JURONG JUNIOR COLLEGE

Preliminary Examinations

MATHEMATICS
Higher 2

9740 / 2

31 August 2016

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together, with the cover page in front.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

[Turn over

Section A: Pure Mathematics [40 marks]

1 **(a)** Given that x and y are related by $\frac{dy}{dx} = \sec^2 y$ and that $y = 0$ when $x = 1$, find x in terms of y. [4]

(b) A medical researcher is investigating the rate of spread of a virus in a group of people of size n at time t weeks. He suggests that n and t are related by the differential equation $\frac{d^2n}{dt^2} = e^{-\frac{t}{5}}$.

(i) Find the general solution of the differential equation, giving your answer in the form $n = f(t)$. [2]

(ii) Explain why all solution curves of the differential equation are concave upwards. [1]

(iii) It is given that initially, the number of people infected with the virus is 50. Sketch on a single diagram, two distinct solution curves for the differential equation to illustrate the following two cases for large values of t :

I. the population of infected people increases indefinitely,

II. the population of infected people stabilizes at a certain positive number. [3]

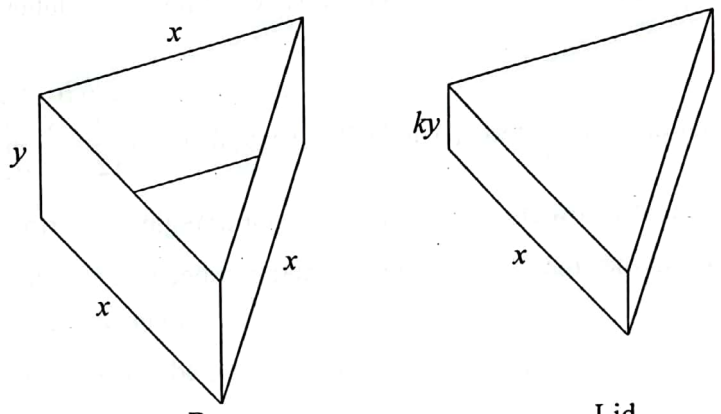
2 **(a)** A parallelogram has two adjacent sides defined by the vectors $\mathbf{a}$ and $2\mathbf{a} + 3\mathbf{b}$. Given that the magnitudes of $\mathbf{a}$ and $\mathbf{b}$ are 4 and 5 respectively and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $30°$, find the area of the parallelogram. [4]

(b) A point P has coordinates $(2, -1, -2)$ and a line l has equation $\frac{x-1}{2} = 1 - z, \quad y = 3$.

(i) Find the perpendicular distance from P to l. [4]

(ii) Find the acute angle between l and the line L that is parallel to the z-axis. [2]

3 A box with volume 250 cm^3 is made of cardboard of negligible thickness. It has a height of y cm and an equilateral triangular base of side x cm. Its lid has depth ky cm, where $0 < k \leq 1$ (see diagram).



Box Lid

(i) Show that the total external surface area of the box and lid can be expressed as

$$\frac{1000\sqrt{3}(1+k)}{x}+\frac{\sqrt{3}}{2}x^2.$$ [4]

(ii) Use differentiation to find, in terms of k, the value of x that gives a minimum total external surface area of the box and lid. [3]

(iii) Find the ratio $\frac{y}{x}$ in this case, in terms of k, simplifying your answer. [2]

(iv) Find the values for which $\frac{y}{x}$ must lie. [2]

[Turn over

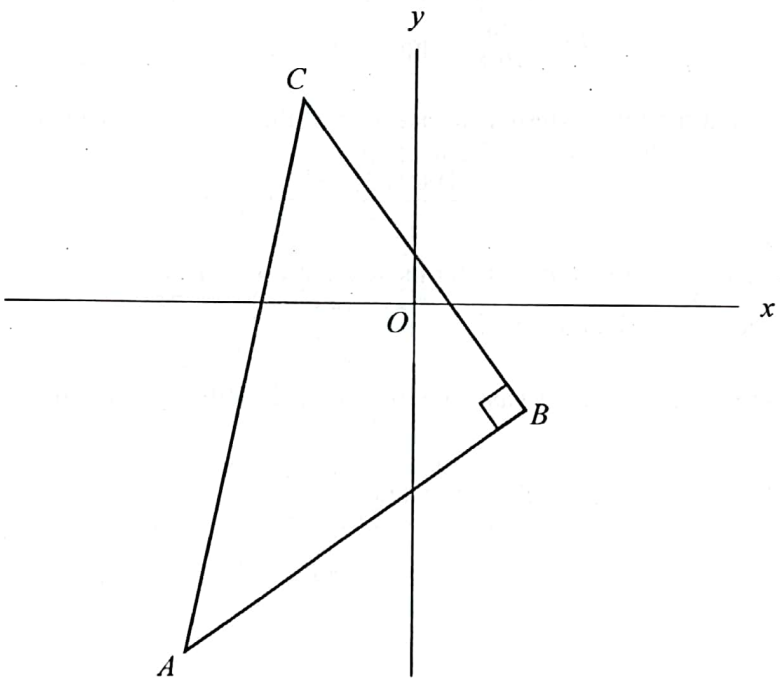
4 The complex numbers a and b are given by $a = -(1+\sqrt{3}\mathrm{i})$ and $b = \frac{1}{2}(1-\mathrm{i})$.

