M1- any two correct]
[M2- all correct]

*If partial fraction form is incorrect, **award 1m** for ability to do substitution method correctly.

$\therefore \dfrac{3x^2 - 14x - 20}{(x-3)^2(2x+1)} = \dfrac{2}{x-3} - \dfrac{5}{(x-3)^2} - \dfrac{1}{2x+1}$ [A1]

*Penalise under "presentation" if students wrote $\dfrac{2}{x-3} + \dfrac{-5}{(x-3)^2} + \dfrac{-1}{2x+1}$

8 The velocity, v m/s, of a particle travelling in a straight line, t seconds after passing through a fixed point O is given by $v = \frac{7}{(t+2)^3}$.

(a) Explain why the particle does not change its direction of motion. [1]

$v = \frac{7}{(t+2)^3}$

When $t > 0$, $\underline{(t+2)^3 > 0}$ ← required.

$\frac{7}{(t+2)^3} > 0$

Since $\underline{v > 0}$, ∴ the particle does not come to instantaneous rest and it <u>does not change its direction of motion</u>. [B1]

↖ req'd

(b) Find the deceleration of the particle when $t = 3$. [2]

$v = 7(t+2)^{-3}$

$a = \frac{dv}{dt}$

$a = 7(-3)(t+2)^{-4}$ [M1]

$a = -\frac{21}{(t+2)^4}$

when $t = 3$, $a = -\frac{21}{(3+2)^4}$

$= -\frac{21}{625}$

∴ the deceleration is $\frac{21}{625}$ m/s^2. [A1- or 0.0336 m/s^2]

8 **(c)** Find the distance travelled by the particle in the third second. [4]

$v = 7(t + 2)^{-3}$

Mtd 1

$s = \int 7(t+2)^{-3}\,\mathrm{d}t$

$s = \dfrac{7(t+2)^{-2}}{-2} + c$ [M1- for $\dfrac{7(t+2)^{-2}}{-2}$]

$s = -\dfrac{7}{2(t+2)^2} + c$

when $t = 0$ and $s = 0$, $-\dfrac{7}{2(2)^2} + c = 0$ [M1 - allow ECF if incorrect integration]

$-\dfrac{7}{2(2)^2} + c = 0$

$c = \dfrac{7}{8}$

$s = -\dfrac{7}{2(t+2)^2} + \dfrac{7}{8}$

third second: from $t = 2$ to $t = 3$

When $t = 2$, $s = -\dfrac{7}{2(2+2)^2} + \dfrac{7}{8}$

$= \dfrac{21}{32}$

When $t = 3$, $s = -\dfrac{7}{2(3+2)^2} + \dfrac{7}{8}$

$= \dfrac{147}{200}$

Dist travelled in the third second

$= \dfrac{147}{200} - \dfrac{21}{32}$ [M1]

$= \dfrac{63}{800}$ m [A1- or 0.07875 m]

Mtd 2:

third second: from $t = 2$ to $t = 3$

Dist travelled in the third second

$= \int_2^3 7(t+2)^{-3}\ \mathrm{d}x$ [M1]

$= \left[\dfrac{7(t+2)^{-2}}{-2}\right]_2^3$ [M1- for $\dfrac{7(t+2)^{-2}}{-2}$]

$= \left[-\dfrac{7}{2(t+2)^2}\right]_2^3$

$= \left(-\dfrac{7}{2(3+2)^2}\right) - \left(-\dfrac{7}{2(2+2)^2}\right)$ [M1]

$= -\dfrac{7}{50} - \left(-\dfrac{7}{32}\right)$

$= \dfrac{63}{800}$ m [A1- or 0.07875 m]

9 The equation of a curve is $y = \dfrac{x^2}{3x+1}$. The tangent to the curve at point P is parallel to the line $4y = x - 8$. The x-coordinate of P is negative.

(a) Find the coordinates of P. [5]

$$y = \frac{x^2}{3x+1}$$

$$\frac{dy}{dx} = \frac{(3x+1)(2x) - x^2(3)}{(3x+1)^2} \qquad \text{[M1]}$$

$$= \frac{6x^2 + 2x - 3x^2}{(3x+1)^2}$$

$$= \frac{3x^2 + 2x}{(3x+1)^2} \qquad \text{[M1]}$$

$4y = x - 8$

$y = \dfrac{1}{4}x - 2$

Gradient of tangent at $P = \dfrac{1}{4}$

$$\frac{3x^2 + 2x}{(3x+1)^2} = \frac{1}{4} \qquad \text{[M1- allow ecf]}$$

$4(3x^2 + 2x) = (3x + 1)^2$

$12x^2 + 8x = 9x^2 + 6x + 1$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$3x - 1 = 0$ or $x + 1 = 0$

$x = \dfrac{1}{3}$ (rej) $\quad \underline{x = -1}$ [M1- for $x = -1$ with mtd to solve quad eqn]

$$y = \frac{(-1)^2}{3(-1)+1}$$

$$= -0.5$$

$\therefore P(-1, -0.5)$ [A1]

9 **(b)** The normal to the curve at P meets the y-axis at Q.

