Name:	Index No.:	Class:

# **PRESBYTERIAN HIGH SCHOOL**



Tuesday

### MATHEMATICS PAPER TWO

4045/02

2 hours

26 July 2022

PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL

# MARK SCHEME

### Section A (52 marks)

Answer **all** the questions in this section.

1 The stem-and-leaf diagram shows the weight, in kilogram, of 20 students.

4	0	1	5	7	9	9			
5	0	1	1	1	4	5	9	9	
6	0	3	6	6					
7	1	2							

Key: 4 | 0 means 40

Find

(a) the number of students with weight more than 59,

Answer 6 students [B1] [1]

(b) the median weight.

median 
$$=\frac{51+54}{2}=52.5$$

Answer 52.5 kg [B1] [1]

2 The amounts of flour, butter and sugar required to bake a cookie are in the ratio 3 : 2 : 1. The total weight of all three ingredients is 450 grams. Find the amount of each ingredient used.

$flour = \frac{3}{6} \times 450 = 225$				
$butter = \frac{2}{6} \times 450 = 150$				
$sugar = \frac{1}{6} \times 450 = 75$				
	Answer	flour:	225 g [B1]	
		butter:	150 g [B1]	
		sugar:	75 g [B1]	[3]

3 (a) Calculate

(i) the exact value of 
$$\frac{9.3 \times 4.62}{5.7 - 2.1}$$

Answer 11.935 [B1] [1]

(ii) 
$$\sqrt[3]{5\frac{1}{7}+2.5^2}$$
.  
Give your answer correct to 1 decimal place.

$$\sqrt[3]{5\frac{1}{7}+2.5^2} = 2.250$$

(b) The table shows the number of coronavirus cases of four countries in Europe.

Country	Number of cases (millions)
France	30.51
Germany	27.77
Russia	18.41
Italy	18.18

<sup>(</sup>Source: https://www.worldometers.info/coronavirus/)

Calculate the total number of coronavirus cases of the four countries. Give your answer in standard form.

total number = 
$$30.51 + 27.77 + 18.41 + 18.18$$
 [M1]  
=  $94.87$  million  
=  $9.487 \times 10^7$   
Answer 9.487×10<sup>7</sup> [A1] [2]

4 (a) Expand and simplify (4x+1)(x-3).

$$(4x+1)(x-3) = 4x^2 + x - 12x - 3$$
 [M1 - 2 correct expansion]  
=  $4x^2 - 11x - 3$   
Answer  $4x^2 - 11x - 3$  [A1] [2]

(b) (i) Solve the inequality 3x < 74.

Answer 
$$x < 24\frac{2}{3}$$
 [B1] [1]

(ii) Write down the largest integer value for *x*.

Answer 24 [B1] [1]

5 (a) Write 1008 as a product of its prime factors.

Answer  $\frac{2^4 \times 3^2 \times 7}{[B2 - 3 \text{ correct}, B1 - 2 \text{ correct}]}$ [2]

(b) What is the largest perfect square that is a factor of 1008?

 $2^4 \times 3^2 = 144$ 

Answer 144 [B1] [1]

(c)  $3780 = 2^2 \times 3^3 \times 5 \times 7$ 

Write down the lowest common multiple of (LCM) of 1008 and 3780. Give your answer as a product of its prime factors.

 $LCM = 2^4 \times 3^3 \times 5 \times 7$ 

Answer 
$$2^4 \times 3^3 \times 5 \times 7$$
 [B1] [1]

6 The diagram shows a quadrilateral *ABDE*. *ACD* and *BCE* are straight lines and *AB* is parallel to *ED*.



- (a) Complete the sentence. The mathematical name of quadrilateral *ABDE* is trapezium [B1] . [1]
- (b) Given that triangle *ABC* is similar to triangle *DEC*, identify all pairs of angles in triangle *ABC* and triangle *DEC* that are equal.

Answer

$$\angle ABC = \angle DEC$$
  

$$\angle BCA = \angle ECD \qquad [B2 - 3 \text{ correct pairs}, B1 - 2 \text{ correct pairs}]$$
  

$$\angle BAC = \angle EDC \qquad [2]$$

- (c) AB = 8 cm, BC = 6.5 cm and DE = 12 cm.
  - (i) Find the scale factor of enlargement of triangle *ABC* to triangle *DEC*.

Answer 
$$\frac{3}{2}$$
 [B1] [1]

(ii) Calculate *EC*.

$$\frac{\frac{8}{12} = \frac{6.5}{EC}}{EC}$$
 [M1]  
$$EC = \frac{12 \times 6.5}{8}$$
  
= 9.75

Answer 9.75 cm [A1] [2]

7 (a) Factorise completely 3ax - 6ay + 2x - 4y.

