ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749

Topic 15: D.C. Circuits

Content

A Circuit symbols and diagrams
B Series and parallel arrangements
C Potential divider
D Balanced potentials

Learning Outcomes

Candidates should be able to:

(a) recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*

(b) draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus

(c) solve problems using the formula for the combined resistance of two or more resistors in series

(d) solve problems using the formula for the combined resistance of two or more resistors in parallel

(e) solve problems involving series and parallel circuits for one source of e.m.f.

(f) show an understanding of the use of a potential divider circuit as a source of variable p.d.

(g) explain the use of thermistors and light-dependent resistors in potential divider circuits to provide a potential difference which is dependent on temperature and illumination respectively

(h) recall and solve problems by using the principle of the potentiometer as a means of comparing potential differences

Additional Notes

Got a question to ask the lecturer, or doubts to clarify after attending the lecture? Key in your questions via the QR code above, or through https://tinyurl.com/j2physicslectureq

The lecturer will address your queries either in the following lecture, or through the link https://tinyurl.com/j2physicslecturea , also accessible via the QR code below.

A Circuit symbols and diagrams

A.1 Introduction

Direct circuit

- Direct current (in short d.c.) refers to current which <u>flows in one direction only</u>. In contrast, alternating current (in short a.c.) refers to current which changes direction periodically.

- Direct current must be one-directional, not necessarily constant magnitude, i.e. <u>it need not be a steady current</u>
  In Fig. 15.1, Graph (a), Graph (b) and Graph (c) show examples of direct current whereas Graph (d) and Graph (e) show examples of alternating current

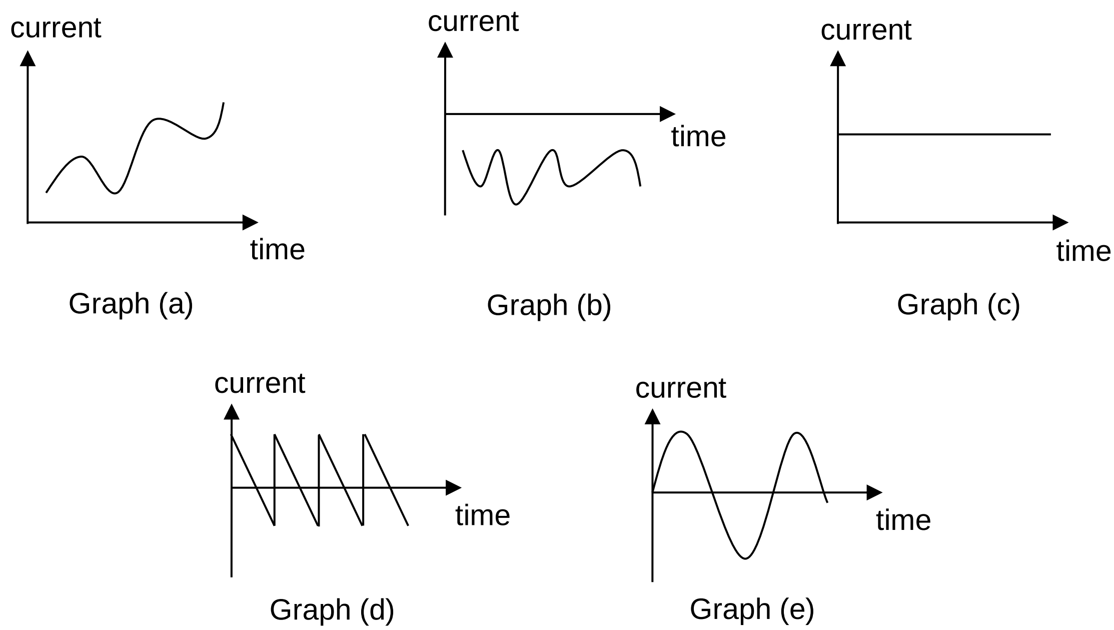
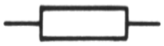
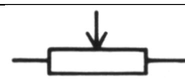
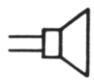
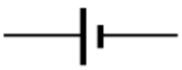
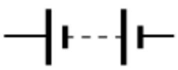
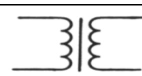
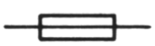
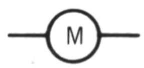
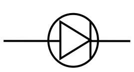
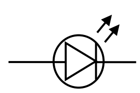
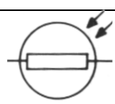
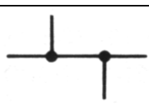


Fig. 15.1

Additional Notes

Circuit symbols

Symbol	Description	Symbol	Description
	Switch		Electric bell
	Fixed resistor		Heater
	Variable resistor		Thermistor
	Voltmeter		Potential divider
	Ammeter		Loudspeaker
	Galvanometer		Lamp
	Cell		Capacitor *
	Battery of cells		Inductor *
	D.C. power supply		Transformer with core
	Aerial *		Microphone *
	Fuse		Motor *
	Diode		Light emitting diode (LED)
	Light Dependent Resistor (LDR)		Earth
	Cross (no connection between wires)		Junction connection (wires physically connected)
	Double junction connection		Frame or chassis connection *

*Less common in the H2 A-Level syllabus

Fig. 15.2

Ammeter

- An ammeter measures the rate at which electrons are flowing through a circuit at a given point.
- It is connected in series with the part of the circuit through which the current is to be measured.
- Ideal ammeters have zero resistance, so its inclusion in the circuit will not increase the effective resistance of the circuit. There will be no p.d. across an ideal ammeter.

Voltmeter

- A voltmeter measures the potential difference between any two points in the circuit.
- It is connected parallel to the two points of the circuit, across which the p.d. is to be measured.
- Ideal voltmeter have infinite resistance, so its inclusion in the circuit will not decrease the effective resistance of the circuit. There will be no current flowing through an ideal voltmeter.

Thermistor

The term "thermistor" usually refers to the Negative Temperature Coefficient (NTC) thermistor.

- The resistance of a thermistor changes with temperature and it can be made of semiconductor material.
- It acts like a variable resistor, except that the variation of the resistance is controlled by temperature. The resistance of a thermistor drops when temperature increases.

Light-Dependent Resistor (LDR)

- The resistance of LDR changes with light intensity.
- It acts like a variable resistor, except that the variation of the resistance is controlled by light. The resistance of a LDR drops when light intensity increases.

A.2 Kirchhoff's Current Law

- Kirchhoff's current law (or junction rule) states that the algebraic sum of currents entering a junction must be equal to the algebraic sum of currents leaving that junction.

- It is based on the conservation of charge which means that the total charges entering a junction must be equal to the total charges leaving that junction.

- Kirchhoff's current law (or junction rule) is a consequence of the conservation of charge. No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal the total charge leaving per unit time.

- Since charge per unit time is current, so the total current entering the junction must equal to the total current leaving the junction.

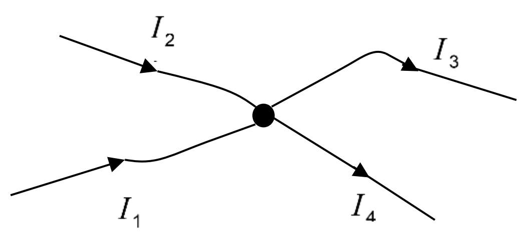
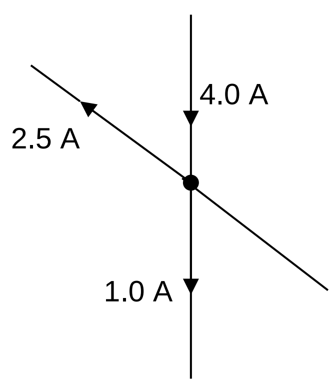


Fig. 15.3

- Hence, for the junction in Fig. 15.3 above, $I_1 + I_2 = I_3 + I_4$.

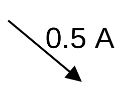
Check your understanding 1

The diagram shows a part of a circuit where four wires are connected at a junction. The current in 3 of the wires are shown in the diagram.

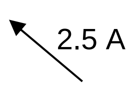


Which of the following shows the current in last wire?

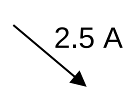
A



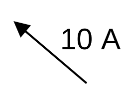
B



C



D



Additional Notes

Kirchhoff's current law is also referred to as Kirchhoff's 1st Law.

There is also Kirchhoff's voltage law, which is also referred to as Kirchhoff's 2nd Law. It states that the algebraic sum of e.m.f. in a loop must equal to the algebraic sum of p.d. in the loop. It is a consequence of the Principle of Conservation of Energy. (Not in syllabus)

Additional Notes

A.3 Potential versus Potential difference

- Consider the circuit as shown in Fig. 15.4 below:

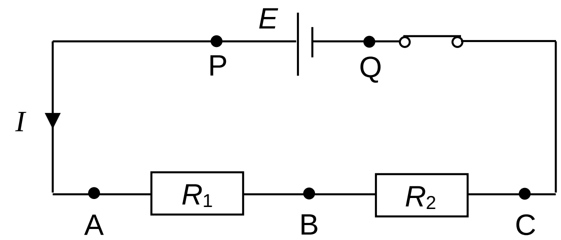


Fig. 15.4

In a circuit where there is no reference point (e.g. earth), we will never be able to determine the potential at various points. However, we can determine the potential difference between 2 points in a circuit.