Find the area of the triangle POQ, where O is the origin. [3]

(grad normal at P) –4 [M1]

(eqn normal at P) $y = -4x + c$

When $x = -1$ and $y = -0.5$, $-0.5 = -4(-1) + c$

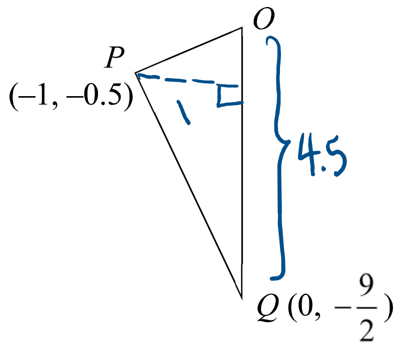
$$c = -\frac{9}{2}$$

$$y = -4x - \frac{9}{2}$$ [M1- for $-\frac{9}{2}$; allow ECF]

$Q\ (0,\ -\frac{9}{2})$

Mtd 1

(area ΔPOQ) $\frac{1}{2} \times 4.5 \times 1 = \frac{9}{4}$ units^2 [A1- or 2.25]



Mtd 2

$$\frac{1}{2}\begin{vmatrix} 0 & -1 & 0 & 0 \\ 0 & -1.5 & -0.45 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[(0 + 4.5 + 0) - (0 + 0 + 0)\right]$$

$= \frac{9}{4}$ units^2 [A1- or 2.25]

10 Water activities are held at a sports centre near Changi Beach. The depth of water, d metres, at time t hours after 0700 is modelled by $d = a \sin bt + c$, where a, b and c are positive constants.

The time between a high tide to a low tide is 6 hours. The depth of the water at high tide is 3.1 metres and the depth of the water at low tide is 0.3 metres.

(a) Show that $b = \frac{\pi}{6}$. [2]

Half a period of the sine graph = 6

$\frac{2\pi}{b} = 12$ [M1]

$2\pi = 12b$

$b = \frac{2\pi}{12}$

$b = \frac{\pi}{6}$ (shown) [A1]

(b) Find the value of a and c. [2]

$a = \frac{3.1 - 0.3}{2}$

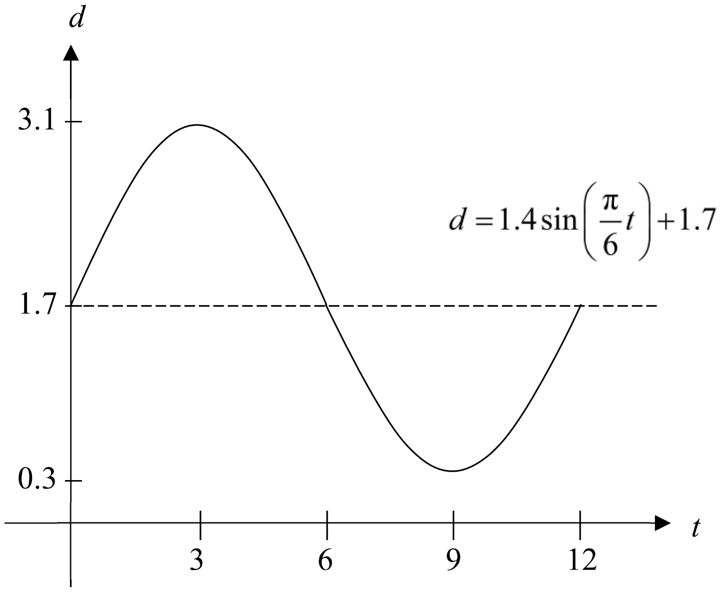
$= 1.4$ [B1]

$c = \frac{3.1 + 0.3}{2}$

$= 1.7$ [B1]