3ax - 6ay + 2x - 4y = 3a(x - 2y) + 2(x - 2y) [M1 - common factor of 2 or 3a] = (3a + 2)(x - 2y)

Answer 
$$(3a+2)(x-2y)$$
 [2]

(b) Rearrange this equation  $y = \frac{xb}{b-x}$  to make *b* the subject.

$$y = \frac{xb}{b-x}$$

$$y = \frac{xb}{b-x}$$

$$y(b-x) = xb$$

$$y(b-x) = xb$$

$$y(b-x) = xb$$

$$yb - yx = xb$$

$$yb - yx = xb$$

$$yb - yx = xb - yb$$

$$b(y-x) = yx$$

$$[M1]$$

$$-yx = b(x-y)$$

$$[M1]$$

$$b = \frac{yx}{y-x}$$

$$b = -\frac{yx}{x-y}$$

Answer 
$$b = \frac{yx}{y-x}$$
 [A1] [3]

- 7
- 8 This table of values is for  $y = x^3 3x^2 6x + 8$ .

x	- 3	- 2	- 1	0	1	2	3	4	5
у	р	0	10	8	0	- 8	q	0	28

- (a) Calculate the value of *p* and the value of *q*.
  - Answer p = -28 [B1] q = -10 [2]
- (b) On the grid provided, plot the graph of  $y = x^3 3x^2 6x + 8$  for  $-3 \le x \le 5$ . *Answer on next page* [3] [P2 - 9 points, P1 - 5 points] [C1 - smooth curve]
- (c) Find the value of x when y = 20.

Answer x = 4.8 [B1] [1]

(d) By drawing a tangent, find the gradient of the curve  $y = x^3 - 3x^2 - 6x + 8$  when x = 4.

tangent at 
$$x = 4$$
 [M1]  
 $m = \frac{15 - (-18)}{4.8 - 3}$   
= 18.3

Answer 18.3 [B1] [2] (range : 17 – 19)

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-3 -2	-1 0 -10-	
-3	= <b>1</b> 0 =10-	
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	- <b>1</b> 0 -10-	
	=1 0 =10-	
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9 *PRS* is a right-angled triangle and *PQR* is a straight line.

PS = 10 cm, SR = 6 cm, angle  $QRS = 90^{\circ}$  and angle  $QSR = 32^{\circ}$ .



(a) Find  $\sin \angle SPR$ . Give your answer as a fraction in its lowest terms.

 $\frac{6}{10} = \frac{3}{5}$ 

Answer 
$$\frac{3}{5}$$
 [B1] [1]

(**b**) Calculate the length of *QS*.

$$\cos 32^{\circ} = \frac{6}{QS}$$
 [M1]  
 $QS = \frac{6}{\cos 32^{\circ}}$   
= 7.0750  
*Answer* 7.08 cm [A1] [2]

(c) Show that the angle PSQ is  $21.1^{\circ}$ .

Answer  

$$\cos \angle PSR = \frac{3}{5} \qquad [M1]$$

$$\angle PSR = \cos^{-1}\left(\frac{3}{5}\right)$$

$$= 53.130^{\circ}$$

$$\angle PSQ = 53.130^{\circ} - 32^{\circ}$$

$$= 21.1^{\circ} \qquad [A1] \qquad [2]$$

(d) Calculate the area of triangle *QPS*.

$$\frac{1}{2}(7.0750)(10)\sin(21.130^{\circ}) \qquad [M1] \quad \frac{1}{2}(7.0750)(10)\sin(21.1^{\circ}) \qquad [M1] \\ = 12.75216 \qquad \qquad = 12.7348 \\ Answer \qquad 12.8 \text{ cm}^2 \text{ or } 12.7 \text{ cm}^2 \text{ [A1]} \qquad [2]$$

- 10 Mr Lo owns a car. He usually drives 18 km from home to work for 30 minutes daily.
  - (a) A car wheel is made up of steel rim and rubber tyre. The outer diameter of the car wheel is 66 cm. The diameter of the steel rim is 41 cm.



(i) Find the section height of the tyre.

$$\frac{66\!-\!41}{2}\!=\!12.5$$

Answer 12.5 cm [B1] [1]

(ii) One revolution of a wheel will make the car move a distance equal to its circumference of the wheel.How many complete revolutions does the car wheel need to make from home to work?

circumference = 
$$\pi$$
 (66) [M1]  
number of complete rotations =  $\frac{1800000}{66\pi}$   
= 8681.17

Answer 8682 revolutions [A1] [2]

(b) On a particular day, he drove a further 6 km in 15 min.Find the average speed, in km/h for the whole of Mr Lo's journey.

average speed = 
$$\frac{18+6}{\frac{1}{2}+\frac{1}{4}}$$
 [M1 - total distance/ total time]  
= 32

10 (c) Mr Lo finds ways to save money as the cost of living rises. He compares prices of petrol between different kiosks.