- For the e.m.f. source, the positive terminal is always at a higher potential than the negative terminal.
  i.e. potential at P > potential at Q
  $V_P > V_Q$
  (In order to determine the exact value of V_P and V_Q, a reference point, e.g. earth, must be specified in the circuit. Do not always assume the negative terminal of the cell to be 0 V.)

- The potential difference across the e.m.f. source would give the terminal p.d. of the source.
  i.e. terminal p.d. = $V_P - V_Q$

In any circuit diagram, all connecting wires are taken to have zero resistance. Hence, for any 2 points connected by wires with no other components in between them, the potential of these 2 points must be the same.

- Since the point P and the point A in the circuit are connected by just a wire with no component in between, the potential at P must be the same as the potential at A.
  i.e. potential at P, V_P = potential at A, V_A

- Similarly, the potential at Q must be the same as the potential at C as they are connected by a wire with no component in between them (switch is assumed to have no resistance).
  i.e. potential at Q, V_Q = potential at C, V_C

- In this closed circuit, there is a drop in potential each time the current flows through a resistor.
  i.e. potential at A > potential at B > potential at C
  $V_A > V_B > V_C$

- The difference between the potential values at each end of the resistor is known as the potential difference.
  e.g. potential difference across R_1, $V_{AB} = V_A - V_B$
  potential difference across R_2, $V_{BC} = V_B - V_C$

Additional Notes

- Consider the same circuit but this time, with the switch open as shown in Fig. 15.5 below:

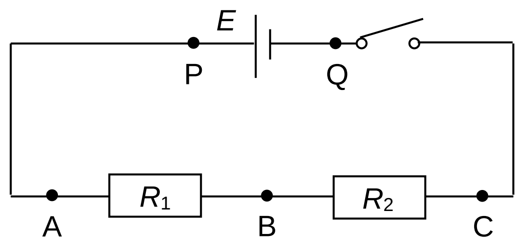


Fig. 15.5

- For the e.m.f. source, $V_P > V_Q$ and terminal p.d. = $V_P - V_Q = E$.

- As the points P and A are connected by a wire with no components in between, the potential at P and A are the same, i.e. $V_P = V_A$.

- In this open circuit, there is no current flow through the resistors. Hence, there is no potential drop across the resistors R_1 and R_2. Thus the potential at A, B and C are the same as they are connected by wire and resistors with no potential drop across them.
  i.e. potential at A = potential at B = potential at C
  $V_A = V_B = V_C$

When there is no current flowing through a component, then the potential at the 2 ends of the component must be the same.

- Since $V_A = V_P$, the potentials at B and C are also the same as the potential at P.
  i.e. $V_A = V_B = V_C = V_P$

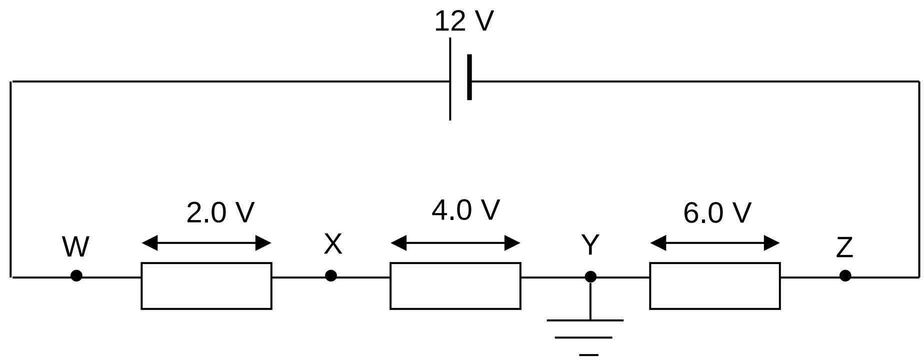
- Hence, $V_{AB} = V_{BC} = V_{PA} = 0$ V.

- Between point Q and point C, there is a break in the connection due to the open switch. Therefore, the potential at Q is not the same as the potential at C, but is lower as the potential at C is equal to the potential at P.
  i.e. potential at Q < potential at C
  $V_Q < V_C$

- Hence, $V_{CQ} = V_{PQ} = E$.

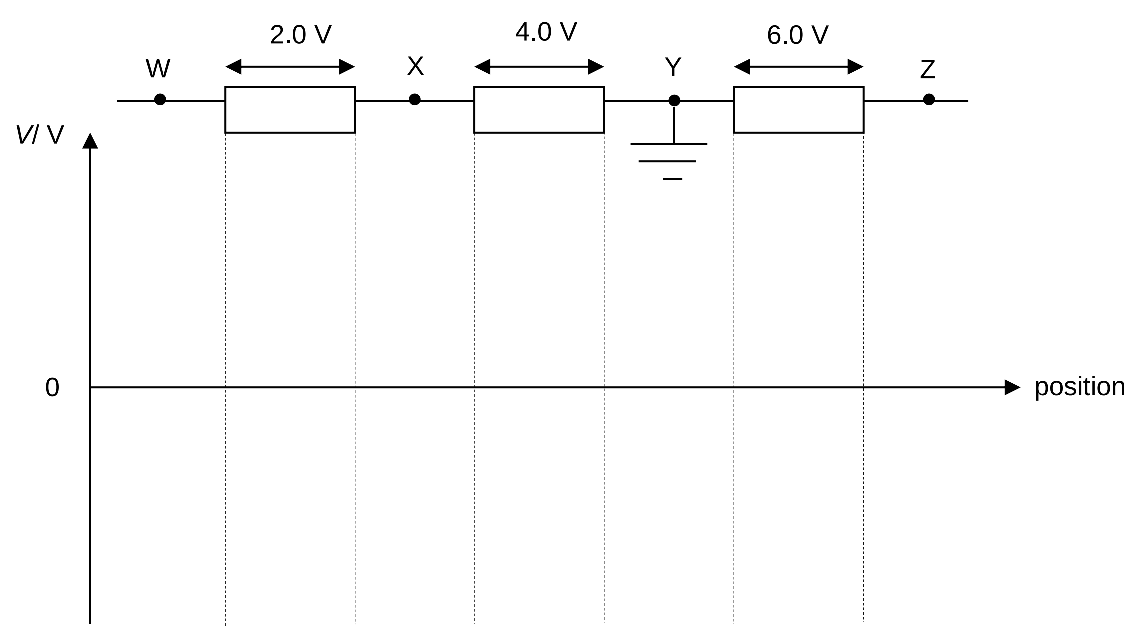
Additional Notes

Example 1
In the circuit below, the potential differences across each of the resistors are given.



Hint:
Start with the point in the circuit where the potential at that point is known. Use this point as the reference.

Sketch a graph of potential values vs positions on the given axes.
<u>Questions to consider:</u>
What is the value of the potential at W?
What is the value of the potential at Z?



Additional Notes

B Series and parallel arrangement

B.1 Resistors in Series

- Resistors connected in series have the <u>same current</u> flowing through them i.e. any charge that flows through one resistor must flow through the others.

- <u>Derivation (not required):</u>
  Consider an e.m.f. E of negligible internal resistance applied across n resistors connected in series, as shown in the circuit diagram on the left of Fig. 15.6.

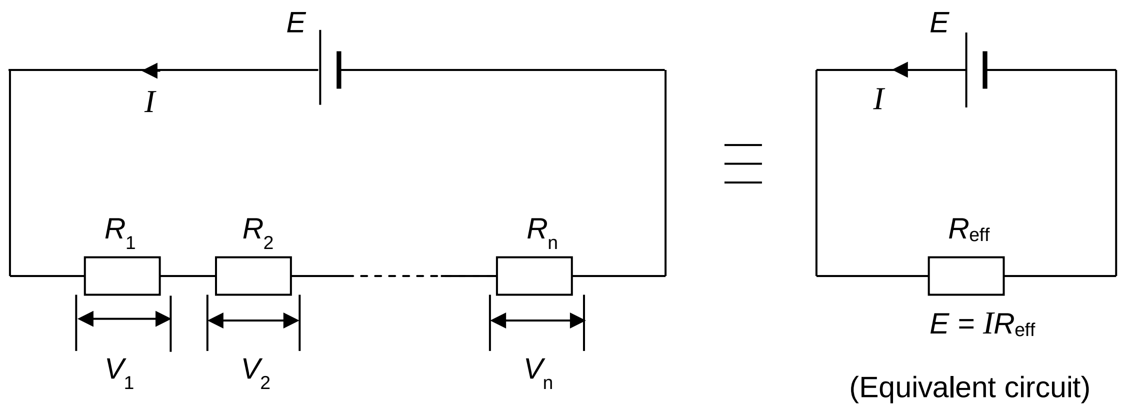


Fig. 15.6

The resistors have resistances R_1, R_2, …. R_n, and the potential difference across each resistor are V_1, V_2, … V_n respectively.

