<u>Oscillation Tutorial Hints & Suggested Solutions to Discussion Questions</u>

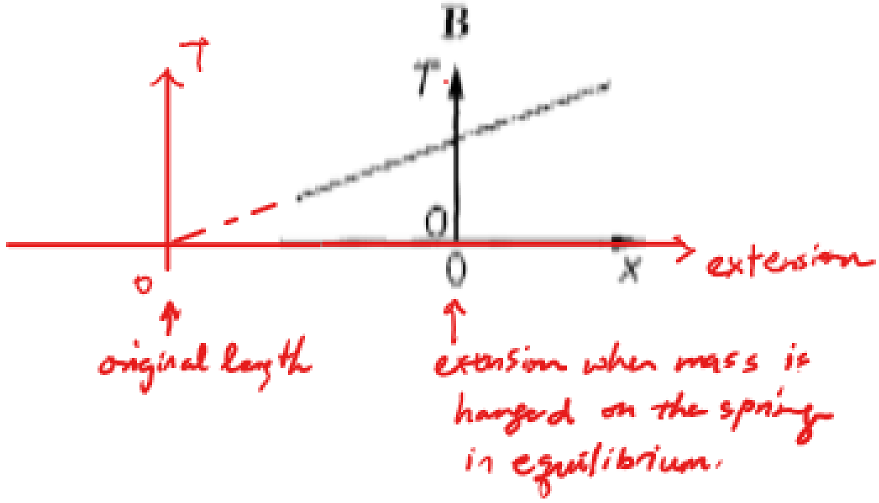
D1 Amplitude of oscillation = 50 mm
Period of oscillation = 2 s
x = 50 sin (2π /2) t (*x* in mm)
[Note: Taking to the right of 650 mm mark as positive direction]

when *x* = +25 mm
25 = 50 sin (2*π /2) t*
Time duration for which shutter remained open, *t* = 1/6 s = <u>0.17 s</u>

Key Concepts: *Characteristics of S.H.M? Able to write down the displacement-time function for a S.H.M. from initial condition (i.e. when t = 0 s).*
Note*: For SHM, displacement is NOT proportional to time.*

D2 ***Hints:*** *Check at equilibrium x = 0, is there tension? How is tension related to the extension of spring?*

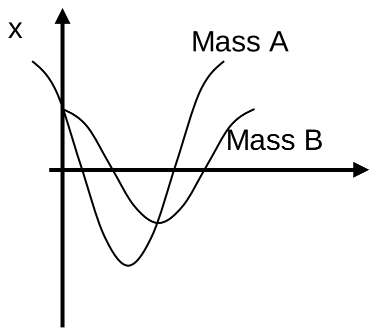
B.



Key Concepts: *Link to topic of Dynamics & Forces.*
Note: *Tension in spring, T is NOT proportional to the displacement, x.*
Qn D2, D10 & D11 is about oscillation of a vertical spring mass system.

D3 ***Hints:*** *Sketch the displacement, x vs phase angle, ϕ graph for the scenario.*
Note that at time t = 0 s, Mass A is released from its maximum displacement position (2 cm below equilibrium position). Mass B is released from it maximum displacement position (1 cm below equilibrium position) only when Mass A is at displacement of 1 cm below equilibrium position.

B.



$$1 = 2\cos\varphi \rightarrow \varphi = \frac{\pi}{3}$$

Key Concepts: Understand phase angle and phase difference for oscillations.

D4 ***Hints:** i) Can you write down the function that relates the acceleration, a to time, t?*
ii) Can you write down the function that relates the displacement, x to time, t?
iii) Can you write down the function that relates the velocity, v to time, t?

A.

From **a – t** graph, $a = a_o \cos \omega t$
We can then infer that displacement, $x = - x_o \cos \omega t$
Hence, velocity, $v = v_o \sin \omega t$

Sketch the $x - t$ and $v - t$ graphs on the $a - t$ graph (same time axis), and determine that at P when a is positive, x is negative and v is positive.

***Key Concepts:** See the relationship between a-t, x-t & v-t graphs of SHM. Know the kinematics of SHM.*

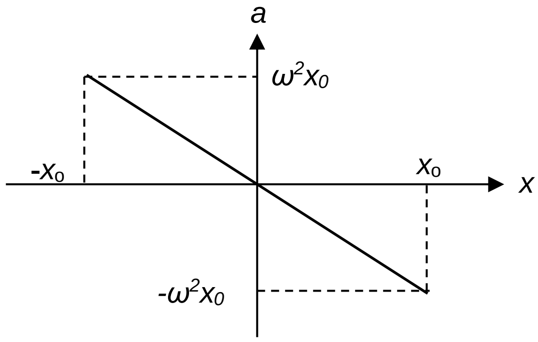
D5 ***Hints:** i) How do you know that the oscillation is SHM?*
ii) Where is the equilibrium position of the SHM?
iii) What is the amplitude of the SHM?
iv) What is the displacement of SHM when the depth of water is 1.5 m?
v) Can you write down the appropriate function that relates displacement, x to the time, t?