The table shows the prices of the petrol at two different petrol kiosks and the discounts offered.

	PCS	OSSE
Price of petrol (per litre)	\$3.86	\$3.91
Credit card discount	14%	12%
Loyalty card discount	-	5%
Additional rebate	\$3 instant rebate with minimum spend of \$80	-

Mr Lo pumps 49 litres every week. He usually pays for his petrol using credit card to get the discount benefits. He has a loyalty card discount at OSSE.

Which petrol kiosk should he pump petrol at? Show all your calculations.

Answer

<u>PCS</u>:  $\cos t = (49 \times \$3.86) \times 86\% - 3$  [M1 - 14% discount, M1- \$3 rebate] = \$159.66

### OSSE:

 $cost = (49 \times \$3.91) \times 83\%$  [M1 - 17% discount] = \$159.02

Mr Lo should choose to pump petrol at **OSSE** because the **cost is lower**. [4] [A1 - conclusion]

## Section B (8 marks)

Answer one question from this section. Each question carries 8 marks.

11 The table shows the heights of 46 plants, grown under experimental condition A.

Height (cm)	$0 < h \le 10$	$10 < h \le 20$	$20 < h \le 30$	$30 < h \le 40$	$40 < h \le 50$
Frequency	3	10	7	12	14

(a) (i) Calculate an estimated of the mean and standard deviation of the height of the plants grown under experimental condition *A*.

$$mean = \frac{3(5)+10(15)+7(25)+12(35)+14(45)}{46}$$
  
=  $\frac{695}{23}$   
=  $30.21739$   
 $SD = \sqrt{\frac{3(5)^2+10(15)^2+7(25)^2+12(35)^2+14(45)^2}{46} - \left(\frac{695}{23}\right)^2}$ [M1]  
=  $\sqrt{\frac{49750}{46} - \left(\frac{695}{23}\right)^2}$   
=  $12.978$ 

Answer mean 30.2 cm [B1]

standard deviation 13.0 cm [A1] [3]

\_\_\_\_\_

(ii) The heights of 46 plants, grown under experimental condition B were observed. The mean height is 35 cm and standard deviation is 9.8 cm.

In which experimental condition are the heights of the plants more consistent? Give a reason for your answer.

Answer

Experimental condition	B	because	the <b>standard deviation</b> of <i>B</i>	
is <b>smaller</b> than that of A.	[B1]			[1]

11 (b) Two plants grown under experimental condition A are chosen at random.

Calculate the probability that

(i) both plants had height of more than 30 cm,

$$\frac{\frac{26}{46} \times \frac{25}{45}}{\frac{65}{207}}$$
 [M1]

Answer 
$$\frac{65}{207}$$
 [A1] [2]

(ii) at least one of the plants had height of more than 10 cm.

$$1 - \frac{3}{46} \times \frac{2}{45}$$
 [M1]  
=  $\frac{344}{345}$ 

Answer  $\frac{344}{345}$  [2]





A, B, C, D and E are points on the circumference of a circle, centre O. AD is a diameter and DFOA is a straight line. Angle  $AEC = 62^{\circ}$ .

Complete these statements by calculating the size of each angle. Give a reason for each statement.

Statement		Reason	
Angle <i>ABC</i> =	180°-62°=118°	Angles in opposite segment	
Angle <i>FDC</i> =		Angles in same segment	
Angle AOC =		Angles at centre = 2 x angles at circumference	  [3]
[B1 – any 2 cor	rrect]		

12 (b) A plane flew from point *P* to point *Q*. *R* is 300 km on a bearing of 075° from *Q*. PQ = 500 km and angle  $PQR = 142^{\circ}$ .



(i) Calculate the bearing of *Q* from *P*.

$$\angle NQP = 360^{\circ} - 142^{\circ} - 75^{\circ} = 143^{\circ}$$
 [M1]  
 $180^{\circ} - 143^{\circ} = 37^{\circ}$   
*Answer* 037° [A1] [2]

(ii) Calculate the distance *PR*.

$$PR^{2} = 500^{2} + 300^{2} - 2(500)(300)\cos 142^{\circ}$$
[M1]  

$$PR = \sqrt{500^{2} + 300^{2} - 2(500)(300)\cos 142^{\circ}}$$
[M1]  
=759.2122  
*Answer* 759 km [A1] [3]

### **END OF MARK SCHEME**