Woodlands Ring Secondary School
Secondary 4E Mathematics Prelim P1 2023 – Worked Solutions

Qns. No.	Working
1(a)	$\sqrt[4]{\frac{7.23}{1.305}} + 8.9^3 = 706.503$ $= 707$ (3 s.f.)
1(b)	7.07×10^2
2(a)	$4a^2 \times 7a^6 = 28a^8$
2(b)	$8 - 5(1-2b) = 8 - 5 + 10b$ $= 3 + 10b$
3(a)	$x^2 - 4x - 12 = (x+2)(x-6)$ $\therefore\ a = 2,\ b = 6$
3(b)	Line of symmetry is $x = 2$
4(a)	Range = 31 – 13 = 18
4(b)	Median = $\frac{19+22}{2}$ = 20.5
5	The line graph for Star-One looks steeper than that for Galaxy as the scales for the vertical axes are different.
6(a)	Diagram 2
6(b)	Height of cylindrical part of the container
7(a)	Sequence: $-2,\ 5,\ 12,\ 19,\ \ldots$ Multiples of 7: $7,\ 14,\ 21,\ 28,\ \ldots,\ 7n$ $\therefore\ n^{\text{th}}$ term $= 7n - 9$
7(b)(i)	1, 7, 17
7(b)(ii)	For any integer n, $2n^2$ is always a positive even number. Since 1 is an odd number, when it is subtracted from $2n^2$, $2n^2 - 1$ will be odd.

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8(a)	$4-x<\frac{2x-5}{3}\le 6$ $12-3x<2x-5\le 18$ $12-3x<2x-5$ **and** $2x-5\le 18$ $-5x<-17$ $\quad$ $2x\le 23$ $x>3\frac{2}{5}$ $\quad$ $x\le 11\frac{1}{2}$ $\quad$ [or $x>3.4$ and $x\le 11.5$] $\therefore \quad 3\frac{2}{5}<x\le 11\frac{1}{2}$ $\quad$ [or $3.4<x\le 11.5$ or $\frac{17}{5}<x\le\frac{23}{2}$]
8(b)	Smallest integer = 4
9(a)	P(cash) = $1-\frac{1}{4}-\frac{1}{5}$ = $\frac{11}{20}$
9(b)	P(at least one of them uses mobile wallet (M) = 1 – P(no M) = $1-\frac{3}{4}\times\frac{3}{4}$ = $\frac{7}{16}$
9(c)	P(only the third customer uses cash) = P(no cash, no cash, cash) = $\frac{9}{20}\times\frac{9}{20}\times\frac{11}{20}$ = $\frac{891}{8000}$
10(a)	Map scale = 5 cm : 2.5 km = 1 cm : 0.5 km = 1 : 50 000
10(b)	On the map, PQ = 8 cm (accept 7.9 – 8.1 cm) Actual length of PQ = 8 × 0.5 = 4 km (accept 3.95 – 4.05 km) <u>Other accepted working</u>: Map scale = 1 cm : 0.5 km = 8 cm : 4 km

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10(c)	Area scale $= 1^2$ cm^2 : 0.5^2 km^2 $=$ 1 cm^2 : 0.25 km^2 Area on the map $= \dfrac{3.5}{0.25}$ $=$ 14 cm^2
11(a)	Gradient of line $l = \dfrac{15-5}{10-2}$ $= \dfrac{5}{4}$
11(b)	Sub (2, 5) into $y = \dfrac{5}{4}x + c$: $5 = \dfrac{5}{4}(2) + c$ $c = 5 - \dfrac{5}{2}$ $= \dfrac{5}{2}$ Equation of line *l*. is $y = \dfrac{5}{4}x + \dfrac{5}{2}$ OR $4y = 5x + 10$ … (1)
11(c)	Length of *AB* $= \sqrt{(10-2)^2 + (15-5)^2}$ $= \sqrt{164}$ $= 12.8$ units (3 s.f.)
11(d)	Since the vertical line passes through the point (18, 2) and point *N*, the *x*-coordinate of *N* is 18. Sub $x = 18$ into (1): $y = \dfrac{5}{4}(18) + \dfrac{5}{2}$ $= 25$ $\therefore$ *N* is (18, 25)
12	$R = \dfrac{k}{x^2}$ New $x = 0.4x$ New $R = \dfrac{k}{(0.4x)^2}$ $= \dfrac{k}{0.16x^2}$ $= \left(\dfrac{1}{0.16}\right)\left(\dfrac{k}{x^2}\right)$ $= 6.25R$

