**Check your Understanding (Inequalities)**

**Section 1: Type of roots of Quadratic Equation**

**1. RVHS JC2 Prelim 8865/2019/Q1**

Find the exact range of values of the constant $k$ for which the equation $kx^2+x+k+2=0$ has 2 distinct real roots. [4]

**RVHS JC2 Prelim 8865/2019/Q1 (Solutions)**

Since the equation has 2 real distinct roots,

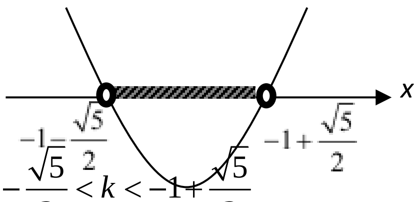
Discriminant $>0$

$$(1)^2-4(k)(k+2)>0$$

$$1-4k^2-8k>0$$

$$4k^2+8k-1<0$$

$$\left[k-\left(-1+\frac{\sqrt{5}}{2}\right)\right]\left[k-\left(-1-\frac{\sqrt{5}}{2}\right)\right]<0$$




$$-1-\frac{\sqrt{5}}{2}<k<-1+\frac{\sqrt{5}}{2}$$

Side working:

Let $4k^2+8k-1=0$.

$$k=\frac{-8\pm\sqrt{64-4(4)(-1)}}{8}$$

$$=\frac{-8\pm\sqrt{80}}{8}$$

$$=-1\pm\frac{\sqrt{5}}{2}$$

**2. CJC JC2 Prelim 8865/2019/Q2**

Find the set of values of $k$ for which the equation $kx^2+(3-k)x+k-3=0$ has real roots.

Without carrying out further calculations, state the set of values of $k$ for which $kx^2+(3-k)x+k-3>0$ for all real values of $x$. [4]

**CJC JC2 Prelim 8865/2019/Q2 (Solutions)**

**2** Since the equation has real roots,

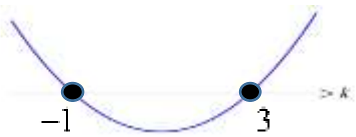
Discriminant $\geq 0$

$$(3-k)^2-4k(k-3)\geq 0$$

$$3k^2-6k-9\leq 0$$

$$3(k+1)(k-3)\leq 0$$

$$\therefore -1\leq k\leq 3$$




The set of values is $\{k\in\mathbb{R}:-1\leq k\leq 3\}$.

If $kx^2+(3-k)x+k-3>0$ for all real values of $x$,

$k>0$ and Discriminant $<0$

$\Rightarrow k>0 \cdots(1)$ and $k<-1$ or $k>3\cdots(2)$

Combining (1) and (2), $k>3$

The set of values is $\{k\in\mathbb{R}: k>3\}$.

**Section 2: Conditions for Quadratic Equation to be always positive or negative**

**3. ASRJC JC2 Prelim 8865/2019/Q1**

Find algebraically the exact set of values of $k$ for which

$$kx^2+(3k+1)x+(4+4k)>0$$

for all real values of $x$. [5]

**ASRJC JC2 Prelim 8865/2019/Q1 (Solutions)**

For $kx^2+(3k+1)x+(4+4k)>0$ for all real values of $x$, 2 conditions need to be satisfied:

(i) the coefficient of $x^2$ must be positive $\Rightarrow k>0$ --- (1) and

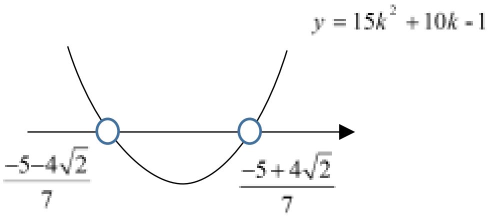
(ii) Discriminant $<0$

$$(3k+1)^2-4k(4+4k)<0$$

$$9k^2+6k+1-16k-16k^2<0$$

$$-7k^2-10k+1<0$$

$$7k^2+10k-1>0$$




$k<\dfrac{-5-4\sqrt{2}}{7}$ or $k>\dfrac{-5+4\sqrt{2}}{7}$ ---(2)

Combining (1) and (2), $k>\dfrac{-5+4\sqrt{2}}{7}$

The set of values of $k$ is $\left\{k\in\mathbb{R}: k>\dfrac{-5+4\sqrt{2}}{7}\right\}$.

