Paya Lebar Methodist Girls' School (Secondary)
Department of Mathematics
2017 Preliminary Examination
Mathematics Paper 2 (4048/2) Worked Solutions

Qns No.	Working
1(ai)	$16x^2y^2 - 40xy + 25 = (4xy-5)^2$
1(aii)	$LHS = (4xy-5)^2 = k$ $(4xy-5) = \pm\sqrt{k}$ $y = \dfrac{\pm\sqrt{k}+5}{4x}$
1(b)	$\dfrac{2x+15}{3} = 4 - \dfrac{2x-3}{6}$ $\dfrac{2x+15}{3} + \dfrac{2x-3}{6} = 4$ $\dfrac{2(2x+15)+2x-3}{6} = 4$ $6x+27 = 24$ $6x = -3$ $x = -\dfrac{1}{2}$ or 0.5
1**(c)**	$\dfrac{8-2x^2}{12+4x-x^2}$ $= \dfrac{2(4-x^2)}{-(x^2-4x-12)}$ or $= \dfrac{2(4-x^2)}{-x^2+4x+12)}$ $= \dfrac{2(2-x)(2+x)}{-(x-6)(x+2)}$ or $= \dfrac{2(2-x)(2+x)}{(-x+6)(x+2)}$ $= \dfrac{2(x-2)}{x-6}$ or $\dfrac{2(2-x)}{6-x}$

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1**(d)**	$\frac{1}{x}-\frac{1}{y}=3$ $y-x=3xy$ ------- (1) Sub (1) into $\frac{2x+3xy\text{-}2y}{x-2xy-y}$ $=\frac{-2(y-x)+3xy}{-(y-x)-2xy}$ $=\frac{-2(3xy)+3xy}{-(3xy)-2xy}$ $=\frac{-3xy}{-5xy}$ $=\frac{3}{5}$
2(ai)	Ext $\angle = x = \frac{360}{n}$ ------(1) Ext $\angle = x-16^\circ = \frac{360}{3n}$ ------(2) Sub (1) into (2) : $\frac{360}{n}-16^\circ = \frac{360}{3n}$ $\frac{360}{n}-\frac{360}{3n}=16^\circ$' $1080-360=48n$ $n=\frac{720}{48}$ $n=15$
2(aii)	$x=\frac{360}{3(15)}=8^\circ$ Int $\angle = 180^\circ - 8^\circ = 172^\circ$ (adj $\angle$s on a str line)
2(b)	In triangle *QRS* and triangle *PST*, *QR* = *PS* (opp sides of rhombus) $\angle SRQ = \angle TEP$ (corresponding $\angle$s, *RQ* // *SP*) *RS* = *ST* (*S* is the midpt of *RT*) Hence, triangle *QRS* is congruent to triangle *PST*. (SAS)
2(c)	*QP* = *ST* and *QP* // *ST* (*RS* & *QP* are opp sides of rhombus and *RST* is a str. line) *QS* = *PT* and *QS* // *PT* (Corresponding sides of congruent triangles, proven in (i)) Hence, *PQST* is a parallelogram. (2 pairs of equal and // sides)

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2(e)	In triangle *POQ* is similar to triangle *RQT*, $\angle QOP = \angle TQR$ (alt $\angle$s, RQ // SP) $\angle OQP = \angle QTR$ (alt $\angle$s, RT // QP) $\angle OPQ = \angle QRT$ (opp angles of rhombus are equal) Hence, triangle *POQ* and triangle *RQT* are similar. (2 pairs of corresponding angles are equal)
3(a)	$a = 90$ $b = 36$ $c = 55$
3(b)	Numbers in the *R* column is made up of the sum of consecutive odd-number factors, i.e. 1 + 3 + 5 + 7 +... And 1 + 3 + 5 + 7 +...+ 17 = 100, hence 99 cannot appear in column *R*.
3(c)	$P = T - R + 1$
3(d)	$T = n(2n+3)$ $\therefore T_n = 2n^2 + 3n$
3(ei)	$T_{p+1} = 2(p+1)^2 + 3(p+1) = 2p^2 + 7p + 3$ $T_p = 2(p)^2 + 3(p) = 2p^2 + 3p$ $\therefore T_{p+1} - T_p = (2p^2 + 7p + 3) - (2p^2 + 3p)$ $= 4p + 3$
3(eii)	$\because T_{p+1} - T_p = 4p + 3$, a common factor 4 cannot be derived from 4*p* + 3.
3(f)	*P* = 275 – 121 + 1 = 155
4(ai)	$\angle AOC$ $= \frac{2\pi}{6} \times 2$ $= \frac{2}{3}\pi$ rad
4(aii)	Area of circle $= \pi r^2$ Area of ΔAOB $= \frac{1}{2} r^2 \sin\left(\frac{\pi}{3}\right)$ $= \frac{1}{2} r^2 \frac{\sqrt{3}}{2}$ Or $0.433r^2$ $= \frac{\sqrt{3}r^2}{4}$ $\therefore$ Area of hexagon *ABCDE* $= 6 \times \frac{\sqrt{3}r^2}{4}$ $= \frac{3\sqrt{3}r^2}{2}$ or $2.598\, r^2$

