2024 4E5N Prelim P1 Solutions

No.	Solution
1(a)	$f(x) = 3x^5 - 11x^3 + 30x^2 + 39 = (x-1)(x+3)Q(x) + ax + b$ When $x = 1$, $61 = a + b$ ----- eqn (1) When $x = -3$, $-123 = -3a + b$ ----- eqn (2) (1) – (2) $184 = 4a$ $a = 46$ $b = 15$
1(b)	$f(x) = (x-1)(x+3)Q(x) + 46x + 15$ $f(x) - 3 = (x-1)(x+3)Q(x) + 46x + 15 - 3$ Remainder = $46x + 15 - 3$ $= 46x + 12$
2(a)	$V = 3\left(\frac{h^2}{4} + \frac{8\pi}{h^3}\right)$ $\frac{dV}{dh} = \frac{3}{2}h - \frac{72\pi}{h^4}$ When $h = 4$, $\frac{dV}{dh} = 6 - \frac{9\pi}{32}$ $= \frac{192-9\pi}{32}$ $(or\ \ 5.1164)$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\frac{dh}{dt} = \frac{32}{192-9\pi} \times 35$ $= \frac{1120}{192-9\pi}\ cm/s$ $(or\ \ 6.84\ cm/s)$
2(bi)	$y = \frac{2-5x}{e^x}$ $\frac{dy}{dx} = \frac{e^x(-5)-(2-5x)e^x}{(e^x)^2}$ $= \frac{5x-7}{e^x}$ For decreasing function, $\frac{5x-7}{e^x} < 0$ $x < \frac{7}{5}$ $(or\ \ 1.4)$
2(bii)	When $y = 0$, $2 - 5x = 0$ $x = 0.4$ $\frac{dy}{dx} = -3.35\ (3s.f)$
3(a)	$\frac{1-3x-3x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $1 - 3x - 3x^2 = A(x+1)^2 + Bx(x+1) + Cx$ When $x = -1$, $C = -1$ When $x = 0$, $A = 1$ When $x = 1$, $B = -4$ $\frac{1-3x-3x^2}{x(x+1)^2} = \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}$