Side working:

Consider $7k^2+10k-1=0$

$$k=\frac{-10\pm\sqrt{10^2-4(7)(-1)}}{2(7)}$$

$$=\frac{-10\pm\sqrt{128}}{14}=\frac{-5\pm4\sqrt{2}}{7}$$

**4. EJC JC2 Prelim 8865/2019/Q1**

Find algebraically the set of values of $k$ for which

$$kx^2+(k-2)x+k>0$$

for all real values of $x$. [4]

**EJC JC2 Prelim 8865/2019/Q1 (Solutions)**

For $kx^2+(k-2)x+k>0$,

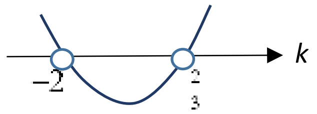
Discriminant $<0$ **and** coefficients of $x^2>0$

$(k-2)^2-4(k)(k)<0$ **and** $k>0\cdots(2)$

$k^2-4k+4-4k^2<0$

$3k^2+4k-4>0$

$(3k-2)(k+2)>0$




$k<-2$ or $k>\frac{2}{3}\cdots(1)$

Combining (1) and (2), the set of values is $\{k\in\mathbb{R}:k>\frac{2}{3}\}$

**5. NYJC JC2 Prelim 8865/2019/Q1**

Find the exact range of values of $k$ for which $(1-2k)x^2-x-k$ is non positive for all values of $x$. [4]

**NYJC JC2 Prelim 8865/2019/Q1 (Solutions)**

For $(1-2k)x^2-x-k\le 0$,

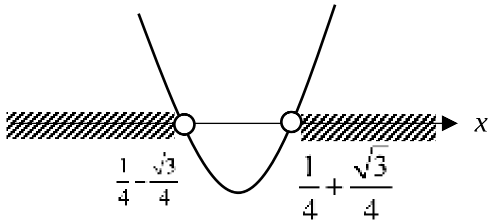
coefficients of $x^2<0$ and Discriminant $\le 0$

$1-2k<0$ & $(-1)^2-4(1-2k)(-k)\le 0$

$k>\frac{1}{2}$ & $1+4k-8k^2\le 0$

$k>\frac{1}{2}$ & $k^2-\frac{1}{2}k-\frac{1}{8}\ge 0$

$k>\frac{1}{2}$ & $\left(k-\frac{1}{4}\right)^2-\frac{3}{16}\ge 0$




$k>\frac{1}{2}\cdots(1)$ & $k\le\frac{1}{4}-\frac{\sqrt{3}}{4}$ or $k\ge\frac{1}{4}+\frac{\sqrt{3}}{4}\cdots(2)$

Combining (1) and (2), $k\ge\frac{1}{4}+\frac{\sqrt{3}}{4}$

**6.** **TMJC JC2 Prelim 8865/2019/Q1**

The equation of a curve is $y=(k-2)x^2+(2k-4)x+(2k-1)$ where $k$ is a real constant. Find the range of values of $k$ for which the curve lies completely above the $x$-axis. [4]

**TMJC JC2 Prelim 8865/2019/Q1 (Solutions)**

For $(k-2)x^2+(2k-4)x+(2k-1)>0$

Discriminant $<0$ and coefficient of $x^2>0 \Rightarrow k-2>0 \Rightarrow k>2\cdots(2)$.

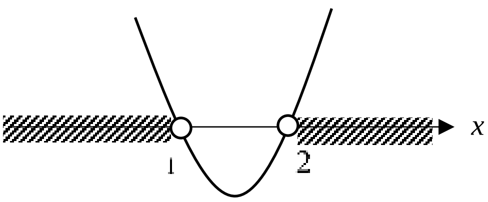
$(2k-4)^2-4(k-2)(2k-1)<0$

$4k^2-16k+16-4(2k^2-5k+2)<0$

$-4k^2+4k+8<0$

$\therefore 4k^2-4k-8>0$

$\Rightarrow (k-2)(k+1)>0$




$\Rightarrow k<-1$ or $k>2\cdots(1)$

Combining (1) and (2), $k>2$

## Section 3: Show Questions

**7.** **ACJC JC2 Prelim 8865/2019/Q1**

Show that there are no real values of $k$ for which $2(x^2-x)+k(x-1)-1$ is always positive. [4]

**ACJC JC2 Prelim 8865/2019/Q1 (Solutions)**

Let $y=2(x^2-x)+k(x-1)-1=2x^2+(k-2)x-k-1$

Assume $y>0$ for all real values of $x$. Since $y>0$ for all real values of $x$, Discriminant $<0$.

**Method 1:**

Discriminant $=(k-2)^2-4(2)(-k-1)$

$=k^2-4k+4+8k+8$

$=k^2+4k+12$

$=(k+2)^2+8>0$ for all real values of $k$

since $(k+2)^2\geq 0 \Rightarrow (k+2)^2+8\geq 0+8>0$ for all real values of $k$.