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	Percentage increase $= \frac{6.25R - R}{R} \times 100\%$ $= \frac{5.25R}{R} \times 100\%$ $= 525\%$
13	$126 = 2 \times 3^2 \times 7$ $90 = 2 \times 3^2 \times 5$ $54 = 2 \times 3^3$ HCF of 126, 90 and 54 $= 2 \times 3^2$ $= 18$ $\therefore$ number of cubes required $= \frac{126}{18} \times \frac{90}{18} \times \frac{54}{18}$ $= 105$
14(a)	$14gh - 35g = 7g(2h - 5)$
14(b)	$(r - 3s)^2 - 8s^2 = r^2 - 6rs + 9s^2 - 8s^2$ $= r^2 - 6rs + s^2$
15	$4c + 3a = 220$ … (1) $3c + 5a = 275$ … (2) (1) × 3: $12c + 9a = 660$ … (3) (2) × 4: $12c + 20a = 1100$ … (4) (4) – (3): $11a = 440$ $a = 40$ Sub $a = 40$ into (1): $4c + 3(40) = 220$ $4c = 100$ $c = 25$ $\therefore$ cost of a ticket for a child = \$25 cost of a ticket for an adult = \$40
16	1 interior angle of the polygon B at point O = 360 – 90 – 115 = 155° (∠s at a point) 1 exterior angle of the polygon B at point O = 180 – 155 = 25° (adj. ∠s on a str. line)

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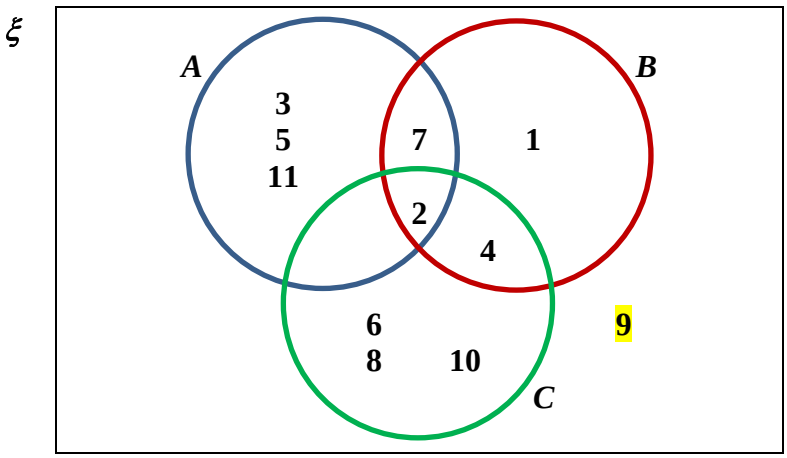
	If polygon B is a regular polygon, number of sides $= \frac{360}{25}$ $= 14.4$ Since n is not an integer, polygon B cannot be a regular polygon.
17	$3\sin x - 2 = 0$ $\sin x = \frac{2}{3}$ $x = 41.8°, 138.2°$ (1 d.p.)
18(a)	1, 4, 6, 8, 9, 10 **or** A' = {1, 4, 6, 8, 9, 10}
18(b)	$A \cup C$ = {2, 3, 4, 5, 6, 7, 8, 10, 11} $(A \cup C)'$ = {1, 9} $\text{n}(A \cup C)'$ = 2
18(c)	
19	$16^{2w} = 64^{1-w}$ $(4^2)^{2w} = (4^3)^{1-w}$ $4^{4w} = 4^{3-3w}$ Comparing the indices: $4w = 3 - 3w$ $7w = 3$ $w = \frac{3}{7}$

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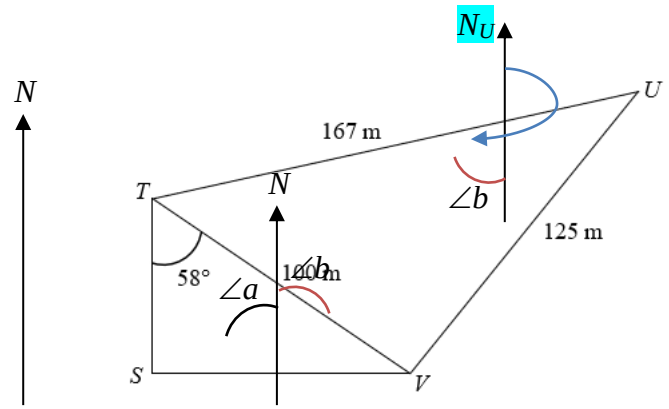
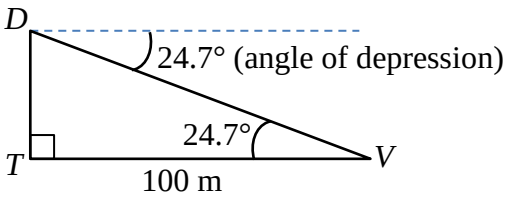
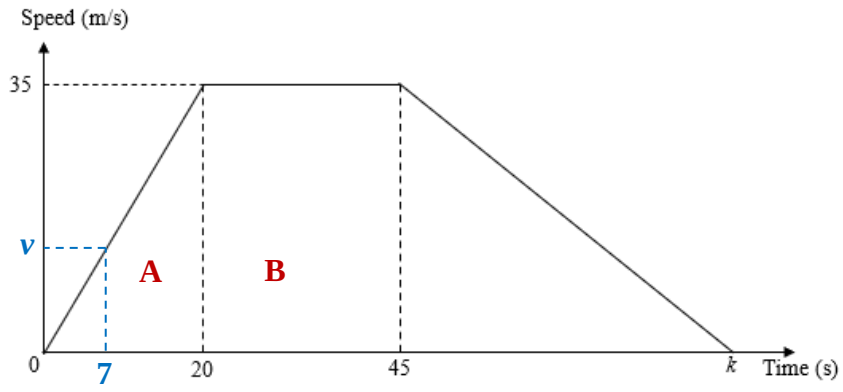
20	1. $BQ = CB$ (given) 2. $\angle ABQ = 180^\circ - \angle ABC$ (adj. $\angle$ on a str. line) $= 180^\circ - \angle ACB$ (base $\angle$s of isos ΔABC) $= \angle PCB$ (adj. $\angle$s on a str. line) 3. $AB = AC$ (given) $= PC$ $\therefore\ \Delta AQB \equiv \Delta PBC$ (SAS)
21	$q = 15 - \frac{2p^2}{3}$ $3q = 45 - 2p^2$ $2p^2 = 45 - 3q$ $p^2 = \frac{45-3q}{2}$ $p = \pm\sqrt{\frac{45-3q}{2}}$
22	$\frac{y}{(2y-5)^2} + \frac{3}{5-2y} = \frac{y}{(2y-5)^2} - \frac{3}{2y-5}$ $= \frac{y-3(2y-5)}{(2y-5)^2}$ $= \frac{y-6y+15}{(2y-5)^2}$ $= \frac{15-5y}{(2y-5)^2}$ $= \frac{5(3-y)}{(2y-5)^2}$
23(a)	In ΔTVU, $\cos\angle TVU = \frac{100^2+125^2-167^2}{2(100)(125)}$ $= -\frac{283}{3125}$ or $-\frac{2264}{25000}$ $\therefore\ \angle TVU = 95.1958°$ $= 95.2°$ (1 d.p.) (shown)