10 **(c)** Sketch the graph of d over the period 0700 to 1900 hours. [3]

$$d = 1.4\sin\left(\frac{\pi}{6}t\right) + 1.7$$

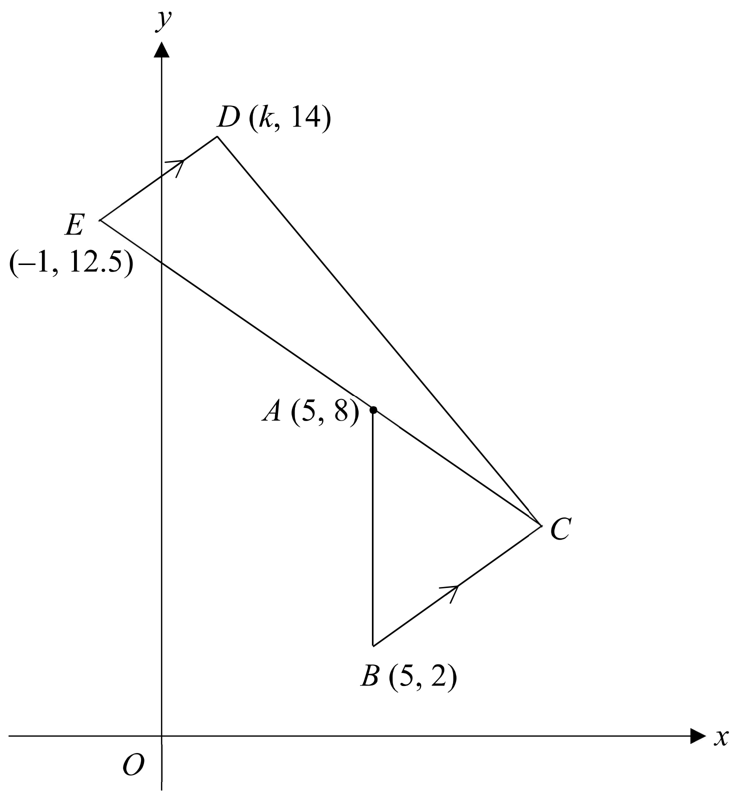


[B1- correct sine curve shape + 1 cycle]
[B1- max at $d = 3.1$, min at $d = 0.3$ and axis of curve at $d = 1.7$]
[B1- all the critical t values correct]

(d) Water activities are only permitted when the depth of water is at least a certain height. As a result, the sports centre closes x hours after 1 pm. Write an expression, in terms of x, for the number of hours that it will be closed for. [1]

Closed after 1 pm → $t > 6$
By symmetry, $6 - x - x = (6 - 2x)$ h [B1]

11



The diagram shows an isosceles triangle ABC in which the point A is (5, 8), B is (5, 2) and $AC = BC$. CA is produced to E such that ED is parallel to BC.
The point D is (k, 14) and E is (–1, 12.5). The area of triangle ABC is 12 units^2.

(a) Show that the coordinates of C is (9, 5). [2]

Let M be the midpoint of AB

$\frac{8+2}{2} = 5$

M (5, 5)

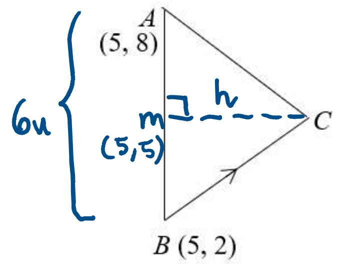
let h be the height of ΔABC

$\frac{1}{2}\times 6\times h = 12$ [M1]

$h = 12 \div \frac{1}{2} \div 6$

$= 4$

$\therefore C$ (9, 5) (shown) [A1]



(b) Find the value of k. [3]

(grad BC) $\frac{5-2}{9-5} = \frac{3}{4}$ [M1]

(grad DE) $\frac{14-12.5}{k-(-1)} = \frac{3}{4}$ [M1- allow ecf for grad BC]

$\frac{1.5}{k+1} = \frac{3}{4}$

$3(k + 1) = 4(1.5)$
$3k + 3 = 6$
$3k = 6$
$k = 1$ (shown) [A1]

Alt Mtd

(grad BC) $\frac{5-2}{9-5} = \frac{3}{4}$ [M1]

(eqn DE) $y = \frac{3}{4}x + c$

When $x = -1$ and $y = 12.5$, $12.5 = \frac{3}{4}(-1) + c$

$c = \frac{53}{4}$

$y = \frac{3}{4}x + \frac{53}{4}$ [M1]

When $x = k$ and $y = 14$,

$14 = \frac{3}{4}k + \frac{53}{4}$

$\frac{3}{4} = \frac{3}{4}k$

$k = 1$ (shown) [A1]