Qns No.	Working
	Total shaded area $= \pi r^2 - \frac{3\sqrt{3}r^2}{2}$ $= r^2(\pi - \frac{3\sqrt{3}}{2})$ units sq. or $0.544\ r^2$
4(b)	Connect point *C* to point *F*, then $\angle ABC + \angle CFA = 180^\circ$ ($\angle$ s in opp seg are suppl) $\angle CDE + \angle EFC = 180^\circ$ ($\angle$ s in opp seg are suppl) Adding up: $\angle ABC + \angle CDE + \angle CFA + \angle EFC = 360^\circ$ $\angle ABC + \angle CDE + \angle EFA = 360^\circ$ (Shown)
5(a)	$\tan 32^\circ = \frac{TB}{DB}$ $TB = DB \tan 32^\circ$ $\tan 24^\circ = \frac{TB}{6 + DB}$ $= \frac{DB \tan 32^\circ}{6 + DB}$ $DB \tan 32^\circ = 6 \tan 24^\circ + DB \tan 24^\circ$ $DB = \frac{6 \tan 24^\circ}{\tan 32^\circ - \tan 24^\circ} \approx 14.87064 \text{ m}$ Height of the flagpole = $14.87064 \tan 32^\circ$ or $20.87064 \tan 24^\circ$ = 9.292209 = 9.29 m (3 s.f)
5(b)	$AB = 6 + 14.87064 = 20.87064$ ≈ 20.9 m By Cosine Rule, $BC^2 = 50^2 + 20.87064^2 - 2(50)(20.87064)\cos 38^\circ$ $BC = 35.92986$ $= 35.9$ m (3 s.f)
5(c)	$\cos 38^\circ = \frac{AE}{20.87064}$ $AE = 20.87064 \cos 38^\circ$ $AE = 16.4463$ $AE = 16.4 \text{ m}$ (3 s.f)

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6(ai)	Since $EF : EO = 4 : 7$, hence $FC : OB = 4 : 7$. Therefore $\overrightarrow{FC} = 4\mathbf{b}$
6(aii)	$\overrightarrow{CB} = \overrightarrow{CF} + \overrightarrow{FO} + \overrightarrow{OB}$ $= -4\mathbf{b} - 4\mathbf{a} + 7\mathbf{b}$ $= 3\mathbf{b} - 4\mathbf{a}$
6(aiii)	$\overrightarrow{EC} = \overrightarrow{EF} + \overrightarrow{FC}$ or $\frac{4}{3}\overrightarrow{CB}$ $= -\frac{16}{3}\mathbf{a} + 4\mathbf{b}$ $= \frac{4}{3}(-4\mathbf{a} + 3\mathbf{b})$ $= \frac{4}{3}(-4\mathbf{a} + 3\mathbf{b})$
6(aiv)	Since $ED // AB$ and $ED : AB = 4 : 3$, Therefore $\overrightarrow{ED} = \frac{8}{3}\mathbf{b}$
6(b)	E, C and B are collinear. $\overrightarrow{EC} = \frac{4}{3}\overrightarrow{CB}$
6(ci)	Since triangle EFC is similar to area of triangle EOB, Therefore $\frac{A_{\Delta EFC}}{A_{\Delta EOB}} = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$ Area of triangle EFC : area of triangle EOB = 16 : 49
6(cii)	Since triangle EDC is similar to area of triangle ABC, Therefore $\frac{A_{\Delta EDC}}{A_{\Delta ABC}} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ Area of triangle EDC : area of triangle ABC = 16 : 9
6(ciii)	$\frac{A_{\Delta EDC}}{A_{\Delta EFC}} = \frac{0.5(ED)(h)}{0.5(FC)(h)} = \frac{8/3}{4} = \frac{2}{3}$ $\frac{A_{\Delta EDC}}{A_{\Delta EFC}} = \frac{2}{3} = \frac{32}{48}$ $\frac{A_{\Delta EFC}}{A_{\Delta EOB}} = \frac{16}{49} = \frac{48}{147}$ Therefore, $\frac{A_{EDCF}}{A_{FCBO}} = \frac{48+32}{147\text{-}48} = \frac{80}{99}$ area triangle $EDCF$: area of triangle $OBCF$ = 80 : 99
7(a)	$38 = \frac{35}{2} + \frac{y}{2}$ $y = 41$
7(bi)	Time taken = $\frac{37}{x}$ h