2.0 m is the equilibrium position.
Amplitude, $x_0 = 1.0$ m
$T = 12$ hrs
Depth of water, $x = - 1.0 \cos (2\pi /12)\, t + 2.0$

$1.5 = - 1.0 \cos (2\pi /12)\, t + 2.0$
Time duration that the boat will have to wait before entering, $t = 2.0$ hrs

***Key Concepts:** Characteristics of S.H.M? Able to write down the displacement-time function for a S.H.M. from initial condition (i.e. when t = 0 s).*

D6 (b) (i)



***Key Concepts:** How to identify SHM? Defining equation for a SHM.*

(ii) 1. $\omega = 2\pi f = 2\pi (13) = 82\ rad\ s^{-1}$
2. when $a_{max} = g$
$| -\omega^2 x_o | = g$
Amplitude of oscillation, $x_o = 9.81 / 82^2$
$= 1.47 \times 10^{-3}$ m

(c) **Hint:** *i) Sketch a diagram of the plate and one particle of sand on plate. What forces act on the particle of sand? Which of these forces is dependent on the relative motion between sand and plate?*

ii) When the plate is at maximum displacement above the equilibrium position, what is the direction of acceleration of the plate? What is the direction of velocity of the plate just before it reach maximum displacement?

Suggested Solution:

If the amplitude of the oscillations of the plate exceeds the value calculated in (b)(ii)2, the maximum acceleration of the plate will be higher than the acceleration of free fall, *g*. Thus the sand will lose contact with the flat horizontal plate on its way up (pass the equilibrium point) as the plate slows down at a rate larger than the sand.

D7 (a) (i) $\theta = \omega t$

(ii) $ST = r sin\omega t$ *(How do you know that it is a sine function?)*

(b) ST is the displacement *x* of the shadow with respect to S. Comparing it with the standard expression for the displacement of an object in SHM ($x = x_0 sin\omega t$), we can see that the shadow of the peg is moving in simple harmonic motion about the point S with amplitude $x_0 = r$.

(c)

(i) amplitude of shadow's SHM is same as radius of the turntable 20 cm.

(ii) Since the turntable has angular speed $\omega = 3.5 = \frac{2\pi}{T}$, the period of the turntable is 1.8s which is also the period of the shadow's SHM.

(iii) Maximum speed of shadow at S is $v_{max} = \omega x_0 = (3.5)(0.2) = 0.7 m/s$

(iv) Maximum acceleration (when shadow at rest) is given by

$a_{max} = \omega^2 x_0 = (3.5)^2(0.2) = 2.45 m/s^2$

D8 **Hint:** *i) What is the expression of the total mechanical energy of a mass in SHM?*

Further qn: *Can you prove that the mass will oscillate in SHM?*

Total mechanical energy for S.H.M = max. K.E.

$= \frac{1}{2} M v^2_{max}$

$= \frac{1}{2} M (\omega a)^2$

$= \frac{1}{2} M (a\, 2\pi/T)^2$

$= 2\pi^2 M a^2 / T^2$

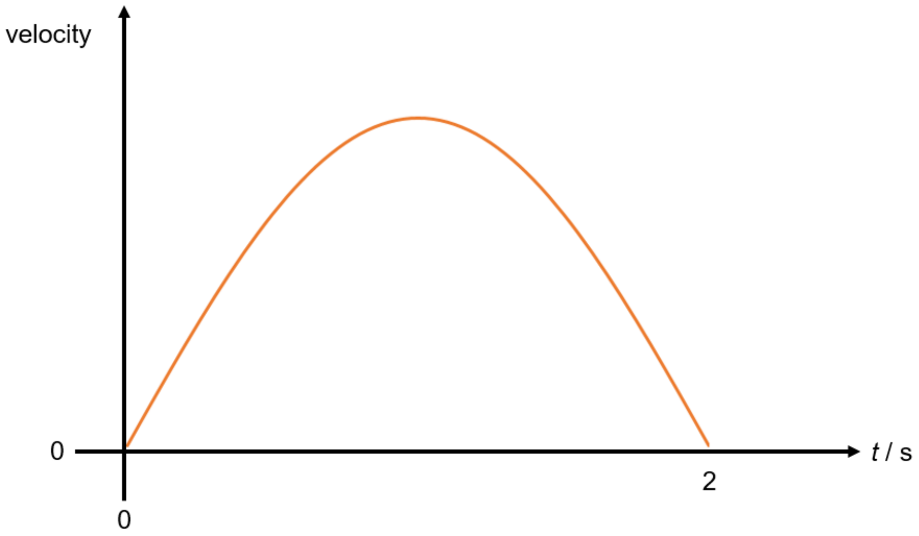
Another method:

Let the effective spring constant for this system be *k*.