Qns. No.	Working
23(b)	 $\angle a$ = 58° (alt $\angle$s, // lines) $\angle b$ = 95.1958 – 58 = 37.1958° $\therefore$ the bearing of V from U = 180 + 37.1958 = 217.2° (1 d.p.)
23(c)	 $\tan 24.7° = \frac{DT}{100}$ $DT = 100 \tan 24.7°$ = 45.99486 m = 46.0 m (3 s.f.)
24(a)	$\mathbf{Q} = \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix}$
24(b)	$\mathbf{R} = \mathbf{PQ} = \begin{pmatrix} 5.2 & 6.6 & 7.3 \\ 5.5 & 6.4 & 7.4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix}$ $= \begin{pmatrix} 5.2\times2+6.6\times4+7.3\times3 & 5.2\times0+6.6\times5+7.3\times4 \\ 5.5\times2+6.4\times4+7.4\times3 & 5.5\times0+6.4\times5+7.4\times4 \end{pmatrix}$ $= \begin{pmatrix} 58.7 & 62.2 \\ 58.8 & 61.6 \end{pmatrix}$

Qns. No.	Working
24(c)	The elements in **R** represent the total amount that Derrick and Elaine need to pay if they purchase the drinks at Café X and Café Y respectively.
24(d)(i)	$\mathbf{S} = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.15 & 0 \\ 0 & 0 & 0.95 \end{pmatrix}$
24(d)(ii)	$\begin{pmatrix} 5.2 & 6.6 & 7.3 \\ 5.5 & 6.4 & 7.4 \end{pmatrix} \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.15 & 0 \\ 0 & 0 & 0.95 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix}$ $= \begin{pmatrix} 5.72 & 7.59 & 6.935 \\ 6.05 & 7.36 & 7.03 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 4 & 5 \\ 3 & 4 \end{pmatrix}$ $= \begin{pmatrix} 62.605 & 65.69 \\ 62.63 & 64.92 \end{pmatrix}$ Derrick spent **\$62.61** at Café **X**. Elaine spent **\$64.92** at Café **Y**.
25(a)	Arc $PQ = 8.4(1.5)$ $= 12.6$ cm $\therefore$ perimeter of the shaded region = 11.5 + 12.6 = 24.1 cm (3 s.f.)
25(b)	Area of the shaded region = Area of sector OPQ – Area of ΔOPQ $= \frac{1}{2}(8.4)^2(1.5) - \frac{1}{2}(8.4)(8.4)\sin 1.5$ $= 52.92 - 35.1916$ $= 17.728$ $= 17.7 \text{ cm}^2$ (3 s.f.)
25(c)	In ΔOPR: $\cos 1.5 = \frac{OR}{8.4}$ $OR = 8.4\cos 1.5$ $= 0.59419$ cm (5 s.f.) $\therefore$ RQ = 8.4 – 0.59419 = 7.80581 (6 s.f.) = 7.81 cm (3 s.f.)

Qns. No.	Working
26(a)	Since speed is constant from the 20th second to the 45th second, acceleration of the object at the 30th second = 0 m/s^2
26(b)	 Let v m/s be the speed at the 7th second. $\frac{v}{7}=\frac{35}{20}$ $v=12.25$ The required speed is 12.25 m/s.
26(c)	Total distance = 1.925 km $\frac{1}{2}[(45-20)+k](35)=1925$ $35(k+25)=3850$ $k+25=110$ $k=85$
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