Since the <u>same current I</u> flows through all the n resistors, using the relation $V = IR$, the potential difference across each resistor can be expressed as IR_1, IR_2, … IR_n.

Since $E = V_1 + V_2 + \ldots + V_n$, we have $E = IR_1 + IR_2 + \ldots. + IR_n = I\,(R_1 + R_2 + \ldots + R_n)$.

If a resistor of equivalent resistance R_{eff} is used in place of the n resistors in series as shown in the circuit diagram on the right of Fig. 15.6, using $V = IR$, we have $E = IR_{eff}$.

From the two equations, we have $E = IR_{eff} = I\,(R_1 + R_2 + \ldots + R_n)$.

Hence, <u>for resistors in series</u>,

$$R_{eff} = R_1 + R_2 + \ldots + R_n$$

This equation allows us to use R_{eff} to represent equivalent resistance of a set of resistors in series.

- The equivalent resistance R_{eff} of resistors in series is always greater than any individual resistance.

- If all the n resistors in series are of the same resistance R, then the equivalent resistance R_{eff} is nR.

- When there are breaks in a series connection, the circuit is no longer closed and no current flows.

B.2 Resistors in Parallel

- Resistors connected in parallel have the <u>same potential difference across the ends of the each resistor</u> i.e. left end of each resistor is connected to a <u>common point</u> and right end of each resistor is connected to another <u>common point</u>.

- <u>Derivation (not required):</u>
Consider an e.m.f. E of negligible internal resistance is applied across n resistors connected in parallel, as shown in the circuit diagram on the left of Fig. 15.7.

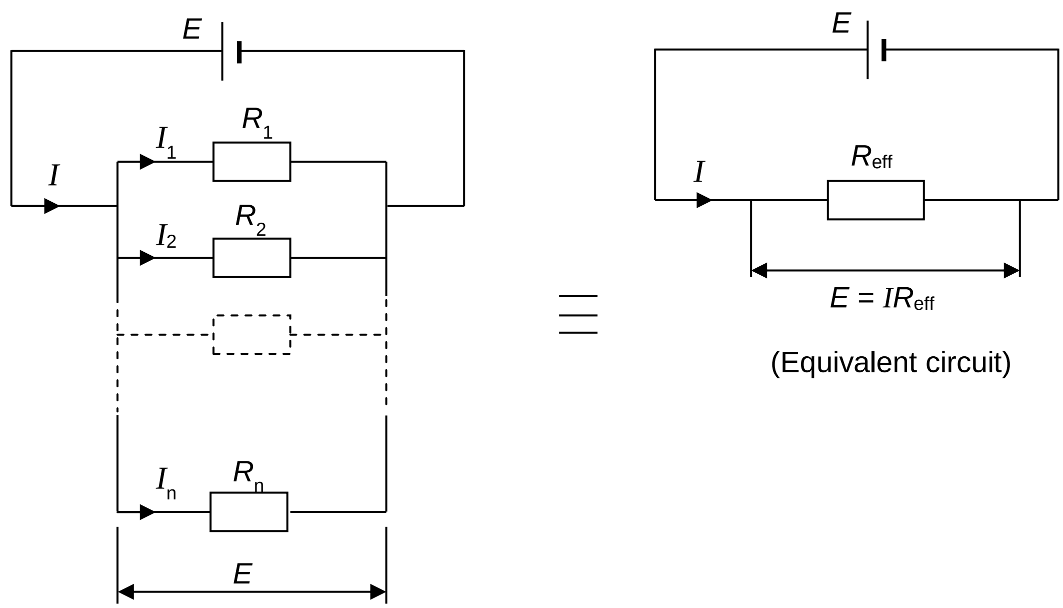


Fig. 15.7

The resistors have resistances R_1, R_2, …. R_n, and the current flowing through each resistor are I_1, I_2, … I_n respectively.

Since the potential difference across each resistor is E, using the relation $V = IR$, the current flowing through each resistor can be expressed as $\frac{E}{R_1}$, $\frac{E}{R_2}$, … $\frac{E}{R_n}$.

Since $I = I_1 + I_2 + \ldots + I_n$, we have $I = \frac{E}{R_1} + \frac{E}{R_2} + \ldots + \frac{E}{R_n} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}\right)$.

Additional Notes

If a resistor of equivalent resistance R_{eff} is used in place of the n resistors in parallel as shown in the circuit diagram on the right of Fig. 15.7, using $V = IR$, we have $I = \frac{E}{R_{eff}}$.

From the two equations, we have $I = \frac{E}{R_{eff}} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}\right)$.

Hence, for resistors in parallel,

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

This equation allows us to use R_{eff} to represent equivalent resistance of a set of resistors in parallel.

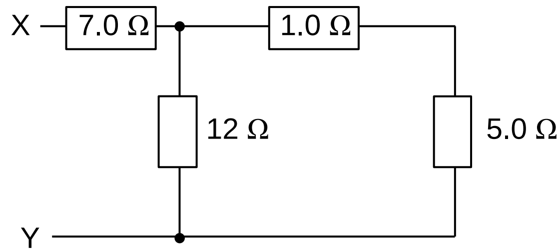
- The equivalent resistance R_{eff} of resistors in parallel is always less than the smallest resistance in the group of resistors.
- If all the n resistors in parallel are of the same resistance R, then the equivalent resistance R_{eff} is $\frac{R}{n}$.
- Current continues to flow through the remaining resistors connected in parallel even when one of resistor fails and has an open circuit across it.

Additional Notes

A network resistor is just a set of resistors connected together using connecting wires. It is not a complete circuit, but we can always find the equivalent resistance of a network resistor.

Example 2

Determine the effective resistance between points X and Y of the following network resistor.



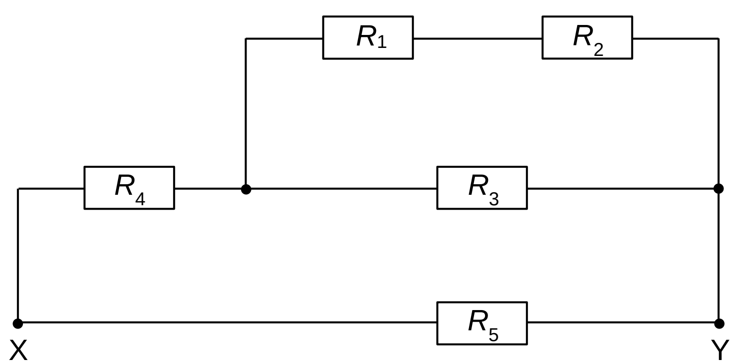
Additional Notes

Practical technique in finding effective resistance of a network of resistors

- Steps to take:
  a) Examine the circuit and redraw circuit (if necessary).
  b) Replace any resistors in series (same current flowing through the resistors) with a single resistor of equivalent resistance.
  c) Sketch the new circuit.
  d) Replace any resistors in parallel (same p.d. across the resistors) with a single resistor of equivalent resistance.
  e) Sketch the new circuit.
  f) Repeat steps b) to e) until a single equivalent resistance is found.
  g) If the current or p.d. across a particular resistor in the network is to be identified, start with the finding the current or p.d. across the equivalent resistor found in part f) and gradually work backwards.

Worked Example

In the circuit shown, each resistor has a resistance of 10 kΩ.

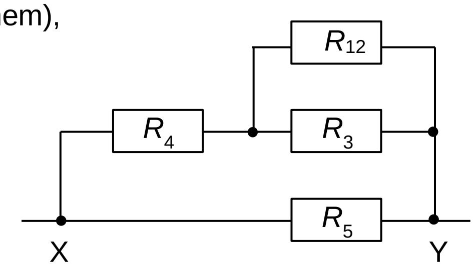


Calculate the resistance across XY.

Since R_1 and R_2 are in series (same current through them),

equivalent resistance $R_{12} = 10 + 10$
$= 20\ \text{k}\Omega$

Redrawing, we have the circuit diagram on the right.



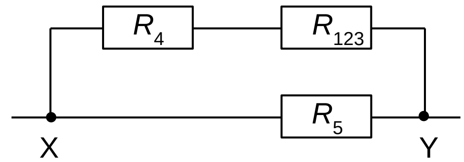
A good practice when finding resistance between 2 points of a network of resistors is to extend the wires from the required points (as shown in this diagram).

The resistors R_{12} and R_3 are in parallel (same p.d. across them),

equivalent resistance $R_{123} = \left(\frac{1}{20} + \frac{1}{10}\right)^{-1}$
$= 6.67\ \text{k}\Omega$

Additional Notes

Redrawing, we have the circuit diagram on the right.

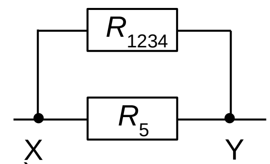


The resistors R_{123} and R_4 are in series (same current through them),

equivalent resistance $R_{1234} = 10 + 6.67$

$= 16.67\ \text{k}\Omega$

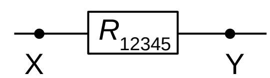
Redrawing, we have the circuit diagram on the right.