From $\omega^2 = \frac{k}{m}$, we get $k = M\omega^2$

Total mechanical energy for SHM = $E_T = \frac{1}{2} k x_0^2 = \frac{1}{2} M\omega^2 a^2 = \frac{1}{2} M \left(\frac{4\pi^2}{T^2}\right) a^2 = \frac{2M\pi^2 a^2}{T^2}$

D9 It is given in the question that the potential energy versus time graph is for half a period. At *t* = 0, the potential energy is maximum. This suggests that the particle is at the maximum displacement where the velocity is zero. Half a period later, the particle is at the other maximum displacement where velocity is again zero. During this time, the velocity is unidirectional. Hence, the velocity-time graph should give a half of sine graph.



D10

$PE_{max} = KE_{max} = ½\, m\, (\omega x_o)^2 = ½\, m\, (x_o\, 2\pi/T)^2$

From graph, when x_o = 0.2 m, U = 1.0 J

$1.0 = ½\,(4)\,(0.2 \times 2\pi/T)^2$

$1.0 = 2.0 \times 0.04 \times 4\pi^2 / T^2$

$T^2 = 2.0 \times 0.04 \times 4\pi^2$

T = 1.8 s

Key Concepts: *Understand and able to determine the potential energy vs displacement relation for a SHM.*

D11 **Hint:** *i) How do you know that the vertical spring-mass system will oscillate in SHM?*

(a)

At the equilibrium point,

$$F_{res} = 0 \rightarrow kx - Mg = 0$$

$$kx = Mg \rightarrow k = \frac{Mg}{x}$$, where x is the extension of spring.

Any of these points or other correct points from the graph;
When a mass of 150 g is hung, the extension on the spring is 10.0 cm.
When a mass of 300 g is hung, the extension is 20.0 cm.
When a mass of 450 g is hung, the extension is 30.0 cm

$$k = \frac{0.150g}{0.100} = 14.7 \text{ N m}^{-1}.$$

(b) **Hint:** *i) What is the unstretched length, l_o, of the spring?*
ii) What is the length of the spring when 450 g is attached? What is the extension of the spring when 450 g is attached?
iii) What is the amplitude of SHM?

$$\omega = 2\pi f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{14.715}{0.450}} = 0.910 \text{ Hz (3 s.f.)}$$

(c) **Hint:** *i) What is the displacement of the oscillation when the length of spring is 80.0 cm?*

ii) What is the relationship (formula) that relates velocity, v with displacement, x?

$$\Delta GPE = Mg(extension) = 0.450 \times 9.81 \times 0.300 = 1.32435 \text{ J}$$

$$\Delta EPE = \frac{1}{2}k(e_2^{\ 2} - e_1^{\ 2}) = \frac{1}{2}(14.715)(0.400^2 - 0.100^2) = 1.103625 \text{ J}$$

$$\Delta KE = 1.32435 - 1.103625 = 0.220725 \text{ J}$$

$$0.220725 = \frac{1}{2}mv^2 \rightarrow v = 0.990 \text{ m s}^{-1}$$

OR
Note that question is asking for *v* for an oscillation with amplitude 20.0 cm at displacement of 10.0 cm.

$$v = \omega\sqrt{x_0^{\ 2} - x^2} = 2\pi(0.910)\sqrt{0.200^2 - 0.100^2} = 0.990 \text{ m s}^{-1}$$

(d) Use $\omega = 2\pi f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, deduce that *f* is inversely proportional to $\sqrt{m}$. When *m* increases, *f* is reduced

OR The effective mass of the spring-mass system increases. The resonant frequency of heavier masses is at lower values of frequencies (or at greater periods). Hence, the frequency of the oscillation would be reduced.

D12 *Qn D2, D11 & D12 is about oscillation of a vertical spring mass system.*

(a) Frequency is the number of oscillations per unit time whereas angular frequency is the rate of change of phase angle. Since each complete oscillation moves through a phase angle of 2π, the angular frequency is given by $2\pi f$.