11 (c) Prove that the angle *CDE* is not a right angle. [2]

(grad *DE*) $\frac{3}{4}$

(grad *DC*) $\frac{14-5}{1-9} = -\frac{9}{8}$

grad *DE* × grad *DC* = $\frac{3}{4} \times -\frac{9}{8}$

$= -\frac{27}{32}$ [M1]

Since grad *DE* × grad *DC* ≠ –1, ∴ the angle *CDE* is not a right angle. [A1]

Alt Mtd

$EC^2 = \left(\sqrt{(-1-9)^2 + (12.5-5)^2}\right)^2$

$= 156.25$ (or $\frac{625}{4}$)

$ED^2 + DC^2 = \left(\sqrt{(-1-1)^2 + (12.5-14)^2}\right)^2 + \left(\sqrt{(1-9)^2 + (14-5)^2}\right)^2$

$= \frac{25}{4} + 145$

$= 151.25$ (or $\frac{605}{4}$) [M1]

Since $EC^2 \neq ED^2 + DC^2$, by the converse of Pythagoras' Theorem, the angle *CDE* is not a right angle. [A1]

(d) Find the area of triangle *CDE*. [2]

Area Δ*CDE*

$= \frac{1}{2}\begin{vmatrix} 9 & 1 & -1 & 9 \\ 5 & 14 & 12.5 & 5 \end{vmatrix}$ [M1]

$= \frac{1}{2}\left[(126 + 12.5 - 5) - (5 - 14 + 112.5)\right]$

$= 15$ units^2 [A1]

12 Since 1980, the number of trees in a forest has been steadily decreasing. The table shows the number of trees, N, remaining in the forest in the decades following 1980. The decade 1980 – 1989 is taken as $t = 1$, and so on.

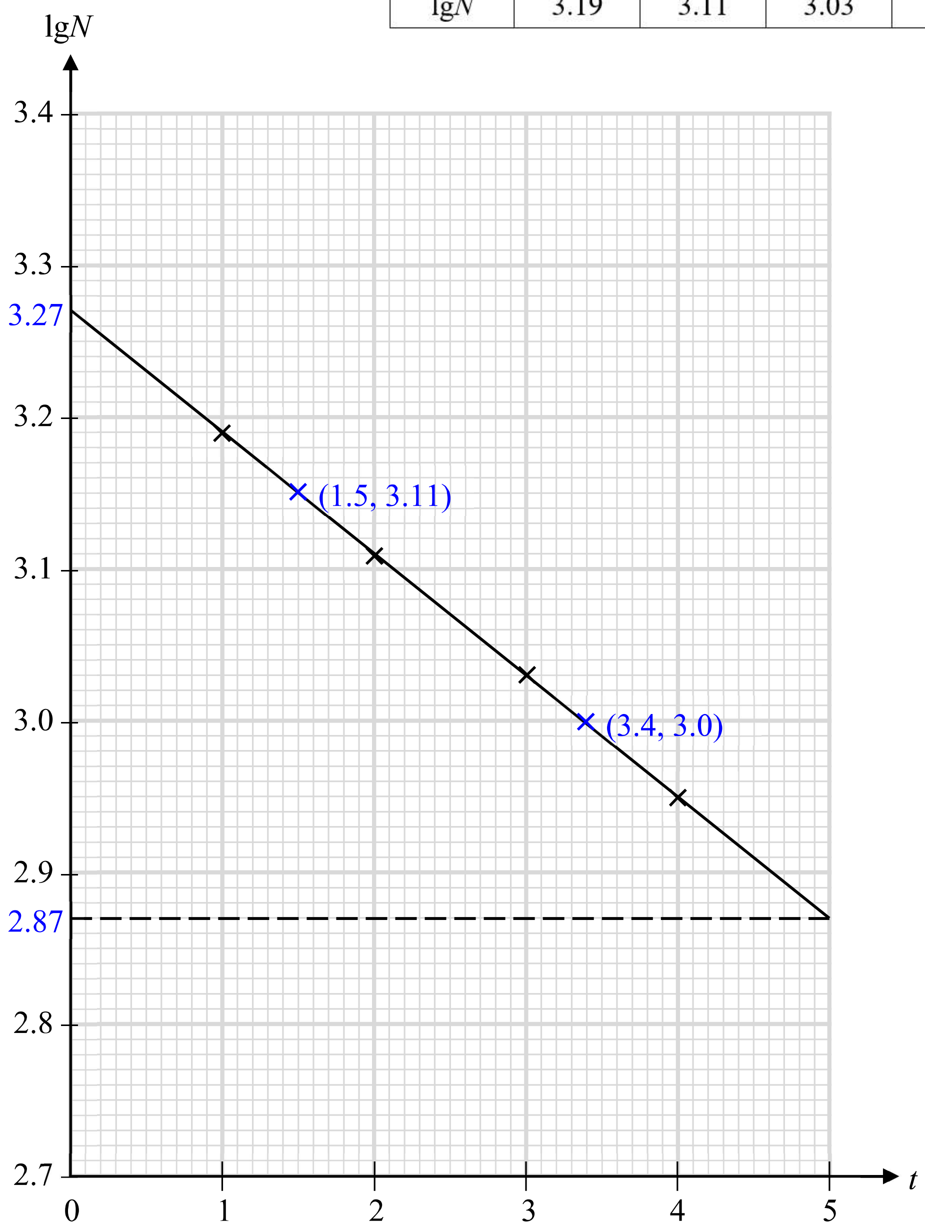
Year	1980 – 1989	1990 – 1999	2000 – 2009	2010 – 2019
Value of t	1	2	3	4
Number of trees N	1560	1300	1080	900