3(b)	$\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx = \frac{1}{2}\int \frac{1-3x-3x^2}{x(x+1)^2}\,dx$ $= \frac{1}{2}\int \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}\ dx$ $= \frac{1}{2}\ln x - 2\ln(x+1) + \frac{1}{2(x+1)} + c$ [Accept $\ln\sqrt{x} - \ln(x+1)^2 + \frac{1}{2(x+1)} + c$]
4(a)	$4^2 + (h-8)^2 = (k-4)^2 + 8^2$ $16 + h^2 - 16h + 64 = k^2 - 8k + 16 + 64$ $h^2 - 16h = k^2 - 8k$ $h^2 - k^2 = 16h - 8k$ (shown)
4(bi)	When $h = 1$, $1 - k^2 = 16 - 8k$ $k^2 - 8k + 15 = 0$ $(k-5)(k-3) = 0$ $k = 5 \quad or\ k = 3\ (rejected\ based\ on\ diagram)$ Let $A(0, y)$ $(1-y)^2 = 5^2 + y^2$ $1 - 2y + y^2 = 25 + y^2$ $y = -12$ $\therefore A(0, -12)$
4(bii)	Area $= \frac{1}{2}\begin{vmatrix} 0 & 5 & 4 & 0 & 0 \\ -12 & 0 & 8 & 1 & -12 \end{vmatrix}$ $= \frac{1}{2}\lvert 44 - (-60)\rvert$ $= 52\ units^2$
5(a)	Area $= \frac{1}{2}(7)(7)\sin\theta + \frac{1}{2}(7)(5.6)\sin(90-\theta)$ $+ \frac{1}{2}(5.6)(8)\sin\theta + \frac{1}{2}(8)(8)\sin(90-\theta)$ $= \frac{49}{2}\sin\theta + \frac{98}{5}\cos\theta + \frac{112}{5}\sin\theta + 32\cos\theta$ $= 51.6\cos\theta + 46.9\sin\theta$ (shown)
5(b)	$Q = 51.6\cos\theta + 46.9\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{51.6^2 + 46.9^2}$ $= 69.729$ $\tan\alpha = \frac{46.9}{51.6}$ $\alpha = 42.268$ $\therefore 51.6\cos\theta + 46.9\sin\theta = 69.7\cos(\theta - 42.3°)$
5(c)	Max value of Q = 69.7 $\cos(\theta - 42.268°) = 1$ $\theta = 42.268$ Corresponding value= 42.3°
5(d)	maximum value of $\frac{1}{Q^2+3} = \frac{1}{0+3}$ $= \frac{1}{3}$
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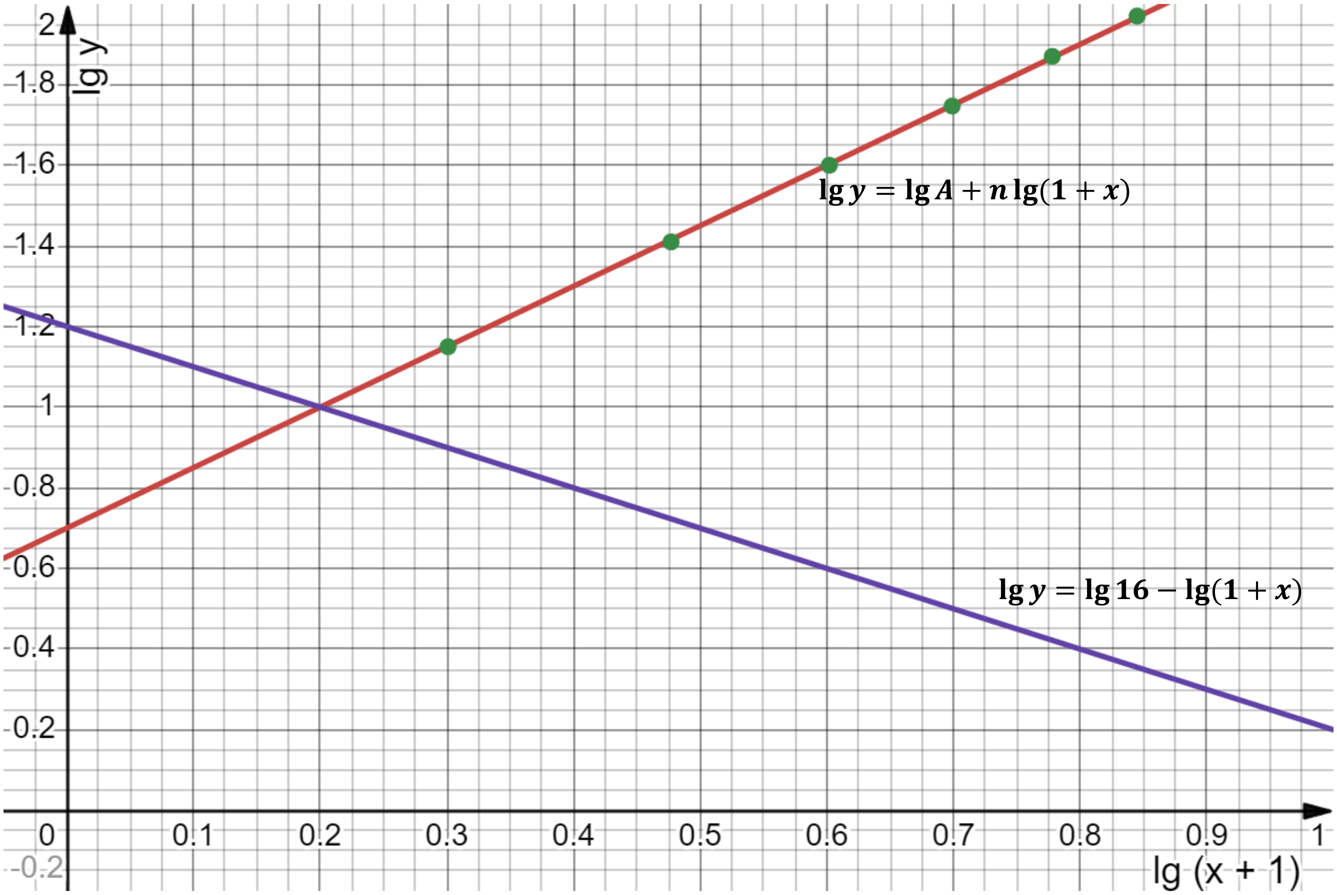
6(a)	$v = 4e^{-t} - \frac{1}{2}e^{2t}$ $a = -4e^{-t} - e^{2t}$ When $t = 0.5, a = -5.14\ m/s^2$
6(b)	$\frac{da}{dt} = 4e^{-t} - 2e^{2t}$ When $\frac{da}{dt} = 0$, $4e^{-t} = 2e^{2t}$ $e^{3t} = 2$ $t = \frac{1}{3}\ln 2$ $\frac{d^2a}{dt^2} = -4e^{-t} - 4e^{2t}$ When $t = \frac{1}{3}\ln 2$, $\frac{d^2a}{dt^2} < 0$ (max)
6(c)	When $v = 0$, $4e^{-t} = \frac{1}{2}e^{2t}$ $e^{3t} = 8$ $t = \frac{1}{3}\ln 8$ $= \ln 8^{\frac{1}{3}}$ $= \ln 2$ (shown)
6(d)	$s = -4e^{-t} - \frac{1}{4}e^{2t} + c$ When $t = 0, s = 0, \therefore c = \frac{17}{4}$ $s = -4e^{-t} - \frac{1}{4}e^{2t} + \frac{17}{4}$ When $t = 0, s = 0$ When $t = \ln 2, s = 1.25$ When $t = 3, s = -96.806$ Total distance travelled $= 1.25 + (96.806 + 1.25)$ $= 99.3m$
7(a)	$\frac{\cot A - \tan A}{\cot A + \tan A} = 2cos^2A - 1$ $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$ $= \frac{cos^2A - sin^2A}{cos^2A + sin^2A}$ $= cos^2A - sin^2A$ $= cos^2A + cos^2A - 1$ $= 2cos^2A - 1$ $= RHS$ <u>Alternative</u> $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{1}{\tan A} - \tan A}{\frac{1}{\tan A} + \tan A}$ $= \frac{1 - tan^2A}{1 + tan^2A}$ $= \frac{1 - tan^2A}{sec^2A}$ $= cos^2A - sin^2A$ $= 2cos^2A - 1$ $= RHS$