(b)(i) loss in gravitational potential energy by mass $= m\,g\,\Delta h$
= (0.400)(9.81)(0.200)= 0.785 J

(b)(ii) At equilibrium, $mg = ke$
Elastic potential energy gained by spring = $\frac{1}{2} ke^2 = \frac{1}{2} (mg)(e)$
= ½ (9.81)(0.400)(0.200) = 0.392 J

(c) **Hint:** *i) The question mention that "A mass of 0.400 kg is attached to the spring and gently lowered until equilibrium is reached".*
ii) Up till this part, the spring-mass system is not in oscillation yet!

An external (variable) upward force is required when the spring mass is stretched gently downwards towards its equilibrium point. Thus some gravitational potential energy is lost due to the negative work done by the external force, while the remainder is converted to elastic potential energy.

(d)(i) $F_{net} = ke' - mg = k(2e) - mg = 2mg - mg = mg$
= (0.400) (9.81) = 3.92 N

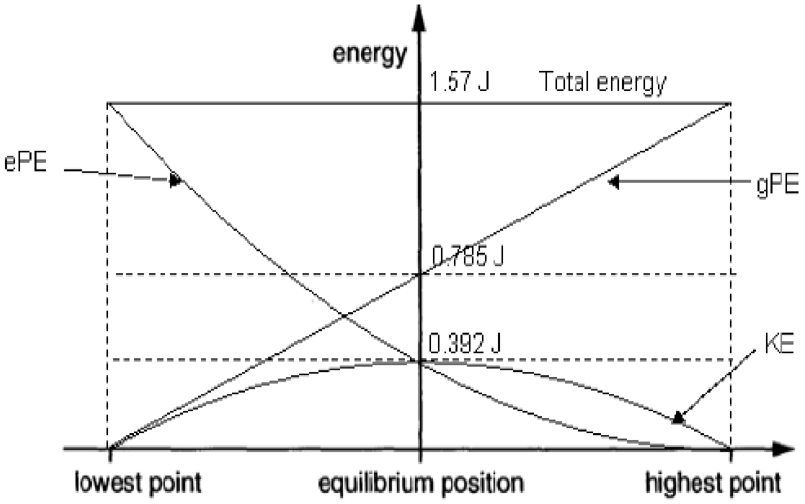
(ii) By Newton's 2nd Law, $F_{net} = ma$
$a = F_{net} / m$
SHM equation: $a = -\omega^2 x$
$\omega = \sqrt{(a / -x)} = \sqrt{(F_{net} / -mx)}$

$= \sqrt{[(0.400)(9.81)/(-0.400)(-0.200)]} = 7.00 \text{ rad s}^{-1}$

(iii) maximum speed of mass = $\omega x_o = 7.00(0.200) = 1.40 \text{ m s}^{-1}$

(e) & (f)

	gravitational potential energy/J	elastic potential energy/J	kinetic energy/J	total energy/J
lowest point	0	1.57	0	1.57
equilibrium position	0.785	0.392	0.392	1.57
highest point	1.57	0	0	1.57



D13 If the natural frequency is within the range of frequencies produced by the loudspeaker, resonance can occur for this frequency when the loudspeaker is being used. This results in a rapid amplification of the sound that will cause discomfort to the ear. Further, the timbre of sound produced by the loudspeaker will also experience distortion as sound waves of this particular frequency are greatly amplified relative to other frequencies.

D14 **(a)** Hydrostatic pressure is proportional to the depth of fluid. When an object displaces a fluid, there is a pressure difference between the lower and upper surface and this pressure difference results in upthrust.

(b) Since the tube is floating, it is displacing its own weight of fluid. (Principle of flotation)

OR

The force on the bottom of the tube due to fluid pressure is equal to the weight of the tube.

$m_{tube}g = m_{fluid}\, g$

$m = V\rho = Ah\rho$

(c) *Extension: Can you derive the expression (of a vs x) given?*

(i) Since *A, h, ρ* and *m* are all constants, the magnitude of the acceleration is proportional to the displacement. The negative sign also indicates that the direction of acceleration is opposite to that of displacement. The equation is of the form *a* = - *kx* where *k* is a constant which describes simple harmonic motion.

