VICTORIA JUNIOR COLLEGE
Preliminary Examination

MATHEMATICS
(Higher 2)

9740/02

Paper 2

September 2015

3 hours

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 5 printed pages

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Section A: Pure Mathematics [40 marks]

1 It is given that $e^y = \frac{1}{2} + \sin 2x$.

(i) Show that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 4 = 2e^{-y}$. [2]

(ii) By repeated differentiation of the result in part **(i)**, find the first four non–zero terms of the Maclaurin series for y, giving the coefficients in exact form. [4]

(iii) Show that the same result in part **(ii)** can be obtained using the standard results given in the List of Formulae (MF15). [3]

2 Relative to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{a}+\mathbf{c}$ and $\mathbf{c}$ respectively. The point X is on AC produced such that $AC : AX$ is $1:4$ and the point Y is such that $AXYB$ is a parallelogram.

(i) The lines AY and BX intersect at the point N. Find, in terms of $\mathbf{a}$ and $\mathbf{c}$, the position vector of N. [2]

(ii) Given that the area of triangle OAB is 2 square units, find the area of triangle AXB. [3]

(iii) Give the geometrical interpretation of $\left|\overrightarrow{AX} \times \frac{\overrightarrow{BX}}{\left|\overrightarrow{BX}\right|}\right|$. Using the results from part **(ii)**, show that $\left|\overrightarrow{AX} \times \frac{\overrightarrow{BX}}{\left|\overrightarrow{BX}\right|}\right| = \frac{k}{|m\mathbf{c} - n\mathbf{a}|}$, where k, m and n are constants to be determined. [4]

3 The complex number z satisfies the equation $|z - 5 + 7i| = 6$.

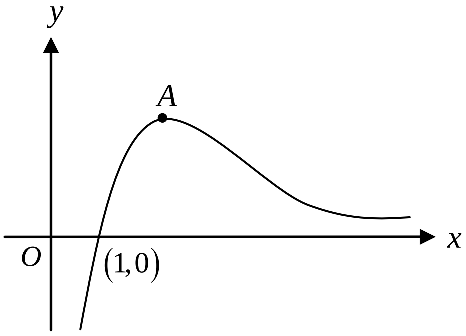
(i) Show the locus of z on an Argand diagram. [2]

(ii) If the locus in part **(i)** intersects the locus $\arg(z - a - 2i) = -\frac{\pi}{2}$ at two distinct points, where a is a real number, find the set of values that a can take. [2]

(iii) Given that the locus in part **(i)** intersects the locus $|z + 2 + 4i| = k$ at exactly one point, find two possible exact values of k. [3]

Using one value of k found, find exactly the value of z represented by the point of intersection, giving your answer in the form $x + iy$. [3]

4 The diagram below shows the curve with equation $y=\frac{\ln x}{x^2}$, $x>0$. The curve cuts the x-axis at $(1,0)$ and has a maximum point at A.



(i) Find the exact coordinates of A. [3]

(ii) Without using a calculator, find the exact area of the finite region bounded by the curve, the x-axis and the line $x=2$. [4]

(iii) Find the volume of the solid generated when the region bounded by the curve, the tangent at A and the line $x=1$ is rotated completely about the x-axis, giving your answer correct to 3 significant figures. [3]

(iv) The geometric series S is defined by

$$1+\frac{2e\ln x}{x^2}+\frac{4e^2(\ln x)^2}{x^4}+\frac{8e^3(\ln x)^3}{x^6}+\dots.$$

A student claims that, if the value of x is larger than 1, then the sum to infinity of S exists. State, with a reason, whether you agree with him. [2]

Section B: Statistics [60 marks]

5 A college has 540 students in Year One and 660 students in Year Two. The college intends to carry out a survey to investigate students' opinions about the gymnasium facilities available at the college.

(i) Describe how to obtain a stratified random sample of 60 students to take part in the survey. [2]

(ii) State how a better stratified random sample of size 60 could have been achieved. [1]

6 A group of 12 people consists of 6 married couples.

(i) The 12 people are to be seated randomly at a round table. Find the number of ways in which the 12 people can be arranged if each married couple is seated together. [2]

(ii) The group is going on a flight and is assigned to sit in three distinct rows of four seats each. Find the number of ways in which the 12 people can be arranged if each row has at least 1 woman. [5]

[Turn over

7 A game is to be played between two players, A and B. A bag contains 5 balls each with A's name and 8 balls each with B's name. Starting from A, the players will take turn to pick 3 balls randomly in a single draw from the bag, note down the names and return all 3 balls to the bag. Assuming that the balls are identical in size, the winner is the first player to get 3 balls of the player's name in a single draw.

