ACJC 2023 FM Promotional Exam Solution

1

$$\cos(5\theta) = \mathrm{Re}\left(\cos(5\theta) + \mathrm{i}\sin(5\theta)\right)$$
$$= \mathrm{Re}\left((\cos\theta + \mathrm{i}\sin\theta)^5\right)$$
$$= \mathrm{Re}\left(\cos^5\theta + 5\mathrm{i}\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10\mathrm{i}\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta +\right.$$
$$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$
$$= \cos^5\theta - 10\cos^3\theta\left(1-\cos^2\theta\right) + 5\cos\theta\left(1-\cos^2\theta\right)^2$$
$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Let $x = \cos\theta$

$$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta - 1 = 0$$
$$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 1$$
$$\cos(5\theta) = 1$$
$$5\theta = 0, 2\pi, 4\pi, \ldots$$
$$\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$$
$$x = \cos 0, \cos\frac{2\pi}{5}, \cos\frac{4\pi}{5}, \cos\frac{6\pi}{5}, \cos\frac{8\pi}{5}$$
$$= 1,\ \cos\frac{2\pi}{5}, \cos\frac{4\pi}{5}, \cos\frac{6\pi}{5}, \cos\frac{8\pi}{5}$$

However, since $\cos\theta = \cos(2\pi - \theta)$,

we have $\cos\frac{2\pi}{5} = \cos\frac{8\pi}{5}$ and $\cos\frac{4\pi}{5} = \cos\frac{6\pi}{5}$ which are repeated.

2

Let P_n be the statement

$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\mathrm{e}^x\sin\left(\sqrt{3}x\right)\right) = 2^n\mathrm{e}^x\sin\left(\frac{n\pi}{3} + \sqrt{3}x\right)$ for all positive integers *n*.

To prove P_1 is true:

LHS:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^x\sin\left(\sqrt{3}x\right)\right) = \sqrt{3}\mathrm{e}^x\cos\left(\sqrt{3}x\right) + \mathrm{e}^x\sin\left(\sqrt{3}x\right)$$
$$= \mathrm{e}^x\left(\sqrt{3}\cos\left(\sqrt{3}x\right) + \sin\left(\sqrt{3}x\right)\right)$$

$$2\mathrm{e}^x\sin\left(\frac{\pi}{3} + \sqrt{3}x\right) = 2\mathrm{e}^x\left(\sin\frac{\pi}{3}\cos\left(\sqrt{3}x\right) + \cos\frac{\pi}{3}\sin\left(\sqrt{3}x\right)\right)$$

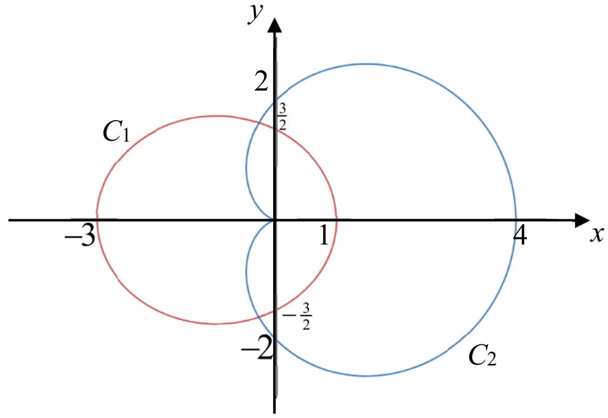
RHS:
$$= 2\mathrm{e}^x\left(\frac{\sqrt{3}}{2}\cos\left(\sqrt{3}x\right) + \frac{1}{2}\sin\left(\sqrt{3}x\right)\right)$$
$$= \mathrm{e}^x\left(\sin\left(\sqrt{3}x\right) + \sqrt{3}\cos\left(\sqrt{3}x\right)\right)$$

Since LHS = RHS, P_1 is true.