It is believed that these figures can be modelled by the formula $N = Ab^{-t}$, where A and b are constants.

(a) On the grid below, plot $\lg N$ against t and draw a straight line graph to illustrate the information. [2]

t	1	2	3	4
$\lg N$	3.19	3.11	3.03	2.95

(to 2dp)



[B1- correct plots]
[B1- line of best fit through $\lg N$ axis]

12 **(b)** Use your graph to estimate

(i) the value of A and b, [4]

$N = Ab^{-t}$
$\lg N = \lg(Ab^{-t})$
$\lg N = \lg A + \lg(b^{-t})$
$\lg N = \lg A - t\lg b$ [M1]
$\lg N = (-\lg b)t + \lg A$
(Y-int) $\lg A = 3.27$ (Accept: 3.26, 3.265, 3.27, 3.275, 3.28)
$A = 10^{3.27}$
$= 1862.08$
$= 1860$ (3sf) [A1] (Accept: 1820, 1840, 1860, 1880, 1910)

(grad) $\dfrac{3.11-3.0}{1.5-3.4} = -0.057894$

$-\lg b = -0.057894$ [M1- for $-\lg b$ = gradient,
Accept: $-0.08 \le \text{gradient} \le -0.05$]

$\lg b = 0.057894$
$b = 10^{0.057894}$
$= 1.1425$
$= 1.14$ (3sf) [A1- accept $10^{\text{grad within the range}}$]

(ii) the number of trees remaining in the forest in the decade 2020 – 2029. [2]

When $t = 5$,
$\lg N = 2.87$ [M1] (Accept: 2.86, 2.865, 2.87, 2.875, 2.88)
$N = 10^{2.87}$
$= 741.31$
$= 741$ (nearest integer) [A1 - accept $10^{\text{accepted } Y \text{ value}}$]

(c) Explain what A represents. [1]

$N = Ab^{-t}$
When $t = 0$, $N = A$
$\therefore$ A represents the number of trees in the forest in 1970 – 1979. [B1]

13 The equation of a curve is $y = 6x^3 + ax^2 + bx + 3$, where a and b are constants.

(a) If y is always increasing, what conditions must apply to the constants a and b? [4]

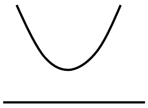
$y = 6x^3 + ax^2 + bx + 3$

$\frac{dy}{dx} = 6(3x^2) + 2ax + b$ [M1]

$= 18x^2 + 2ax + b$

Mtd 1

When $\frac{dy}{dx} > 0$, $18x^2 + 2ax + b > 0$ [M1]



No real roots: $D < 0$

$(2a)^2 - 4(18)(b) < 0$ [M1]

$4a^2 - 72b < 0$

$4a^2 < 72b$

$a^2 < 18b$ [A1- accept equivalent ans]

Mtd 2

$\frac{dy}{dx} = 18x^2 + 2ax + b$

$= 18\left(x^2 + \frac{a}{9}x + \frac{b}{18}\right)$

$= 18\left(x^2 + \frac{a}{9}x + \left(\frac{a}{18}\right)^2 - \left(\frac{a}{18}\right)^2 + \frac{b}{18}\right)$

$= 18\left(\left(x + \frac{a}{18}\right)^2 - \frac{a^2}{324} + \frac{b}{18}\right)$