(i) Without using a calculator, find the value of a^2b in the form $x+\mathrm{i}y$. [2]

(ii) By using the moduli and arguments of a and b, find the modulus and argument of a^2b. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to show that $\sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$. [2]

(iv) The diagram below shows an isosceles right triangle ABC, where the points A, B and C represent the complex numbers a, b and c respectively. Find the exact value of c. [2]



Section B: Statistics [60 marks]

5 A group of 11 people consists of 6 men and 5 women, 3 of whom are sisters. A committee consisting of six people is to be selected. Find the number of ways the committee can be formed if

(i) it consists of exactly two men, [1]

(ii) it includes at least one of the sisters. [2]

Given that the chosen committee consists of 2 sisters, Sue and Suzy, together with 3 other men, Muthu, Mark, Michael and 1 other woman, Wina. They are seated at a round table meant for six people. Find the number of possible arrangements if

(iii) one of the men is to be seated between the two sisters, [2]

(iv) the two sisters are sitting directly opposite each other. [2]

6 The table below shows the number of male and female students studying Chemistry, Physics and Biology at a private school.

	Chemistry	Physics	Biology
Male	200	130	70
Female	250	300	50

One of the students is chosen at random. Events C, B and M are defined as follows:

C : The student chosen is studying Chemistry.

B : The student chosen is studying Biology.

M : The student chosen is a male.

Find

(i) $\mathrm{P}(C \mid M)$, [1]

(ii) $\mathrm{P}(M \cup C)$, [1]

(iii) $\mathrm{P}(M' \cap B')$. [1]

Determine whether C and M are independent. [2]

It is given that 20% of Chemistry students, 30% of Physics students and 5% of Biology students are international students.

(iv) One of the students selected at random is an international student. What is the probability that this student studies Chemistry? [2]

(v) Three students are chosen at random. Find the probability that there is exactly one international student who studies Physics. [2]

[Turn over

7 In order to investigate whether there is a correlation between rainfall and crop yields, the total rainfall, x mm, and the weights of a particular crop per square metre, y kg, were recorded in a number of fields. The data are shown below.

x	36	72	44	74	64	50
y	2.2	8.4	1.8	7.4	4.3	2.2

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between x and y is $y = a + bx$. [2]

(iii) By comparing the product moment correlation coefficients, explain whether $y = a + bx$ or $y = c + dx^2$ is a better model. [2]

(iv) Using a suitable regression line, estimate the yield of crop per square metre when the total rainfall is 55mm. Comment on the reliability of your estimation. [3]

8 It is known that 8% of the population of a large city use a particular web browser called Voyager. A researcher wishes to interview people from the city who use Voyager and selects people at random, one at a time.

(i) Find the probability that the first person that he finds uses Voyager is the third person selected. [2]

A random sample of n people is now selected.

(ii) State two conditions needed for the number of people in the sample who use Voyager

to be well modelled by binomial distribution. [2]

(iii) Given that $n = 80$, use a suitable approximation to find the probability that, fewer than 10 people use Voyager. [3]

(iv) Find the least value of n such that the probability of at least 10 people use Voyager is more than 0.2. [3]

9 A supermarket sells boxes of a particular brand of biscuits in two flavours, chocolate and strawberry. The mean number of boxes of chocolate biscuits sold in a day is 2.2.

(i) Find the probability that in a day, no boxes of chocolate biscuits were sold. [1]

(ii) In a week of 7 days, find the expected number of days that no boxes of chocolate biscuits were sold. [2]

The mean number of boxes of strawberry biscuits sold in a day is denoted by λ.

(iii) Given that the probability of less than 2 boxes of strawberry biscuits sold in a day is 0.6, write down an equation for the value of λ, and find λ numerically, correct to 1 decimal place. [3]

(iv) Find the probability that in a week of 7 days, the total number of boxes of chocolate and strawberry biscuits sold exceeds 25 boxes. [2]

(v) Use a suitable approximation to find the probability that, in a month of 30 days, the number of boxes of chocolate biscuits sold is more than the number of boxes of strawberry biscuits. [4]

10 A researcher is running a trial of a new variety of potato. A field contains 20 rows of the new variety of potato plants, with 80 plants in each row. A researcher intends to dig up 8 plants and measure the mass of potatoes produced by each plant.

(i) Describe how he could choose a systematic sample of 8 plants from a single row of 80 plants and state the advantage of this sampling method. [3]

The researcher claims that the average mass of the new variety of potato is at least 150g. The mass of a new variety of potato is denoted by X grams. The masses of a random sample of 80 new variety potatoes are summarized by

$$\sum(x-150) = -160, \quad \sum(x-150)^2 = 5520.$$

(ii) Calculate the unbiased estimates of the population mean and variance. [2]

(iii) Test at the 1% significance level, whether the researcher's claim is valid. [4]

(iv) Explain what you understand by the phrase "at the 1% significance level" in the context of this question. [1]

Another random sample of 8 potatoes was chosen with mean mass 148.5g and standard deviation k g. Find the range of values that k can take such that at 1% level of significance, this sample would indicate that the researcher's claim is invalid. [3]

BLANK PAGE