The resistors R_{1234} and R_5 are in parallel (same p.d. across them),

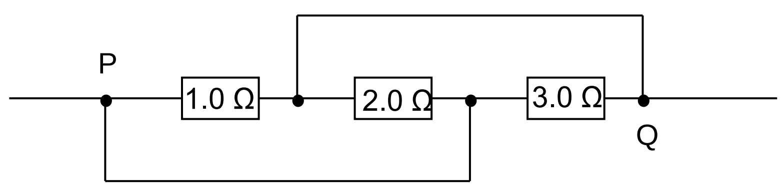
equivalent resistance $R_{12345} = \left(\frac{1}{16.67} + \frac{1}{10}\right)^{-1}$

$= 6.25\ \text{k}\Omega = R_{XY}$



Example 3

Determine the effective resistance between the points P and Q of the following network resistor.



Hint:
Along connecting wires, there is no potential drop as they are assumed to have negligible resistance.

Starting at P, use a coloured pen/pencil and draw on the wires that have the same potential as P.

Then, use a different coloured pen/pencil and draw on the wires that have the same potential as Q.

What do you observe about each resistor?

Additional Notes

General approach (non-linear) to solve circuit problems

- When solving a circuit problem, take note of the following:
  - Check if cell has internal resistance. Draw onto the circuit diagram if it has but not present in the diagram given.
  - Label conventional current directions in all wires.
  - Label all known values of *V*, *I* and *R*.
  - Use the following to determine unknown values of *V*, *I* and *R*:
    - Kirchhoff's current law
    - $V = IR$
    - Potential divider (See Section 15.3)

Check your understanding 2

When four identical lamps P, Q, R and S are connected as shown in diagram 1, they have normal brightness.

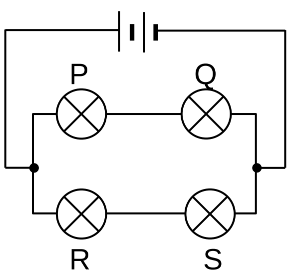


diagram 1

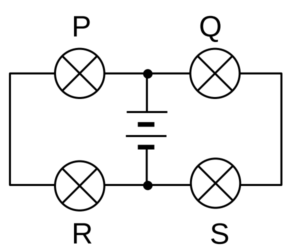


diagram 2

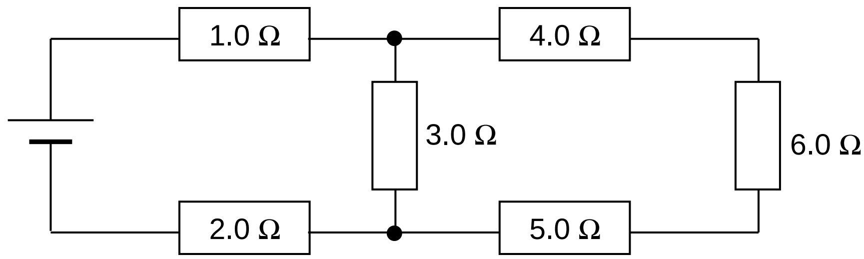
Note that brightness of a bulb is determined by the Power dissipated at the bulb.

When the four lamps are connected as shown in diagram 2, which statement is correct?

A The lamps do not light up.
B The lamps are less bright than normal.
C The lamps have normal brightness.
D The lamps are brighter than normal.

Example 4

An e.m.f. source is connected to a network of resistors as shown in the diagram below.



Calculate

(a) the equivalent resistance of the circuit between the terminals of the source

(b) the potential difference between the terminals of the battery if 1.0 A of current flows through the 6.0 Ω resistor.

(a)

Additional Notes

(b)

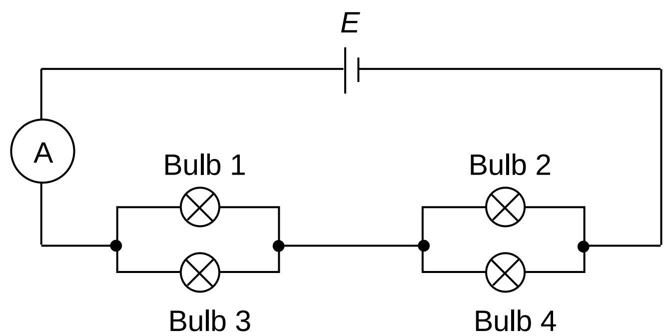
Additional Notes

Since brightness of a bulb is determined by the Power dissipated at the bulb, we should compare brightness by looking at what happens to the current through the bulb.

We should NOT compare by looking only at the current through the circuit.

Check your understanding 3

Four similar bulbs are connected to a constant-voltage d.c. supply as shown below. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current.



The filament of Bulb 2 breaks. What happens to the ammeter reading and the brightness of the remaining lamps?

	Ammeter reading	Bulb 1	Bulb 3	Bulb 4
A	increases	increases	increases	increases
B	increases	increases	increases	decreases
C	decreases	decreases	decreases	decreases
D	decreases	decreases	decreases	increases

C. Potential divider

- A potential divider setup is an arrangement of resistances in series to obtain a fraction of the potential difference across the branch of resistances in series.
- For such a setup, we can apply the Potential Divider rule to find the p.d. across the resistors directly using the resistance of the resistors, without having to find current I.
- Consider the following circuit in Fig. 15.8 of two resistors arranged in series.

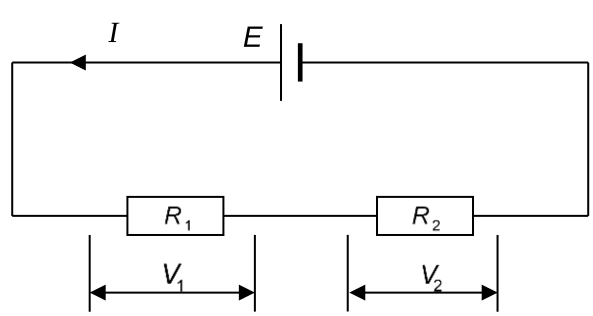


Fig. 15.8

Additional Notes

- As the resistors are connected in series, the effective resistance of the circuit is $R_{eff} = R_1 + R_2$.

- Using $V = IR$, the current through the circuit, $I = \dfrac{E}{R_1 + R_2}$.

- Therefore, p.d. across R_1, $V_1 = IR_1 = \dfrac{E}{R_1 + R_2} R_1$

$$V_1 = \frac{R_1}{R_1 + R_2} E$$

Similarly, p.d. across R_2, $V_2 = \dfrac{R_2}{R_1 + R_2} E$

- In general, the p.d. across a resistor R in a <u>branch of resistors in series</u> is given by

$$\text{p.d. across resistor } R,\ V_R = \frac{R}{\text{total resistance of the branch}} \times \text{total p.d. across the branch}$$

Note that the Potential Divider rule applies to any number of components <u>in series</u>, and not just 2.

The resistors need not be connected directly to an e.m.f source in order to use the Potential Divider rule. We can use it as long as we know the p.d. across the branch of components in series.

Practical uses of a potential divider circuit

- In order to <u>vary</u> the p.d. across two resistors R_1 and R_2 in a potential divider circuit, one of the resistors should have variable resistance.

- Ways to setup a potential divider circuit include:

a) a fixed resistor connected in series with a resistor with variable resistance (e.g. rheostat, thermistor, LDR)

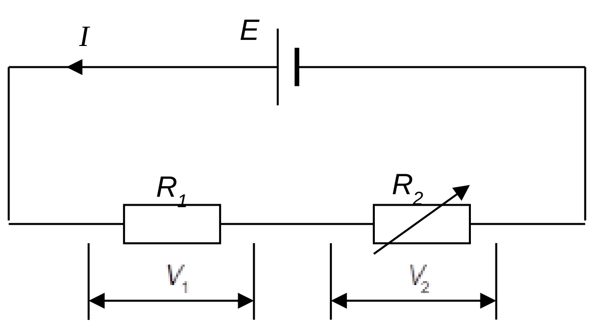


Fig. 15.9

As the resistance of R_2 in Fig. 15.9 varies, the p.d. across both resistors, V_1 and V_2 will vary.

The resistance of thermistor decreases with temperature increase.
Thermistor in a potential divider circuit can be used as a temperature sensor to detect fire and activate water sprinkler system, switch on the heater of a room when outside temperature drops and many more applications.