	Assume P_k is true for some positive integer k. Try to prove P_{k+1} is true. $\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left(e^x \sin\left(\sqrt{3}x\right)\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(e^x \sin\left(\sqrt{3}x\right)\right)\right)$ $=\frac{\mathrm{d}}{\mathrm{d}x}\left(2^k \mathrm{e}^x \sin\left(\frac{k\pi}{3}+\sqrt{3}x\right)\right)$ $=2^k\left(\sqrt{3}\mathrm{e}^x \cos\left(\frac{k\pi}{3}+\sqrt{3}x\right)+\mathrm{e}^x \sin\left(\frac{k\pi}{3}+\sqrt{3}x\right)\right)$ $=2^{k+1}\mathrm{e}^x\left(\frac{\sqrt{3}}{2}\cos\left(\frac{k\pi}{3}+\sqrt{3}x\right)+\frac{1}{2}\sin\left(\frac{k\pi}{3}+\sqrt{3}x\right)\right)$ $=2^{k+1}\mathrm{e}^x\left(\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{k\pi}{3}+\sqrt{3}x\right)+\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{k\pi}{3}+\sqrt{3}x\right)\right)$ $=2^{k+1}\mathrm{e}^x \sin\left(\frac{\pi}{3}+\frac{k\pi}{3}+\sqrt{3}x\right)$ $=2^{k+1}\mathrm{e}^x \sin\left(\frac{(k+1)\pi}{3}+\sqrt{3}x\right)$ Since P_1 is true and $P_k \Rightarrow P_{k+1}$, by Mathematical Induction, P_n is true for all positive integers n.
3(i)	$z^5=-1=\mathrm{e}^{i\pi}$ Let $z=\mathrm{e}^{\mathrm{i}\theta}$ $\mathrm{e}^{\mathrm{i}5\theta}=\mathrm{e}^{\mathrm{i}(2k+1)\pi}$ $5\theta=(2k+1)\pi$ $\theta=\frac{(2k+1)\pi}{5}$ $z=\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{5}},\mathrm{e}^{\pm\mathrm{i}\frac{3\pi}{5}},\mathrm{e}^{\mathrm{i}\pi}$ $=\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{5}},\mathrm{e}^{\pm\mathrm{i}\frac{3\pi}{5}},-1$
3(ii)	$\mathrm{p}(z)=\frac{1}{z^2}-\frac{1}{z}+1-z+z^2$ $=\frac{1}{z^2}\frac{\left(1-(-z)^5\right)}{1-(-z)}$ $=\frac{z^5+1}{z^2(z+1)}$

	The solutions of $\mathrm{p}(z)=0$ are the solutions of $z^5+1=0$ which are not solutions of $z+1=0$ Therefore, the solutions of $\mathrm{p}(z)=0$ are $z=\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{5}},\mathrm{e}^{\pm\mathrm{i}\frac{3\pi}{5}}$.
3(iii)	$\mathrm{p}(z)=\frac{1}{z^2}-\frac{1}{z}+1-z+z^2$ $=\left(\frac{1}{z^2}+2+z^2\right)-\left(\frac{1}{z}+z\right)-1$ $=w^2-w-1$ $w^2-w-1=0$ $w=\frac{1\pm\sqrt{5}}{2}$
3(iv)	If $z=\mathrm{e}^{\mathrm{i}\frac{\pi}{5}}$ then $\frac{1}{z}=\mathrm{e}^{-\mathrm{i}\frac{\pi}{5}}$ so $w=2\cos\frac{\pi}{5}$. Similarly, if $z=\mathrm{e}^{\mathrm{i}\frac{3\pi}{5}}$ then $\frac{1}{z}=\mathrm{e}^{-\mathrm{i}\frac{3\pi}{5}}$ so $w=2\cos\frac{3\pi}{5}$. Since $0<\frac{\pi}{5}<\frac{\pi}{2}$, $\cos\frac{\pi}{5}>0$ so $2\cos\frac{\pi}{5}=\frac{1+\sqrt{5}}{2}\Rightarrow\cos\frac{\pi}{5}=\frac{1+\sqrt{5}}{4}$ and $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$
4(i)	$3(x+1)^2+4y^2=12$ $3(r\cos\theta+1)^2+4r^2\sin^2\theta=12$ $3r^2\cos^2\theta+6r\cos\theta+3+4r^2\left(1-\cos^2\theta\right)=12$ $r^2\left(4-\cos^2\theta\right)+6r\cos\theta=9$ $r^2+\frac{6r\cos\theta}{4-\cos^2\theta}=\frac{9}{4-\cos^2\theta}$ $\left(r+\frac{3\cos\theta}{4-\cos^2\theta}\right)^2-\frac{9\cos^2\theta}{\left(4-\cos^2\theta\right)^2}=\frac{9}{4-\cos^2\theta}$ $\left(r+\frac{3\cos\theta}{4-\cos^2\theta}\right)^2=\frac{9\left(4-\cos^2\theta\right)+9\cos^2\theta}{\left(4-\cos^2\theta\right)^2}$ $r+\frac{3\cos\theta}{4-\cos^2\theta}=\pm\frac{6}{4-\cos^2\theta}$ $r=\frac{6-3\cos\theta}{4-\cos^2\theta}$ or $r=-\frac{6+3\cos\theta}{4-\cos^2\theta}$ $\left(\text{rejected}\because r>0\right)$ $\therefore r=\frac{6-3\cos\theta}{4-\cos^2\theta}$