$\Rightarrow$ Discriminant can never be negative for all real values of $k$

$\Rightarrow 2(x^2-x)+k(x-1)-1=0$ will always have 2 real and distinct roots, i.e.

$\Rightarrow$ There are no real values of $k$ for which $2(x^2-x)+k(x-1)-1$ is always positive.

**Method 2:**

Solving Discriminant $< 0$,

$(k-2)^2 - 4(2)(-k-1) < 0$

$k^2 - 4k + 4 + 8k + 8 < 0$

$k^2 + 4k + 12 < 0$

$(k+2)^2 + 8 < 0$

But $(k+2)^2 \geq 0$ for all real values of $k$ so $(k+2)^2 + 8 > 0$.

$\Rightarrow$ There are no real values of $k$ for which $2\left(x^2 - x\right) + k\left(x-1\right) - 1$ is always positive.

**8. MI PU2 Prelim 8865/2019/Q2**

Show that there are no real values of $k$ for which $2x^2 + \left(2k+1\right)x - k - 1$ is always positive. [4]

**MI PU2 Prelim 8865/2019/Q2 (Solutions)**

Assume that that there are real values of $k$ such that $2x^2 + \left(2k+1\right)x - k - 1$ is always positive.

Since $2x^2 + \left(2k+1\right)x - k - 1$ is always positive, Discriminant $< 0$

Discriminant

$= \left(2k+1\right)^2 - 4\left(2\right)\left(-k-1\right)$

$= 4k^2 + 4k + 1 + 8k + 8$

$= 4k^2 + 12k + 9$

$= \left(2k+3\right)^2 \geq 0$ for all real values of $k$

This contradicts with the original assumption. Hence there are no real values of $k$ such that $2x^2 + \left(2k+1\right)x - k - 1$ is always positive.

## Section 4: Intersection of a curve and a line

**9. SAJC JC2 Prelim 8865/2019/Q2**

The curve $C$ has equation $y = 2x^2 - kx + 5$ and the line $L$ has equation $y = 3x + k$.

Find the exact range of values of $k$ such that $C$ intersects $L$. [4]

**SAJC JC2 Prelim 8865/2019/Q2 (Solutions)**

When $C$ intersects $L$,

$2x^2 - kx + 5 = 3x + k$

$2x^2 - (k+3)x + (5-k) = 0$

Since $C$ intersects $L$, Discriminant $\geq 0$

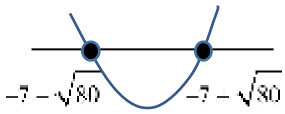
$\left[-(k+3)\right]^2 - 4\left(2\right)(5-k) \geq 0$

$k^2 + 14k - 31 \geq 0$

$\left(k+7\right)^2 - 80 \geq 0$

$\left(k+7\right)^2 - \left(\sqrt{80}\right)^2 \geq 0$

$\left(k + 7 + \sqrt{80}\right)\left(k + 7 - \sqrt{80}\right) \geq 0$




$k \le -7-\sqrt{80}$ or $k \ge -7+\sqrt{80}$

$k \le -7-4\sqrt{5}$ or $k \ge -7+4\sqrt{5}$

**10. NJC JC2 Prelim 8865/2019/Q1**

Find the range of values of $m$ such that the line $y = 2x + m$ and the curve $y = 2mx^2 - (m+2)x - 4$ intersect at two distinct points. [4]

**NJC JC2 Prelim 8865/2019/Q1 (Solutions)**

When the line intersects the line,

$2mx^2 - (m+2)x - 4 = 2x + m$

$2mx^2 - (m+4)x - m - 4 = 0$

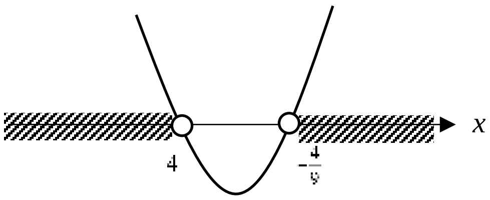
Since the line and the curve intersect at 2 distinct points, $\therefore$ discriminant $> 0$

$(-(m+4))^2 - 4(2m)(-m-4) > 0$

$(m+4)^2 + 8m(m+4) > 0$

$(m+4)(m+4+8m) > 0$

$(m+4)(9m+4) > 0$




$$m < -4 \text{ or } m > -\frac{4}{9}$$

When $m$ = 0, the curve becomes a line $y$ = –2$x$ – 4 and the other line will have equation $y$ = 2$x$. These 2 lines intersect at a single point.

Hence, for the lines to intersect at two distinct points, $m < -4$ or $m > -\frac{4}{9}, m \ne 0$.