(i) Find the probability that A wins the game. [4]

(ii) Find the probability that A wins on A's first draw given that A wins the game. [3]

Suppose A eventually wins the game on A's 4^{th} turn. Let (a_1, a_2, a_3, a_4) denote A's draw sequence where a_i denote the number of balls with A's name picked by A on A's i^{th} turn.
Find the total number of possible draw sequence for A. [2]

8 Each night in the month of August, Amy observes the number of meteors from the telescope set up in her laboratory. Amy observes meteors at an average rate of 2 per minute.

(i) State two conditions needed for the number of meteors observed in a randomly chosen period of 1 minute to be well modelled by a Poisson distribution. [2]

Assume that the conditions in **(i)** are satisfied.

(ii) Find the probability that Amy observes exactly 6 meteors in a randomly chosen period of 4 minutes. [1]

(iii) Find the probability that Amy observes exactly 3 meteors in each of the two successive 2-minute intervals. [2]

(iv) Explain why the answer to part **(ii)** is greater than the answer to part **(iii)**. [1]

(v) In a randomly selected period of n minutes (where $n > 10$), the probability that Amy sees more than $3n$ meteors is less than 0.005. Using a suitable approximation, determine an inequality satisfied by n and, hence, find algebraically, the smallest possible integer value of n. [5]

9 Past records shows that female students in tertiary institutions have a mean height of 162 cm. A random sample of 150 female students is taken from a particular institution and the height, x cm, of each female student is measured. The results are summarised by

$$\sum(x-160) = 480, \qquad \sum(x-160)^2 = 8837.$$

Test, at the 5% significance level, whether the mean height of 162 cm is an understated value for this institution. [5]

Explain the meaning of 5% significance level in the context of this question. [1]

Two tests, each with a sample size of 10 taken from this particular institution, are carried out with the same hypotheses as above. Assume that the unbiased estimates of the population variance are the same for both the samples.

(i) What change would there be in carrying out the two tests as compared to the previous test? State whether any assumption is needed for the two tests to be valid. [2]

(ii) The first test has a sample mean height of m cm. Based on this test, the null hypothesis is not rejected at the 5% significance level. The second test has a sample mean height less than m cm. Determine with a reason, whether the second test will yield the same conclusion as the first test. [2]

10 The following table shows the population, y (in millions) of a certain country in year, t

Year , t	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990
Population, y (in millions)	13.5	15.1	17.7	20.9	24.7	29.2	34.5	40.8	48.2	56.9

(i) Draw the scatter diagram to illustrate the relationship between y and t. [1]

(ii) Calculate the product moment correlation coefficient between t and y, giving your answer to 5 decimal places. Explain why its value does not necessarily mean that the best model for the relationship between t and y is $y = c + dt$. [2]

(iii) It is known that if environmental factors remain constant, the relation $y = ae^{bt}$, where a and b are constants gives a reasonable model for this data.

(a) Express the given relation in a linear equation of the form $Y = A + Bt$. [1]

(b) Calculate the product moment correlation between t and Y, giving your answer to 5 decimal places. Explain how to use this value and the value calculated in part **(ii)** to decide, for this data, whether $y = c + dt$ or $Y = A + Bt$ is the better model. [2]

(c) Find the estimated regression line of Y on t and use it to estimate the population in the year 2020. Comment on the reliability of your estimation. [3]

11 [In this question you should state clearly the values of the parameters of any normal distribution you use.]

A supermarket sells 2 different types of apples – Granny Smith and Fuji. The masses, in grams, of each type of apples follow normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (g)	Standard deviation (g)
Granny Smith	150	11.7
Fuji	180	15.2

(i) Two Fuji apples are chosen. Find the probability that their masses differs by at least 20 g. [3]

(ii) Granny Smith apples are sold at \$3.50 per kg and Fuji apples at \$5 per kg.
Find the probability that six Granny Smith apples and four Fuji apples cost more than \$6.50. [3]

(iii) State an assumption needed for your calculations in parts **(i)** and **(ii)**. [1]

(iv) Some Granny Smith apples are packed in 6, at random, into bags with negligible mass. Fifty-five such bags are randomly selected. By using a suitable approximation, find the probability that there are less than 48 bags whereby each bag has a mass less than 950 g. [4]