4(ii)

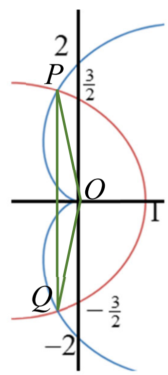


4(iii)

To find points of intersection, let

$$\frac{6-3\cos\theta}{4-\cos^2\theta}=2+2\cos\theta$$

$$\frac{6-3\cos\theta}{4-\cos^2\theta}-2-2\cos\theta=0$$



Using GC,

$\theta=1.7489$ or $\theta=4.5343$

Sub. $\theta=1.7489$ into equation of C_2:

$OP=2+2\cos(1.7489)=1.6457$

Area of triangle *OPQ*

$$=\frac{1}{2}(OP)(OQ)\sin\angle QOP$$

$$=\frac{1}{2}(1.6457)^2\sin(4.5343-1.7489)$$

$=0.472$ units^2 (3 s.f.)

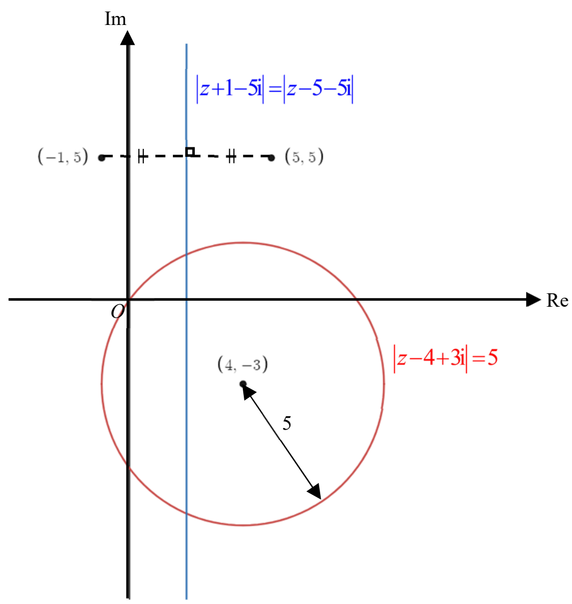
5(i)

Auxiliary equation: $m^2-2\sqrt{3}m+4=0$

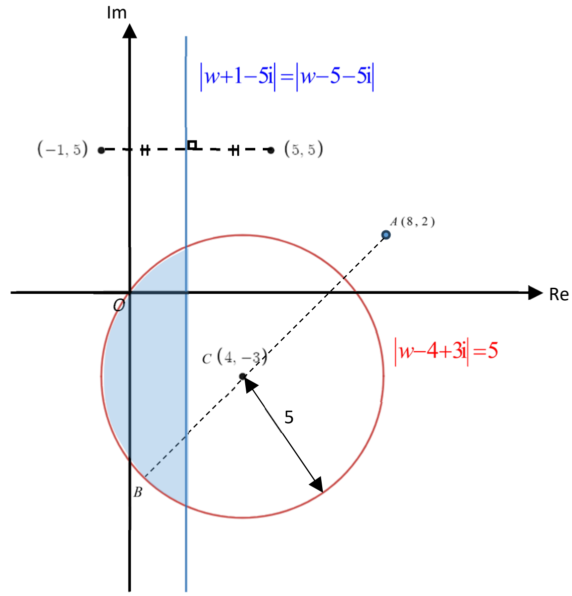
Roots are $m=\sqrt{3}\pm\mathrm{i}=2\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{6}}$