**11. TJC JC2 Prelim 8865/2019/Q1**

Find algebraically the range of values of $k$ for which the curve $y = -4x^2 + 2(k-1)x - 9$ intersects the $x$-axis. [4]

**TJC JC2 Prelim 8865/2019/Q1 (Solutions)**

When the curve intersects the the $x$-axis,

$-4x^2 + 2(k-1)x - 9 = 0$

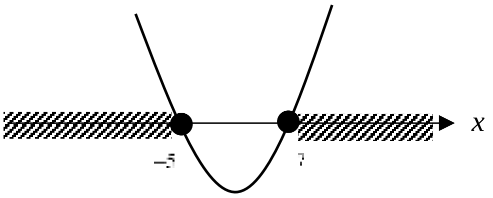
Since the curve must intersect the $x$-axis (either at one or two points),

Discriminant $\ge 0$

$4(k-1)^2 - 4(-4)(-9) \ge 0$

$\Rightarrow \quad k^2 - 2k - 35 \geq 0$

$\Rightarrow \quad (k+5)(k-7) \geq 0$




$\Rightarrow \quad k \leq -5 \quad \text{or} \quad k \geq 7$

**12. RVHS JC1 Promo 8865/2019/Q7**

(i) Show that the expression $-\left(k^2+1\right)x^2 + 2kx - 1$ is negative for all real values of *x* and *k*. [3]

(ii) Hence find the values of *x* for which $\dfrac{x^2-5x+4}{-\left(k^2+1\right)x^2+2kx-1} > 0$. [3]

(iii) Given that $k > 0$, deduce the range of value of *x* which satisfies $\dfrac{-k^2x^2+5kx-4}{-\left(k^2+1\right)k^2x^2+2k^2x-1} < 0$, leaving your answer in terms of *k*. [4]

**RVHS JC1 Promo 8865/2019/Q7 (Solutions)**

(i) For $-\left(k^2+1\right)x^2+2kx-1<0$,

Coefficient of $x^2 = -\left(k^2+1\right)$

Since $k^2 \geq 0$, $k^2+1>0$ thus $-\left(k^2+1\right)<0$

$$\begin{aligned}\text{Discriminant} &= (2k)^2 - 4\left(-k^2-1\right)(-1) \\ &= 4k^2 + 4\left(-k^2-1\right) \\ &= -4 < 0\end{aligned}$$

Therefore $-\left(k^2+1\right)x^2+2kx-1<0$ for all real values of *x* and *k*.

(ii) $\dfrac{x^2-5x+4}{-\left(k^2+1\right)x^2+2kx-1} > 0$

Since $-\left(k^2+1\right)x^2+2kx-1<0$ for all real values of *x* and *k*, this implies that $x^2-5x+4<0$.

$x^2-5x+4<0$

$(x-4)(x-1)<0$

$1<x<4$

(iii) $\dfrac{-k^2x^2+5kx-4}{-\left(k^2+1\right)k^2x^2+2k^2x-1} < 0$

$\dfrac{k^2x^2-5kx+4}{-\left(k^2+1\right)k^2x^2+2k^2x-1} > 0$

$\dfrac{(kx)^2-5(kx)+4}{-\left(k^2+1\right)(kx)^2+2k(kx)-1} > 0$

Replace $x$ by $kx$

$1 < kx < 4$

$\frac{1}{k} < x < \frac{4}{k}$

**13. DHS JC1 Promo 8865/2019/Q1**

By using an algebraic method, find the set of values of $k$ such that $x^2 + 3kx + 4k$ is positive for all real values of $x$.

Hence solve $\frac{x^2 + 3x + 4}{2e^x + 3xe^x} \le 0$. [4]

**DHS JC1 Promo 8865/2019/Q1 (Solutions)**

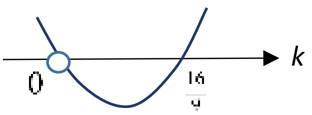
For $x^2 + 3kx + 4k > 0$,

Discriminant < 0

$(3k)^2 - 4(1)(4k) < 0$

$9k^2 - 16k < 0$

$k(9k - 16) < 0$




$\left\{k \in \mathbb{R} : 0 < k < \frac{16}{9}\right\}$

$\frac{x^2 + 3x + 4}{2e^x + 3xe^x} \le 0$

$x^2 + 3x + 4 > 0$ since $k = 1$

$\therefore\ 2e^x + 3xe^x < 0$

$e^x(2 + 3x) < 0$

Since $e^x > 0$ for all real values of $x$,

$2 + 3x < 0$

$x < -\frac{2}{3}$