$= 18\left(x + \frac{a}{18}\right)^2 - \frac{a^2}{18} + b$ [M1]

When $\frac{dy}{dx} > 0$,

$18\left(x + \frac{a}{18}\right)^2 - \frac{a^2}{18} + b > 0$ [M1]

Since $\left(x + \frac{a}{18}\right)^2 \geq 0$,

$\therefore -\frac{a^2}{18} + b > 0$ [A1- accept equivalent ans]

13 **(b)** In the case where $a = 13$ and $b = -16$, find the x-coordinate of the points at which the curve intersects the line $y + 2x = 0$. [5]

Mtd 1: sub y

$y = 6x^3 + 13x^2 - 16x + 3$ ---(1)

$y + 2x = 0$

$y = -2x$ ---(2)

(1)=(2),

$6x^3 + 13x^2 - 16x + 3 = -2x$ [M1]

$6x^3 + 13x^2 - 14x + 3 = 0$

Let $f(x) = 6x^3 + 13x^2 - 14x + 3$

$f(-3) = 6(-3)^3 + 13(-3)^2 - 14(-3) + 3$

$= 0$ [M1]

$\therefore (x + 3)$ is a factor of $f(x)$

$$\begin{array}{r l}
 & 6x^2 - 5x + 1 \\
x+3 \,\big) & 6x^3 + 13x^2 - 14x + 3 \\
 & -(6x^3 + 18x^2) \\
 & -5x^2 - 14x \\
 & -(-5x^2 - 15x) \\
 & x + 3 \\
 & -(x+3) \\
 & 0
\end{array}$$

$f(x) = (x + 3)(6x^2 - 5x + 1)$ [M1- with correct mtd shown]

when $f(x) = 0$,

$(x + 3)(6x^2 - 5x + 1) = 0$

$x + 3 = 0$ or $6x^2 - 5x + 1 = 0$

$x = -3$ $(3x - 1)(2x - 1) = 0$

$3x - 1 = 0$ or $2x - 1 = 0$

$x = \frac{1}{3}$ $x = \frac{1}{2}$

$\therefore x = -3$ or $\frac{1}{3}$ or $\frac{1}{2}$ [A1, A1- must show mtd to solve quadratic eqn]

Mtd 2: sub x

$y = 6x^3 + 13x^2 - 16x + 3$ ---(1)

$y + 2x = 0$

$2x = -y$

$x = -\frac{1}{2}y$ ---(2)

sub (2) into (1),

$y = 6\left(-\frac{1}{2}y\right)^3 + 13\left(-\frac{1}{2}y\right)^2 - 16\left(-\frac{1}{2}y\right) + 3$ [M1]

$y = 6\left(-\frac{1}{8}y^3\right) + 13\left(\frac{1}{4}y^2\right) + 8y + 3$

$y = -\frac{3}{4}y^3 + \frac{13}{4}y^2 + 8y + 3$

$\frac{3}{4}y^3 - \frac{13}{4}y^2 - 7y - 3 = 0$

$3y^3 - 13y^2 - 28y - 12 = 0$

Let $f(y) = 3y^3 - 13y^2 - 28y - 12$

$f(6) = 3(6)^3 - 13(6)^2 - 28(6) - 12$
$= 0$
$\therefore (y - 6)$ is a factor of $f(y)$ [M1]

$$\begin{array}{r l} & 3y^2 + 5y + 2 \\ y - 6 \,\big) & 3y^3 - 13y^2 - 28y - 12 \\ & -(3y^3 - 18y^2) \\ & \quad 5y^2 - 28y \\ & \quad -(5y^2 - 30y) \\ & \qquad 2y - 12 \\ & \qquad -(2y - 12) \\ & \qquad\qquad 0 \end{array}$$

$f(y) = (y - 6)(3y^2 + 5y + 2)$ [M1- with correct mtd shown]

when $f(y) = 0$,

$(y - 6)(3y^2 + 5y + 2) = 0$

$y - 6 = 0$ or $3y^2 + 5y + 2 = 0$

$y = 6$ $\quad (3y + 2)(y + 1) = 0$

$3y + 2 = 0$ or $y + 1 = 0$

$y = -\frac{2}{3}$ $\quad y = -1$

sub into (2),

$\therefore x = -3$ or $\frac{1}{3}$ or $\frac{1}{2}$ [A1, A1- must show mtd to solve quadratic eqn]

*If fractional factors are used and whole qn correct, deduct 1m from the qn itself.

END OF PAPER