Hence solutions are

	$x_n = X\left(2e^{\pm i\frac{\pi}{6}}\right)^n + Y\left(2e^{\pm i\frac{\pi}{6}}\right)^n$ $= 2^n\left(Xe^{i\frac{n\pi}{6}} + Ye^{-i\frac{n\pi}{6}}\right)$ $= 2^n\left(X\left(\cos\left(\frac{n\pi}{6}\right) + i\sin\left(\frac{n\pi}{6}\right)\right) + Y\left(\cos\left(\frac{n\pi}{6}\right) - i\sin\left(\frac{n\pi}{6}\right)\right)\right)$ $= 2^n\left((X+Y)\cos\left(\frac{n\pi}{6}\right) + (X-Y)\left(i\sin\left(\frac{n\pi}{6}\right)\right)\right)$ $= 2^n\left(A\cos\left(\frac{n\pi}{6}\right) + B\sin\left(\frac{n\pi}{6}\right)\right)$ Where $A = X+Y$, $B = (X-Y)i$
5(ii)	**Method 1** $x_{n+6} = 2^{n+6}\left(A\cos\left(\frac{(n+6)\pi}{6}\right) + B\sin\left(\frac{(n+6)\pi}{6}\right)\right)$ $= 2^{n+6}\left(A\cos\left(\frac{n\pi}{6}+\pi\right) + B\sin\left(\frac{n\pi}{6}+\pi\right)\right)$ $= 2^{n+6}\left(-A\cos\left(\frac{n\pi}{6}\right) - B\sin\left(\frac{n\pi}{6}\right)\right)$ $= (-2^6)2^n\left(A\cos\left(\frac{n\pi}{6}\right) + B\sin\left(\frac{n\pi}{6}\right)\right)$ $= -64x_n$ **Method 2** $x_{n+6} = 2\sqrt{3}x_{n+5} - 4x_{n+4}$ $= 2\sqrt{3}\left(2\sqrt{3}x_{n+4} - 4x_{n+3}\right) - 4\left(2\sqrt{3}x_{n+3} - 4x_{n+2}\right)$ $= 12x_{n+4} - 16\sqrt{3}x_{n+3} + 16x_{n+2}$ $= 12\left(2\sqrt{3}x_{n+3} - 4x_{n+2}\right) - 16\sqrt{3}\left(2\sqrt{3}x_{n+2} - 4x_{n+1}\right) + 16\left(2\sqrt{3}x_{n+1} - 4x_n\right)$ $= 24\sqrt{3}x_{n+3} - 144x_{n+2} - 32\sqrt{3}x_{n+1} - 64x_n$ $= 24\sqrt{3}\left(2\sqrt{3}x_{n+2} - 4x_{n+1}\right) - 144x_{n+2} + 96\sqrt{3}x_{n+1} - 64x_n$ $= 144x_{n+2} - 96\sqrt{3}x_{n+1} - 144x_{n+2} + 96\sqrt{3}x_{n+1} - 64x_n$ $= -64x_n$
5(iii)	When $n=0$: $A = \frac{\sqrt{3}}{2}$ $n=1$: $2\left(\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + B\frac{1}{2}\right) = 1 \Rightarrow B = -\frac{1}{2}$
6(a)	$\lvert z-4+3i\rvert = 5$ represents a circle centred at $(4,-3)$ and has a radius of 5 units.