The resistance of LDR decreases as the light intensity falling on the LDR increases.
LDR in a potential divider circuit can be used as a light sensor to switch on light when the surrounding is dark, activate burglar alarm and many more applications.

b) a resistance wire (usually of uniform cross-sectional area and resistivity) providing variable resistance

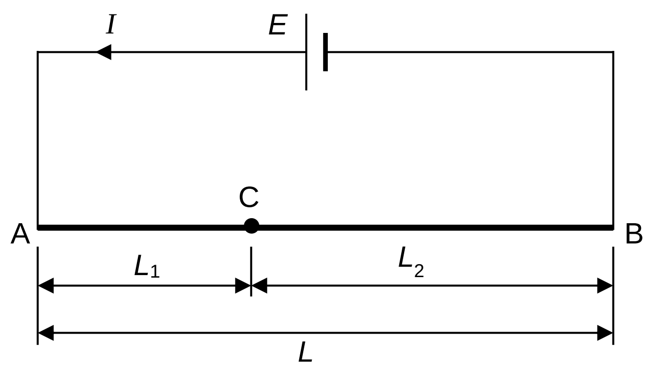


Fig. 15.10

For a resistance wire of uniform cross-sectional area and resistivity, its resistance is proportional to its length, since R = $\frac{\rho L}{A}$.

In Fig. 15.10, the resistance of section AC, $R_1 = \frac{\rho}{A} L_1$.

Similarly, the resistance of section CB, $R_2 = \frac{\rho}{A} L_2$.

As the two sections of wire AC and CB are in series with each other, they form a potential divider setup.

Hence, p.d. across section AC, $V_1 = \frac{R_1}{R_1 + R_2} E$

$$= \frac{\frac{\rho}{A} L_1}{\frac{\rho}{A} L_1 + \frac{\rho}{A} L_2} E = \frac{L_1}{L_1 + L_2} E$$

Since $L_1 + L_2 = L$,

p.d. across section AC, $V_1 = \frac{L_1}{L} E$.

Additional Notes

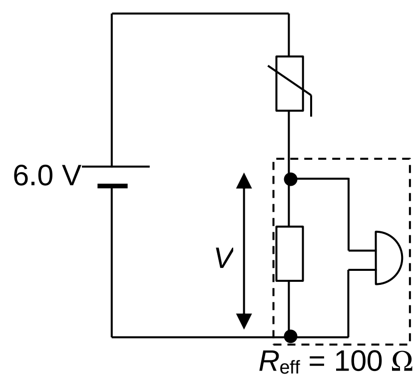
Similarly, p.d. across section CB, $V_2 = \frac{L_2}{L} E$.

The p.d. V_1 and V_2 are sometimes expressed as $\frac{E}{L} L_1$ and $\frac{E}{L} L_2$ respectively, where $\frac{E}{L}$ is a constant called the potential gradient (i.e. the potential drop per unit length).

Worked Example

A thermistor has a resistance of 1000 Ω at room temperature but quickly drops to 300 Ω when the temperature is at 80°C. The thermistor is connected to a buzzer arranged in parallel to a fixed resistor as shown in the circuit below. The effective resistance of the buzzer and fixed resistor is 100 Ω.

Determine the range of values of the p.d., V, across the 100 Ω resistor.



This setup shows a practical application of a thermistor as a fire alarm in a potential divider circuit.

When the thermistor is at a low temperature (room temperature), the resistance of the thermistor is high. Hence, p.d. across the thermistor is high and the p.d. across the buzzer is low, and the buzzer would not ring.

However, in the event of a fire, the thermistor is at a high temperature and the p.d. across the buzzer would now be high. The buzzer would then ring.

As the setup is a potential divider,

$$V = \frac{100}{100 + R_{thermistor}} (6.0)$$

Hence, V_{min} occurs when the resistance of the thermistor is the largest and V_{max} occurs when the resistance of the thermistor is the smallest.

$$V_{min} = \frac{100}{100 + 1000} (6.0) = 0.55 \text{ V}$$

$$V_{max} = \frac{100}{100 + 300} (6.0) = 1.5 \text{ V}$$

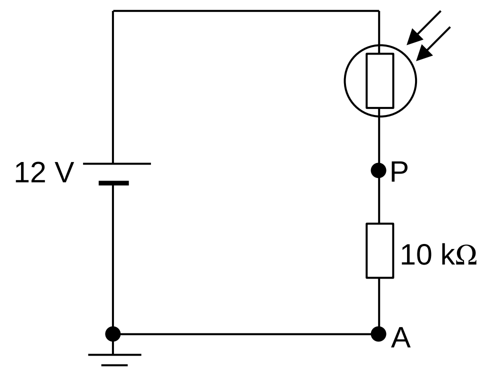
Therefore, the range of values of p.d. across the 100 Ω resistor is from 0.55 V to 1.5 V.

Example 5

A light-dependent resistor (LDR) is connected in series with a 10 kΩ resistor and a 12 V d.c. supply of negligible internal resistance as shown.

(a) Determine the potential at P when the LDR is
   (i) in the dark and has resistance of 50 kΩ,
   (ii) in bright sunlight and has resistance of 2.0 kΩ.

(b) Calculate the resistance of the LDR if the potential at P is 4.0 V.



Additional Notes

D. Balanced potential

- To determine the e.m.f. of a cell, there are two methods:

  a) using a voltmeter

  - It is a fast and efficient method.
  - However, a voltmeter draws some current from the circuit under investigation as it does not have infinite resistance, thus affecting the circuit to be measured. The accuracy of the results obtained depends on the resistance of the voltmeter used.

  b) using a potentiometer (uses balanced potential method)

  - It is a bulky setup that requires the use of a resistance wire and a moving coil galvanometer.
  - The galvanometer is used as a current detector. Its needle is normally at the zero mark in the centre and deflects to either side when current flows.
  - Measurements are made on the potentiometer when no current passes through the galvanometer (or null deflection).
  - This method measures the e.m.f. of a source accurately as no current is drawn from the circuit to be measured.

D.1 Condition for null deflection on the galvanometer

- Consider two e.m.f. sources, E_1 and E_2, with their high potential ends connected together as shown in the circuit in Fig. 15.11 below. A galvanometer G is connected to detect the presence of current in the circuit, when the switch is closed.

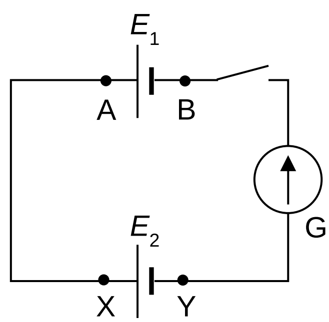
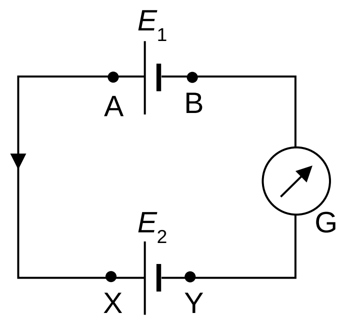


Fig. 15.11

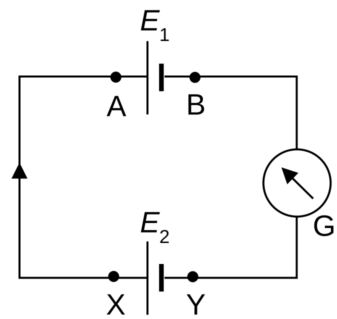
- Depending on the e.m.f. of E_1 and E_2, the galvanometer will deflect according to the direction of current flow in the circuit when the switch is closed.



$E_1 > E_2$

- Current flows in an anticlockwise direction
- Galvanometer deflects to one side

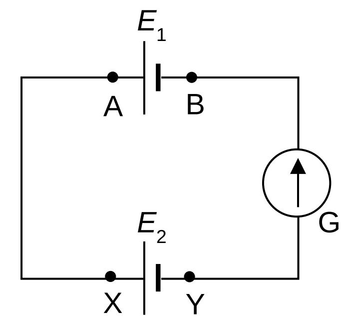
(a)



$E_1 < E_2$

- Current flows in a clockwise direction
- Galvanometer deflects to the opposite side

(b)



$E_1 = E_2$

- No current flows
- Galvanometer shows no deflection, i.e. null deflection

(c)

Fig. 15.12

D.2 Potentiometer

- A potentiometer makes use of the balanced potential method to determine the potential difference across two points.

- It is more accurate than a voltmeter as all practical voltmeters do not have infinite resistances, and hence, draws current. A potentiometer acts like a voltmeter with infinite resistance as it draws no current from the circuit to be measured.

- A potentiometer setup is shown in the circuit in Fig. 15.13 below:

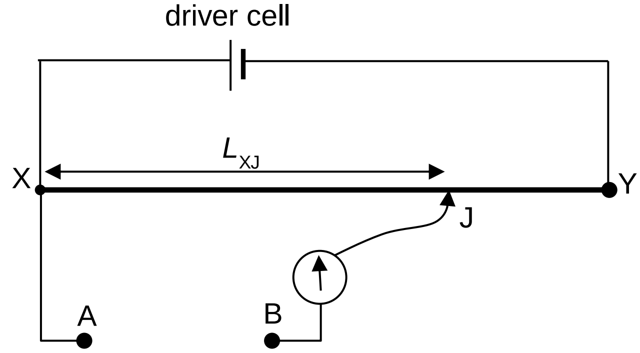


Fig. 15.13

Additional Notes

- A potentiometer typically comprises of the following components
  (i) a driver cell
  (ii) a resistance wire (XY)
  (iii) a jockey (J) and
  (iv) a galvanometer

The jockey J is a movable junction so that the length L_{XJ} is adjustable via tapping the jockey on the resistance wire.