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7(bii)	Time taken = $\frac{37}{x+4}$ h
7(biii)	$\frac{37}{x}-\frac{37}{x+4}=\frac{15}{60}$ $\frac{37(x+4)-37x}{x(x+4)}=\frac{1}{4}$ $4(148)=x^2+4x$ $x^2+4x-592=0$ (Shown)
7(biv)	Using quadratic formula or completing the square $x^2+4x-592=0$ $x=\frac{-4\pm\sqrt{(4)^2-4(1)(-592)}}{2(1)}$ $x=\frac{-4\pm\sqrt{2384}}{2}$ $x=22.4$ or $x=-26.4$ (3 s.f)
7(bv)	Time taken = $\frac{37}{x}=\frac{37}{22.4}=1.65=1$ hour 39 minutes (Correct to nearest minute)
8(a)	p = 5.71
8(b)	3
8(ci)	Min value $=-0.2$
8(cii)	When y = 4, x = 0.5 or x = 3
8(d)	Draw tangent at @ (2, 1) m = $\frac{3.5-1}{3-2}$ = 2.5 (Accept 0.473 to 2.52)
8(e)	Drawing line $y=-2x+5$ a = 5 and b = 2
9(aia)	20 + 16 + x + 10 + x = 50 2x = 4 x = 2
9(aib)	Percentage = $\frac{30}{50}\times100$ = 60%
9(aiia)	Mean = $\frac{5420}{50}$ = 108.4 cm
9(aiib)	Standard deviation = $\sqrt{\frac{603400}{50}-108.4^2}$ = 17.8 cm
9(aiii)	Second group of students have a higher mean than first group of students. Hence, 2nd group of students jumped further/longer distance. Also, 2nd group of students has a lower standard deviation. This implies that the distance achieved by all the students is more consistent than students in the 1st group.

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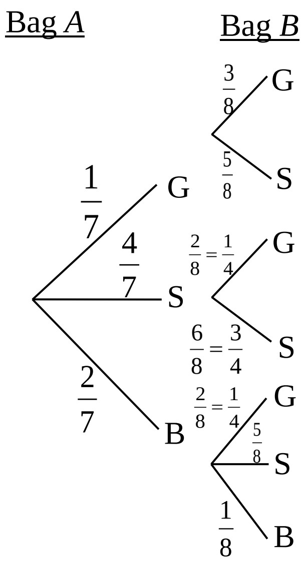
9(bi)	
9(biia)	P(Gold from bag *B*) $=\frac{1}{7}\left(\frac{3}{8}\right)+\frac{4}{7}\left(\frac{2}{8}\right)+\frac{2}{7}\left(\frac{2}{8}\right)$ $=\frac{15}{56}$
9(biib)	P(Bronze from bag *B*) $=\frac{2}{7}\left(\frac{1}{8}\right)=\frac{1}{28}$
10(a)	U-Taxi offers the cheapest fare. Percentage difference between U-taxi & C-taxi $=\frac{9.80-6.02}{9.80}\times 100$ $= 38\frac{4}{7}\%$ or 38.6%
10(bi)	Base Fare = \$3.00 Distance Fare = \$0.45 (8 km) = \$3.60 Time Fare = \$0.20 (12 min) = \$2.40 Total Fare = \$3.00 + \$3.60 + \$2.40 = \$9.00
10(bii)	The driver can get reach the destination in time because of the relatively clear roads during midnight.
10(c)	**<u>C-Taxi</u>** Base Fare = \$3.20 Metered Fare = $0.22\left(\frac{8500}{400}\right)+0.22\left(\frac{5\times 60}{45}\right)$ = 0.22 (21 **or** 21.25 **or** 22) + 0.22**(7)** = \$4.675 + \$1.54 = \$6.215 or \$6.16 or \$6.38

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	Peak Hour Surcharge = ($3.20 + ***$6.215***)(25%) = $2.35 or $2.34 or $2.95 Total Fare = $3.20 + ***$6.215*** + $2.35 **= $11.77** or $11.70 or $11.96 **G-Taxi** Base Fare = $3.00 Distance Fare = $0.80 (9.5 km) = $7.60 Total Fare = 7.60 + 3 **= $10.60 ≈ $11** **U-Taxi** Base Fare = $3.00 Distance Fare = $0.45 (9.5 km) = $4.275 Time Fare: $0.20 (15 min) = $3.00 Total Fare = $3.00 + $4.275 + $3 = **$10.28** Ms Seet should use G-Taxi. Although U-taxi is slightly cheaper than G-taxi, the 15 minutes journey time was based on no traffic jam during the morning peak hour when she commutes to work.