	$\lvert z+1-5\mathrm{i}\rvert = \lvert z-5-5\mathrm{i}\rvert$ represents a perpendicular bisector of the line segment joining the points $(-1,5)$ and $(5,5)$.  Cartesian equation of circle is $(x-4)^2+(y+3)^2=25$ ---(1) Cartesian equation of perpendicular bisector is $x=2$ ---(2) Sub. (2) into (1): $(2-4)^2+(y+3)^2=25$ $y^2+6y-12=0$ $\therefore y=\dfrac{-6\pm\sqrt{36-4(1)(-12)}}{2}=-3\pm\sqrt{21}$ The complex numbers are $2-\left(3+\sqrt{21}\right)\mathrm{i}$ and $2+\left(\sqrt{21}-3\right)\mathrm{i}$.

6(b)(i)



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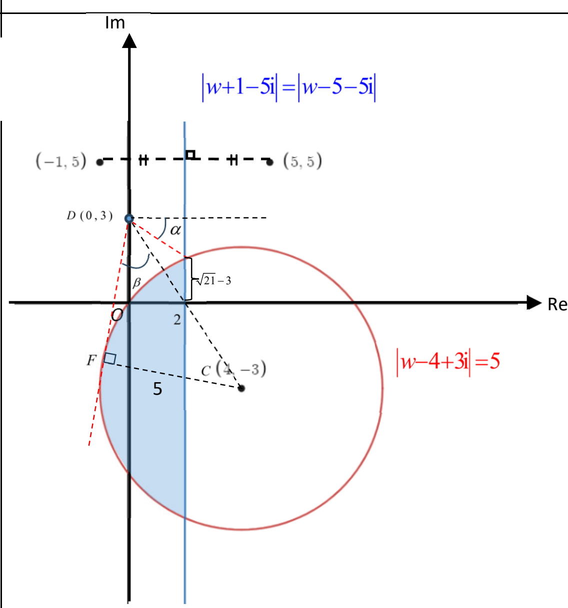
greatest value of $|w-8-2\mathrm{i}| = AB$

$= AC + 5$

$= \sqrt{(8-4)^2 + (2-(-3))^2} + 5$

$= \left(\sqrt{41} + 5\right)$ units (or 11.4 units)

6(b)(ii)



From diagram,

$$\tan\alpha = \frac{3-\left(\sqrt{21}-3\right)}{2} = \frac{6-\sqrt{21}}{2} \Rightarrow \alpha = 0.61655 \text{ rad}$$

	$CD=\sqrt{(4-0)^2+(-3-3)^2}=\sqrt{52}$ units $\sin\beta=\frac{CF}{CD}=\frac{5}{\sqrt{52}}\Rightarrow\beta=0.76616$ rad $\therefore -\left(\beta+\tan^{-1}\frac{3}{2}\right)\le\arg\left(w-3\text{i}\right)\le-\alpha$ i.e. $-1.75\le\arg\left(w-3\text{i}\right)\le-0.617$ (3 s.f.)
7(i)	Amount of salt in the beaker after 10 g of salt is added is $u_{n-1}+10$. After it is mixed and k ml is removed, there is $\left(\frac{100-k}{100}\right)\left(u_{n-1}+10\right)$ Adding k ml of pure water does not change the amount of salt. Hence, $u_n=\left(\frac{100-k}{100}\right)\left(u_{n-1}+10\right)$.
7(ii)	$u_n=\left(1-\frac{k}{100}\right)\left(u_{n-1}+10\right)$ $\quad=\left(1-\frac{k}{100}\right)u_{n-1}+\left(10-\frac{k}{10}\right)$ Let $p=1-\frac{k}{100}$ $u_n=pu_{n-1}+10p$ $\quad=p\left(pu_{n-2}+10p\right)+10p$ $\quad=p^2u_{n-2}+10p\left(1+p\right)$ $\quad=p^2\left(pu_{n-3}+10p\right)+10p\left(1+p\right)$ $\quad=p^3u_{n-2}+10p\left(1+p+p^2\right)$ $\quad\vdots$ $\quad=p^nu_0+10p\left(1+p+p^2+\ldots+p^{n-1}\right)$ $\quad=10p\frac{1-p^n}{1-p}$ $\quad=\left(10-\frac{1}{10}k\right)\frac{1-\left(1-\frac{1}{100}k\right)^n}{1-\left(1-\frac{1}{100}k\right)}$ $\quad=\left(10-\frac{1}{10}k\right)\frac{100}{k}\left[1-\left(1-\frac{1}{100}k\right)^n\right]$ $\quad=\left(\frac{1000}{k}-10\right)\left[1-\left(1-\frac{1}{100}k\right)^n\right]$
7(iii)	**Method 1** As $n\to\infty$, $u_n\to L$ and $u_{n-1}\to L$