- The unknown p.d. to be measured is to be connected to the ends AB with the point of higher potential connected to A.

- The jockey is then tapped on the wire, causing $V_{AB} = V_{XJ}$.

- When the jockey is moved along the length XY, we want to identify a point on XY where there is null deflection in the galvanometer, which means that no current flows through the galvanometer.

- The point on the wire where null deflection occurs is called the balanced point.

- At balanced point, and the length L_{XJ} is called the balance length.

- Therefore, p.d. across XJ, $V_{XJ} = \frac{L_{XJ}}{L_{XY}} V_{XY}$

where V_{XY} = p.d across wire XY
L_{XY} = length of wire XY

- Since $V_{AB} = V_{XJ}$, we can therefore work out that $V_{AB} = \frac{L_{XJ}}{L_{XY}} V_{XY}$.

For the potentiometer to work, the p.d. across XY must be larger than the p.d. of the branch AB to be measured. Otherwise, we can never achieve balance point (or balance length).

Case 1 (single branch connected across AB)

- Consider the following potentiometer setup in Fig. 15.14 where a single branch containing a secondary cell and a fixed resistor is connected across AB.

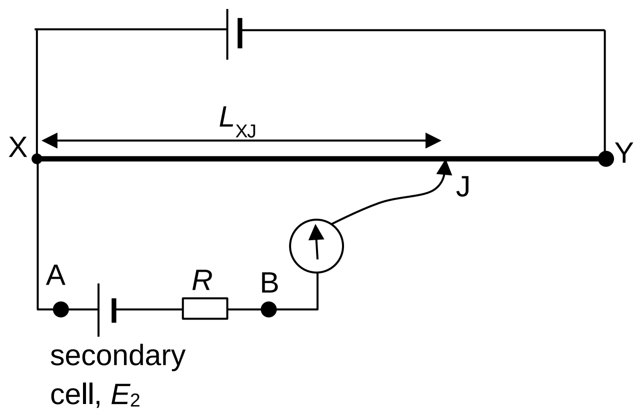


Fig. 15.14

Additional Notes

- At <u>balance point (or balance length)</u>, no current flows through the galvanometer.

- Therefore, p.d. across XJ, $V_{XJ} = \frac{L_{XJ}}{L_{XY}} V_{XY}$.

- Since no current flows through the galvanometer, this also implies that there is no current flowing through the secondary cell E_2 and resistor R. Hence, there is no potential drop across R.

- The p.d. across AB V_{AB} is therefore simply the e.m.f. of the secondary cell E_2.

- Since $V_{AB} = V_{XJ}$, this means that $E_2 = \frac{L_{XJ}}{L_{XY}} V_{XY}$.

The advantage a potentiometer has over a voltmeter when measuring potential differences is that <u>the potentiometer, at balance length, does not take current from the lower circuit as a non-ideal voltmeter will</u>.

Case 2 (<u>2 or more branches</u> connected across AB)

- Consider the following potentiometer setup in Fig. 15.15 where an additional branch PQ is connected across AB.

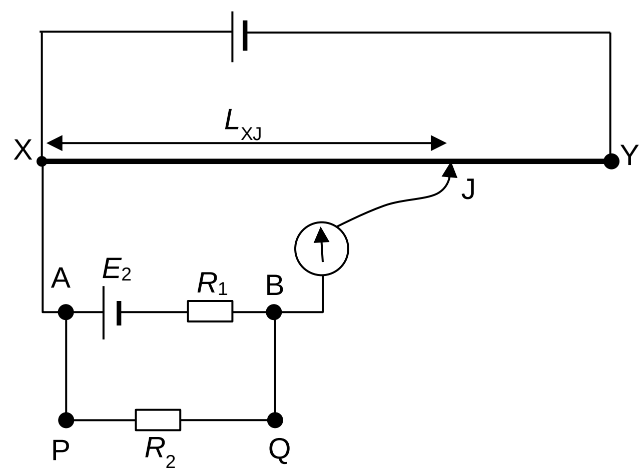


Fig. 15.15

- At balance point (or balance length), no current flows through the galvanometer. i.e. no current through BJ and XA.

- Therefore, p.d. across XJ, $V_{XJ} = \frac{L_{XJ}}{L_{XY}} V_{XY}$.

- However, the loop ABQPA is a closed circuit, and thus current flows within this loop.

- Suppose the current in the loop ABQPA is I, then the p.d. across AB, $V_{AB} = E_2 - IR_1$.

Additional Notes

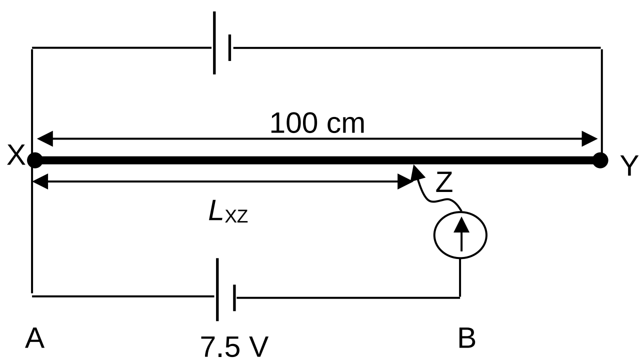
- The p.d. across AB is also equal to the p.d. across PQ. Thus, $V_{AB} = V_{PQ} = IR_2$.

- Since $V_{AB} = V_{PQ} = V_{XJ}$, this means that $E_2 - IR_1 = IR_2 = \frac{L_{XJ}}{L_{XY}} V_{XY}$.

Again, the potentiometer setup does not draw current from the loop ABQPA at balance length and thus act like an ideal voltmeter.

Worked Example

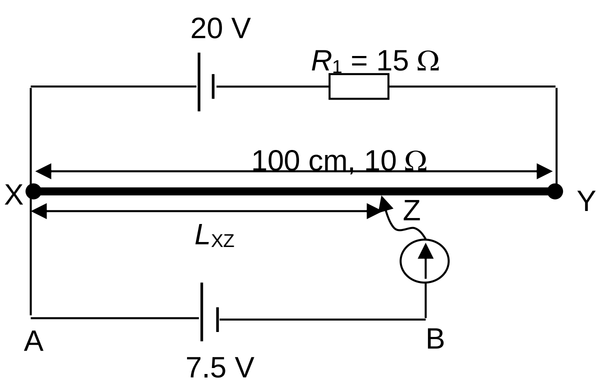
In the circuit below, the resistance wire XY is 100 cm. The secondary cell has e.m.f. 7.5 V, while the driver cell of e.m.f. 20 V has negligible internal resistance.



(a) Calculate the balance length XZ.
(b) A 15 Ω resistor is placed in series with the 20 V driver cell.
Calculate the new balance length XZ if the resistance wire XY has a total resistance of 10 Ω.

(a) $V_{XZ} = V_{AB}$
At balance length, there is no current through the galvanometer.
Hence, $V_{XZ} = \frac{L_{XZ}}{L_{XY}} \times 20 = 7.5$
balance length, $L_{XZ} = (7.5/20) \times 100$
$= 37.5$ cm

(b)

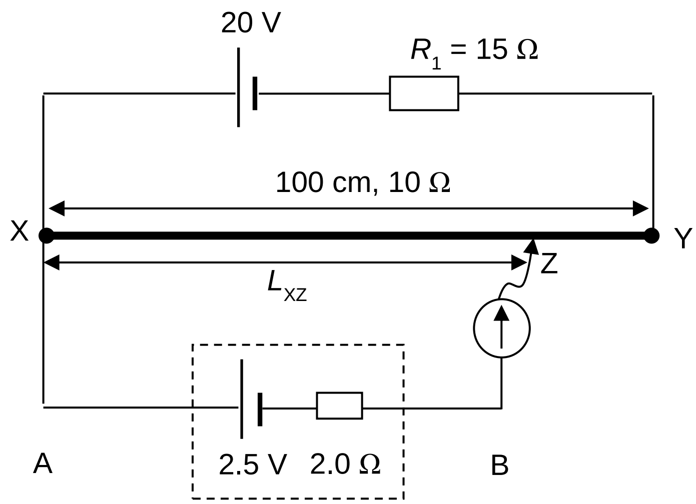


$V_{XZ} = V_{AB}$
At balance length,
$V_{XY} = \frac{R_{XY}}{R_{XY} + R_1} \times 20 = \frac{10}{10 + 15} \times 20 = 8.0$ V

➔ $\frac{L_{XZ}}{L_{XY}} \times 8.0 = 7.5$ V
$L_{XZ} = (7.5/8.0) \times 100$
$= 93.8$ cm

Example 6

In the circuit below, the resistance wire XY has resistance of 10 Ω and has length 100 cm. The secondary cell has e.m.f. 2.5 V with 2.0 Ω of internal resistance, while the driver cell of e.m.f. 20 V has negligible internal resistance. Resistor R_1 of 15 Ω is connected in series with the driver cell and XY.