(ii) Comparing with defining equation:

$$\omega^2 = \frac{A\rho g}{m} \Rightarrow 4\pi^2 f^2 = \frac{A\rho g}{m}$$

$$\Rightarrow f = \sqrt{\frac{A\rho g}{4\pi^2 m}} = \sqrt{\frac{(4.2\times10^{-4})(1.0\times10^{3})(9.81)}{4\pi^2(0.032)}} = 1.8 \text{ Hz (2 s.f.)}$$

(d) **(i)1**

$$f = \frac{1}{T} = \frac{1}{1.5\div 3} = 2.0 \text{ Hz}$$

(i)2 From (c)(ii):

$$\rho = \frac{4\pi^2 f^2 m}{Ag} = \frac{4\pi^2(2.0)(0.032)}{(4.2\times10^{-4})9.81} = 1226 = 1200 \text{ kg m}^{-3} \text{ (2 s.f.)}$$

(ii)1 The fluid is viscous and exerts a force on the tube which opposes its motion. It removes energy from the oscillating tube.

There is also friction along the sides of the tube (skin friction). This results in a constant loss of energy and hence the amplitude decreases with time.

(ii)2 Read from Figure, Amplitudes when t = 0 s and t = 1.0 s.

Loss in energy = $E_0 - E_1$

$$= \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x_1^2$$

$$= \frac{1}{2}(0.032)(4\pi^2 2^2)(0.0150^2 - 0.0085^2) = 3.86\times10^{-4} \text{ J}$$

D15 (a)(i) A forced oscillation is an oscillatory system driven into oscillation by a periodic force.

(b)(i) 1. Maximum linear speed $v_{max} = \omega x_0 = 2\pi f x_0$

At resonance, x_0= 1.60 cm, *f* = 12.0 Hz

$v_{max} = 2\pi f x_0$

$= 2\pi (12.0)(1.60 \times 10^{-2})$

$= 1.21$ m s^{-1}

2. Maximum acceleration $a_0 = | -\omega^2 x_0 | = 4\pi^2 (12.0)^2(1.60 \times 10^{-2}) = 91.0$ m s^{-2}

3. resonance.

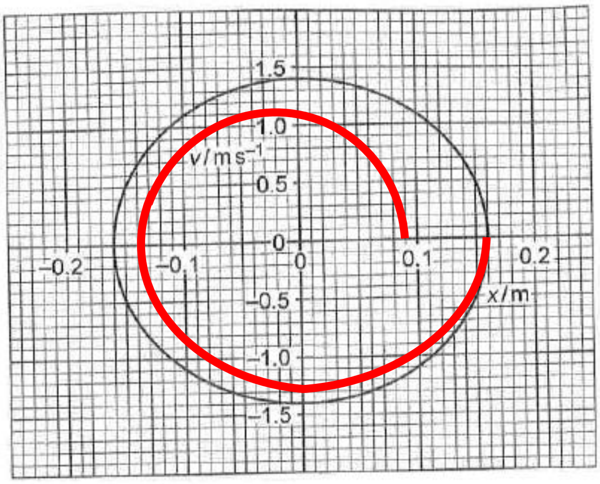
(ii) Time interval = *T* / 4 = 1/12.0 (1/4) = 0.0208 s. Velocity and acceleration are $\pi/2$ out of phase with each other.

D16 (a) Maximum acceleration $a = \frac{v_0^2}{x_0} = \frac{1.4^2}{0.16} = 12.3\ m\ s^{-2}$

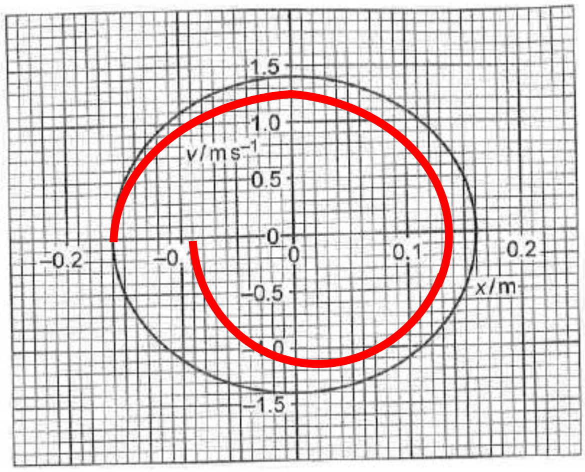
(b) When the light piece of card is added, the oscillations will be damped. The velocity and displacement will decrease gradually as mechanical energy is lost to the system due to the work done against air resistance.

There are two possible solutions to this question.

Solution 1:



Solution 2:



Solution 1 assumes that **downward is positive**, hence when the mass is displaced downwards, it has a positive x displacement of 0.16. When the mass moves **upwards**, **its velocity will be negative**. Hence, as the oscillation progresses, the red line is drawn in a clockwise direction, with energy being lost continuously.

Solution 2 assumes that downward is negative, and hence its initial displacement is negative. However when the mass moves upwards, its velocity will be positive. This way, the red line is also drawn in a clockwise direction.