	$L=(0.9)(L+10)$ $0.1L=9$ $L=90$ **Method 2** $u_n=90\left[1-(0.9)^n\right]$ As $n\to\infty$, $(0.9)^n\to 0$ $u_n\to 90$
7(iv)	$\lvert u_n-90\rvert<9$ $\left\lvert 90\left[1-(0.9)^n\right]-90\right\rvert<9$ $90(0.9)^n<9$ $(0.9)^n<0.1$ $n>\dfrac{\ln 0.1}{\ln 0.9}=21.8$ Smallest n is 22.
7(v)	$u_n=\left(\dfrac{1000}{k}-10\right)\left[1-\left(1-\dfrac{k}{100}\right)^n\right]$ As $n\to\infty$, $\left(1-\dfrac{k}{100}\right)^n\to 0$ $u_n\to\dfrac{1000}{k}-10$ $\dfrac{1000}{k}-10<50$ $k>\dfrac{1000}{60}$ Hence the range of values is $\dfrac{50}{3}<k<100$.
8(i)	[Diagram] Light ray coming from *Y* hits mirror at *A*, reflects and intersects *y*-axis at *S*. Let *R* be at $(0,r)$ and $\angle RAY=\theta$. Since *RA* is a radius of the circle, $\sin\theta=\dfrac{k}{r}$. *RA* is perpendicular to the tangent at *A*, so $\angle RAS=\angle RAY=\theta$ By alternate angles, $\angle SRA=\angle RAY=\theta$. Therefore, ΔRAS is an isosceles triangle with $RS=AS$. By drawing the perpendicular from *S* to *RA*, we see that $\cos\theta=\dfrac{r}{2(SR)}$ so $SR=\dfrac{r}{2\cos\theta}$. Therefore, the required distance *OS* is

	$OS = OR - RS$ $= r\left(1 - \frac{1}{2\cos\theta}\right)$ $= r\left(1 - \frac{1}{2\sqrt{1-\sin^2\theta}}\right)$ $= r\left(1 - \frac{1}{2\sqrt{1-\frac{k^2}{r^2}}}\right)$
8(ii)	$(y-r)^2 = r^2 - x^2$ $y - r = \pm\sqrt{r^2 - x^2}$ $y = r \pm \sqrt{r^2 - x^2}$ $y = r - \sqrt{r^2 - x^2}$ Because the part of the cross-section in the neighbourhood of the origin is where $y < r$. $y = r - \left(r^2 - x^2\right)^{\frac{1}{2}}$ $= r - r\left(1 - \frac{x^2}{r^2}\right)^{\frac{1}{2}}$ $= r - r\left(1 - \frac{x^2}{2r^2} + \dots\right)$ $= \frac{x^2}{2r} + \dots$
8(iii)	Compare the equation $y = \frac{x^2}{2r}$ with $x^2 = 4ay$ $4a = 2r \Rightarrow a = \frac{r}{2}$ The focus is at $(0, a) = \left(0, \frac{r}{2}\right)$.
8(iv)	Using the answer from (i), $r\left[1 - \frac{1}{2}\left(1 - \frac{k^2}{r^2}\right)^{-\frac{1}{2}}\right]$ $= r\left[1 - \frac{1}{2}\left(1 + \frac{k^2}{2r^2} + \dots\right)\right]$ $= r\left(\frac{1}{2} - \frac{k^2}{4r^2} - \dots\right)$ $\approx \frac{r}{2}$