Since the potentiometer circuit remains the unchanged, V_{XY} remains unchanged.

(a) Calculate the balance length XZ.

(b) Explain whether the balance length would be affected if a 3.0 Ω resistor is added in series with the 2.5 V secondary cell.

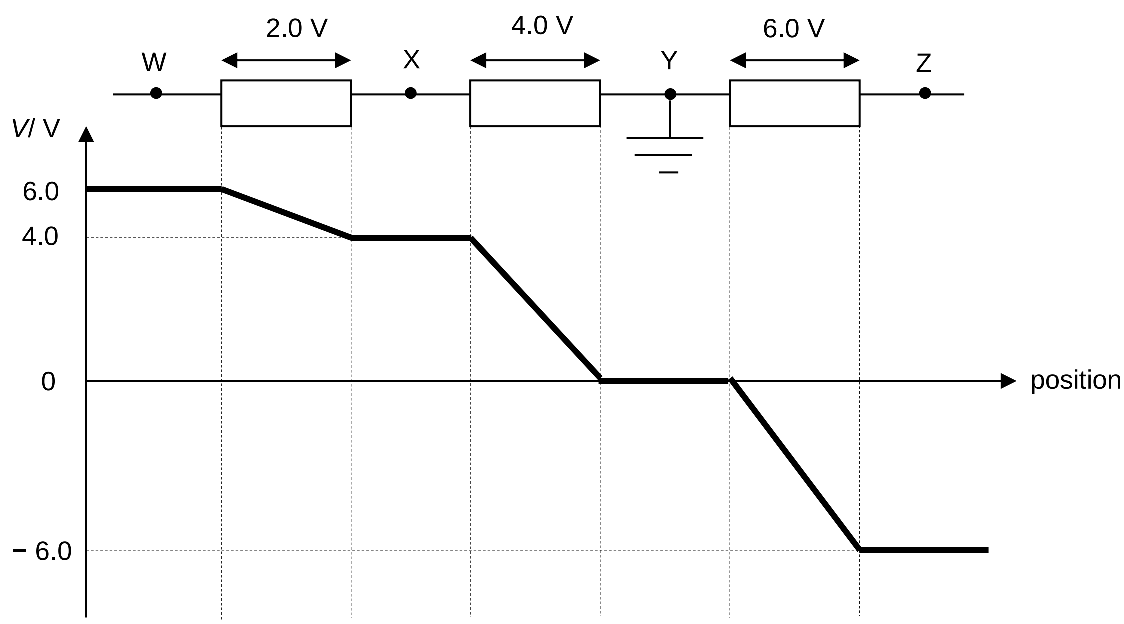
(c) Calculate the new balance length when a 3.0 Ω resistor is added in parallel to the 2.5 V cell.

Additional Notes

Solutions to Examples

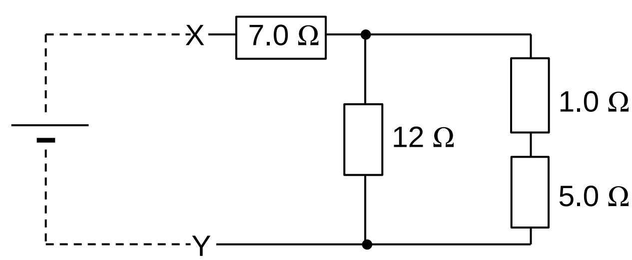
Example 1

- Start at point Y as this point is earthed. Thus, the potential at Y, $V_Y = 0$ V.
- As current is flowing in an anticlockwise direction, potential at point Z, V_Z is 6.0 V lower than that of point Y.
  Hence, $V_{YZ} = V_Y - V_Z$
  $6.0 = 0 - V_Z$
  $V_Z = -6.0$ V
- Potential at point X, V_X is higher than V_Y by 4.0 V.
  Hence, $V_{XY} = V_X - V_Y = 4.0$ V,
  $V_X = 4.0$ V
- Potential at point W, V_W is higher than V_X by 2.0 V.
  Hence, $V_{WX} = V_W - V_X = 2.0$ V,
  $V_W = 4.0 + 2.0$
  $= 6.0$ V



Example 2

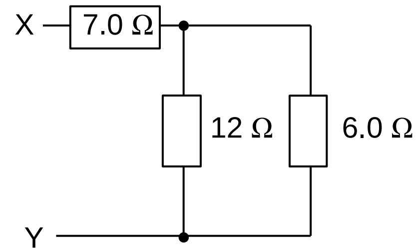
The network of resistors can be redrawn by rearranging some resistors and you can imagine that the ends XY are connected to an e.m.f. source as shown.



Additional Notes

Since the 1.0 Ω and 5.0 Ω resistors are in series (same current through them), equivalent resistance, R_{eff} = 6.0 Ω.

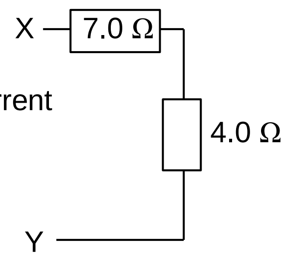
Redrawing, we have the new circuit diagram on the right.



Since the 12 Ω and 6.0 Ω resistors are in parallel (same p.d. across them),

equivalent resistance, $R_{eff} = \left(\frac{1}{12} + \frac{1}{6.0}\right)^{-1}$
$= 4.0\ \Omega$

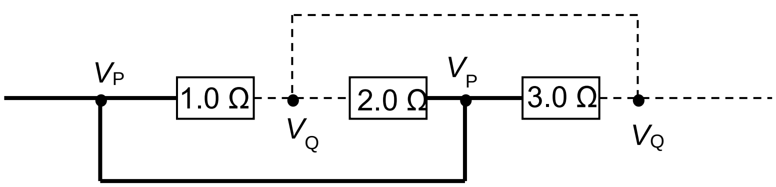
Redrawing, we have the new circuit diagram on the right.



Since the 7.0 Ω and 4.0 Ω resistors are in series (same current through them),
equivalent resistance, R_{eff} = 7.0 + 4.0
= 11 Ω
= R_{XY}

Hence, the effective resistance between X and Y is 11 Ω.

Example 3

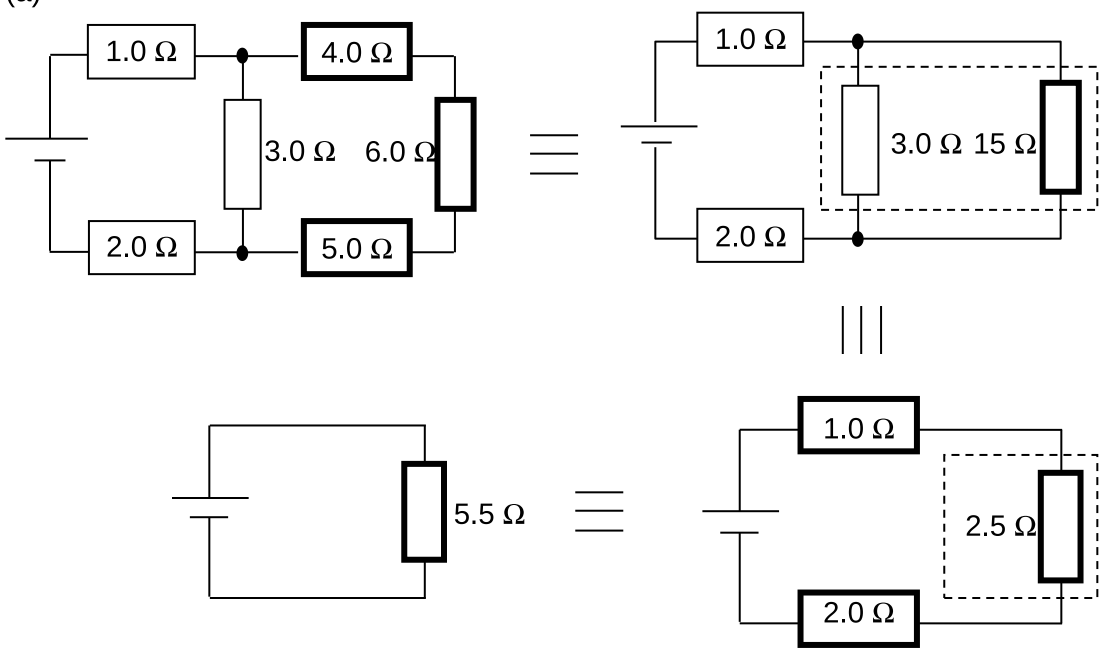


By identifying connecting wires that are of the same potential as that of P and Q, we see that for each resistor, one end has a potential of V_P and the other end has a potential of V_Q.

Hence, the potential difference across each resistor is the same (V_{PQ}) and therefore the 3 resistors are actually arranged in parallel as shown.