7(b)	$\frac{\cot A - \tan A}{\cot A + \tan A} = \cos A \qquad -\pi < A < \pi$ $2cos^2 A - 1 = \cos A$ $(2\cos A + 1)(\cos A - 1) = 0$ $\cos A = -\frac{1}{2} \qquad or \quad \cos A = 1$ $ref\ angle = \frac{\pi}{3} \quad or\ ref\ angle = 0$ $A = \frac{2\pi}{3}, \frac{-2\pi}{3} \qquad or \qquad A = 0$ $A = \frac{-2\pi}{3}, 0, \frac{2\pi}{3}$
8(a)	$y = \tan x$ $\frac{dy}{dx} = sec^2 x$
8(b)	When $x = \frac{\pi}{6}, \quad y = \frac{\sqrt{3}}{3}, \quad \frac{dy}{dx} = \frac{4}{3}$ $y = mx + c$ $\frac{\sqrt{3}}{3} = \frac{4}{3}\left(\frac{\pi}{6}\right) + c$ $c = \frac{\sqrt{3}}{3} - \frac{2\pi}{9}$ $y = \frac{4}{3}x + \frac{\sqrt{3}}{3} - \frac{2\pi}{9}$ Accept $\left(y = \frac{4}{3}x + \frac{3\sqrt{3}-2\pi}{9}\right)$ or $\left(9y = 12x + 3\sqrt{3} - 2\pi\right)$
8(c)	$\frac{d^2y}{dx^2} = -2cos^{-3}x(-\sin x)$ $= \frac{2\sin x}{cos^3 x}$ At $x = \frac{\pi}{6}, \frac{d^2y}{dx^2} = 1.5396$ $= 1.54\ (3s.f)$
8(d)	$\frac{dy}{dx} = sec^2 x$ When $\frac{dy}{dx} = 0, \quad \frac{1}{cos^2 x} = 0$ Since $sec^2 x = 0$ is not defined, ∴ the above conclusion is wrong.
9	$\int_0^{\frac{4}{3}} (3x^2 - 16x + 16)\,dx + \int_{\frac{4}{3}}^{4} (3x^2 - 16x + 16)\,dx$ $= [x^3 - 8x^2 + 16x]_0^{\frac{4}{3}} + [x^3 - 8x^2 + 16x]_{\frac{4}{3}}^{4}$ $= \frac{256}{27} + \left(0 - \frac{256}{27}\right)$ $= 0$ The **area above the x-axis**, bounded from $x = 0$ to $x = \frac{4}{3}$ is the **same** as the **area below the x-axis**, bounded from $x = \frac{4}{3}$ to $x = 4$.