	if k is small enough that powers of $\left(\frac{k}{r}\right)$ higher than the second power can be ignored. This agrees with the results from (ii) and (iii), where the approximation of the parabola is valid for small values if x, where powers of x higher than the third can be ignored. Therefore, the approximation is suitable as long as the light rays are not too far away from the y-axis / the diameter of the cross-section.
9(i)	Gradient of PQ: $\frac{2ap-2aq}{ap^2-aq^2}=\frac{2a(p-q)}{a(p+q)(p-q)}=\frac{2}{p+q}$ Gradient of tangent at $(ar^2, 2ar)$: $y(2ar)=2a(x+ar^2)$ so the gradient is $\frac{2a}{2ar}=\frac{1}{r}$. Therefore, $\frac{2}{p+q}=\frac{1}{r}\Rightarrow r=\frac{p+q}{2}$.
9(ii)	Similar to (i), $s=\frac{p+r}{2}=\frac{3p+q}{4}$. Equation of line ST is $y=2as=2a\left(\frac{3p+q}{4}\right)=a\left(\frac{3p+q}{2}\right)$ Equation of line PQ is $\frac{y-2ap}{x-ap^2}=\frac{2}{p+q}$ $y-2ap=\frac{2(x-ap^2)}{p+q}$ $y=\frac{2(x-ap^2)}{p+q}+2ap$ Substitute $y=a\left(\frac{3p+q}{2}\right)$: $a\left(\frac{3p+q}{2}\right)=\frac{2(x-ap^2)}{p+q}+2ap$ $\frac{a}{2}(q-p)=\frac{2(x-ap^2)}{p+q}$ $x-ap^2=\frac{a(p+q)(q-p)}{4}$ $x=\frac{a(q^2-p^2)}{4}+ap^2$ $=\frac{a(3p^2+q^2)}{4}$

	Hence the coordinates of T are $\left(\frac{a}{4}\left(3p^2+q^2\right),\frac{a}{2}\left(3p+q\right)\right)$
9(iii)	Equation of line PR is $\frac{y-2ap}{x-ap^2}=\frac{2ar-2ap}{ar^2-ap^2}=\frac{2}{r+p}=\frac{4}{3p+q}$ $y-2ap=\frac{4\left(x-ap^2\right)}{3p+q}$ $y=\frac{4\left(x-ap^2\right)}{3p+q}+2ap$ Substitute $y=a\left(\frac{3p+q}{2}\right)$: $a\left(\frac{3p+q}{2}\right)=\frac{4\left(x-ap^2\right)}{3p+q}+2ap$ $\frac{a}{2}\left(q-p\right)=\frac{4\left(x-ap^2\right)}{3p+q}$ $x-ap^2=\frac{a\left(3p+q\right)\left(q-p\right)}{8}$ $x=\frac{a\left(3p+q\right)\left(q-p\right)}{8}+ap^2$ $=\frac{a\left(5p^2+2pq+q^2\right)}{8}$ Hence, the coordinates of X are $\left(\frac{a}{8}\left(5p^2+2pq+q^2\right),\frac{a}{2}\left(3p+q\right)\right)$ $SX=\frac{a}{8}\left(5p^2+2pq+q^2\right)-as^2$ $=\frac{a}{8}\left(5p^2+2pq+q^2\right)-a\left(\frac{3p+q}{4}\right)^2$ $=\frac{a}{16}\left(p^2-2pq+q^2\right)$ $XT=\frac{a}{4}\left(3p^2+q^2\right)-\frac{a}{8}\left(5p^2+2pq+q^2\right)$ $=\frac{a}{8}\left(p^2-2pq+q^2\right)$ $=2SX$
9(iv)	Since $XT=2SX$

	Area of ΔPXT = 2(Area of ΔPSX) = Area of ΔPRS Area of ΔPQR = 2(Area of ΔPRY) = 2(4(Area of ΔPXT)) = 8(Area of ΔPRS) Hence the ratio is 8:1
9(v)	Let the area of ΔPQR be A. Area of region bounded by parabola and PQ $= A+2\left(\frac{A}{8}\right)+4\left(\frac{A}{8^2}\right)+8\left(\frac{A}{8^3}\right)+\ldots$ $= A\left(1+\frac{2}{8}+\frac{2^2}{8^2}+\ldots\right)$ $= A\left(\frac{1}{1-\frac{2}{8}}\right)$ $=\frac{4}{3}A$