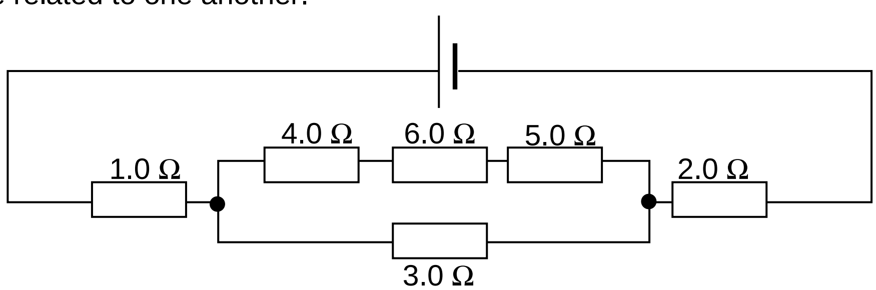
Example 4

(a)



Equivalent resistance R = 5.5 Ω

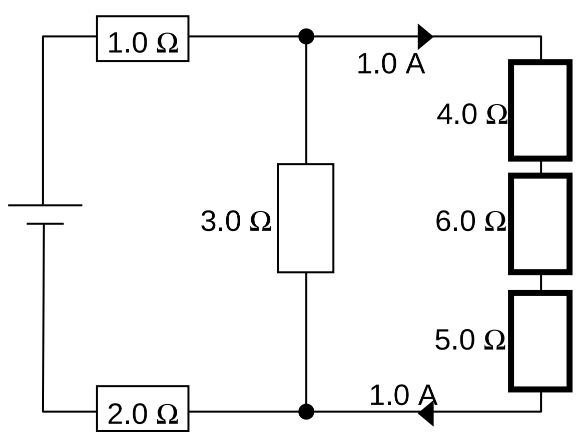
Alternatively, the question can be redrawn as follows, which makes it easier to see how the resistors are related to one another.



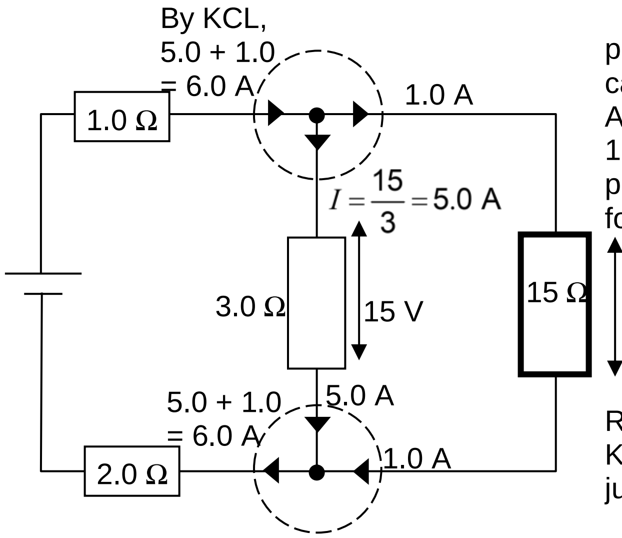
(b)

Appreciate that the 4.0 Ω, 6.0 Ω and 5.0 Ω resistors are in series.

R_{eff} = 4.0 + 6.0 + 5.0 = 15 Ω



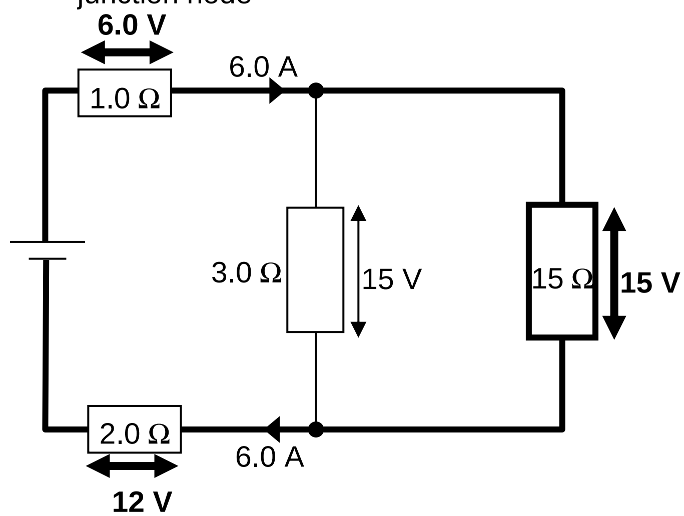
Additional Notes



p.d. across the effective resistance of 15 Ω can be found.
Appreciate that the effective resistance of 15 Ω is parallel to the 3.0 Ω resistor.
p.d. across the 3.0 Ω resistor can then be found.

V = 1.0 × 15
= <u>15 V</u>

Remember that currents must obey Kirchhoff's Current Law (KCL) at each junction node



With the current value, now find the p.d. across the 1.0 Ω and 2.0 Ω resistors.

Appreciate that the terminal p.d. equals to the total p.d. across the external resistors along any one complete loop from its terminals
Terminal p.d.
= 6.0 + 15 + 12 = 33 V

Example 5

(a)(i) In the dark, p.d. across 10 kΩ resistor is

$$V_{PA} = \left(\frac{10}{10+50}\right)12 = 2.0\,\text{V}$$

Since $V_{PA} = V_P - V_A = 2.0$ V,
⇒ potential at P, $V_P = 2.0$ V
(∵ $V_A = 0$ V as it is connected to a point in the circuit that is earthed)

(ii) In bright sunlight, p.d. across 10 kΩ resistor is

$$V_{PA} = \left(\frac{10}{10+2.0}\right)12 = 10\,\text{V}$$

Since $V_{PA} = V_P - V_A = 10$ V
⇒ potential at P, $V_P = 10$ V

(b) Let R = resistance of the LDR when the potential at P is 4.0 V.

As the p.d. across resistor 10 kΩ is 4.0 V,

$$V_{PA} = \left(\frac{10}{10+R}\right)12 = 4.0\,\text{V}$$

Hence, 10 + R = 30 $\Rightarrow$ R = 20 kΩ

Example 6

(a)

$V_{XZ} = V_{AB}$

At balance length,

$$V_{XY} = \frac{R_{XY}}{R_{XY}+R_1}\times 20 = \frac{10}{10+15}\times 20 = 8.0\,\text{V}$$

$$V_{XZ} = \frac{L_{XZ}}{L_{XY}}\times V_{XY} = V_{AB}$$

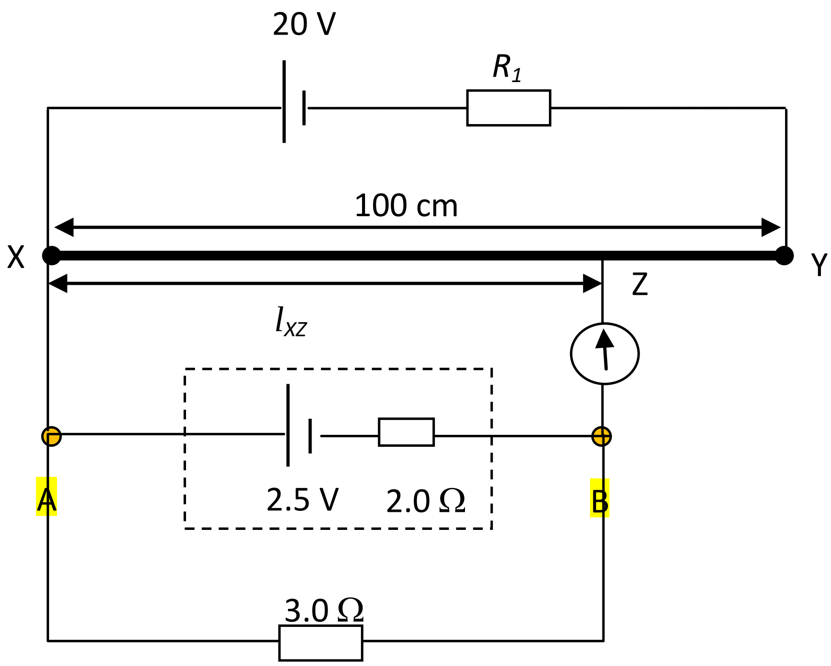
$$L_{XZ} = \frac{2.5}{8.0}\times 100$$

$$L_{XZ} = 31.3\,\text{cm}$$

(b) At balance length, there is no current through the galvanometer, so no current flows through the cell and the 3.0 Ω resistor.
Hence, there is no potential drop across the 3.0 Ω resistor.
Since the potential difference across the lower branch remains as e.m.f. value of the secondary cell, the p.d. across the balance length remains unchanged.
So balance length remains the same.

Additional Notes

(c)



$V_{XZ} = V_{AB}$

At balance length,

$$V_{XZ} = \frac{L_{XZ}}{L_{XY}} V_{XY} = \frac{L_{XZ}}{100} \times 8.0$$ (looking at the driver cell circuit)

$$V_{AB} = \frac{3.0}{3.0 + 2.0} \times 2.5 = 1.5\,\text{V}$$ (looking at the bottom loop)

Since $V_{XZ} = V_{AB}$, $$\frac{L_{XZ}}{100} \times 8.0 = 1.5$$

$$L_{XZ} = \frac{1.5}{8.0} \times 100 = 18.8 \text{ cm}$$