10(a)	When $y = e,\ e = \frac{4}{x}$ $x = \frac{4}{e}$ $\therefore A\left(\frac{4}{e}, e\right)$ When x= $2e,\ y = \frac{4}{2e}$ $y = \frac{2}{e}$ $\therefore B\left(2e, \frac{2}{e}\right)$
10(b)	$Area = \int_{\frac{4}{e}}^{2e} \frac{4}{x}\,dx \ + \left(\frac{4}{e}\right)(e)$ $= 4[\ln x]_{\frac{4}{e}}^{2e} + 4$ $= 4\left[\ln 2e - \ln\frac{4}{e}\right] + 4$ $= 4[\ln 2 + \ln e - \ln 4 + \ln e] + 4$ $= 4[\ln 2 - \ln 4] + 12$ $= 4[-\ln 2] + 12$ $= 12 - \ln 16$
10(c)	Area of rect from from y-axis to A $= e \times \frac{4}{e}$ $= 4\ units^2$ Area of whole rect= $2e \times e$ $= 2e^2\ units^2$ $\therefore 4 < area\ of\ shaded\ region < 2e^2$ (explained)
11(a)	Refer to graph. $y = A(1+x)^n$ $\lg y = \lg A + n\lg(1+x)$ lg(1+x): 0.301, 0.477, 0.602, 0.699, 0.778, 0.845 lg y: 1.15, 1.41, 1.60, 1.75, 1.87, 1.97 Correct points plotted Straight line Plot table
11(b)	From the graph, $\lg A = 0.7$ [Accept $0.68 \leq \lg A \leq 0.72$] $A = 5.01$ $n = grad = 1.51$ [Accept $1.49 \leq n \leq 1.51$]
11(c)	$y = \frac{16}{1+x}$ $\lg y = \lg 16 - \lg(1+x)$ Draw the line $\lg y = \lg 16 - \lg(1+x)$
11(d)	$A(1+x)^{n+1} = 16$ $A(1+x)^n = \frac{16}{1+x}$ $\therefore \lg(x+1) = 0.2$ $x = 0.585$

Table in 11(a):

lg(1+x)	0.301	0.477	0.602	0.699	0.778	0.845
lg y	1.15	1.41	1.60	1.75	1.87	1.97


lg y
lg (x + 1)
$\lg y = \lg A + n \lg(1 + x)$
$\lg y = \lg 16 - \lg(1 + x)$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

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