FINAL EXAMINATION 2021

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 2 SOLUTIONS

1. Required Height $= \dfrac{2\left(16+6\sqrt{2}\right)}{4-2\sqrt{2}} = \dfrac{4\left(8+3\sqrt{2}\right)}{2\left(2-\sqrt{2}\right)}$

$$= \frac{2\left(8+3\sqrt{2}\right)}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{2\left(16+8\sqrt{2}+6\sqrt{2}+6\right)}{4-2}$$

$$= 22+14\sqrt{2} \quad \text{cm}$$

2. $8x^2 + Ax - 5 \equiv (3-2x)^2 + B(x-1)^2 - 3(C-x)$

$8x^2 + Ax - 5 \equiv 9 - 12x + 4x^2 + Bx^2 - 2Bx + B - 3C + 3x$

$8x^2 + Ax - 5 \equiv (4+B)x^2 - (2B+9)x + (9+B-3C)$

Comparing corresponding coefficients:

x^2: $\quad 4 + B = 8 \quad \therefore B = 4$

x : $\quad A = -(2\times 4 + 9) \quad \therefore A = -17$

x^0: $\quad -5 = 9 + 4 - 3C \quad \therefore C = 6$

3a. LHS: $(\sec\theta - \tan\theta)^2 = \left(\dfrac{1-\sin\theta}{\cos\theta}\right)^2 = \dfrac{(1-\sin\theta)^2}{\cos^2\theta}$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = \text{RHS (proven)}$$

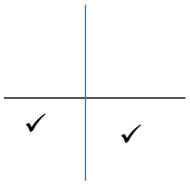
3b. $(\sec\theta - \tan\theta)^2 = 3$ for $0^o \le \theta \le 360^o$.

$$\Rightarrow \frac{1-\sin\theta}{1+\sin\theta} = 3$$

$$1-\sin\theta = 3+3\sin\theta$$

$$4\sin\theta = -2$$

$$\sin\theta = -\frac{1}{2}$$



$$\alpha = \sin^{-1}\frac{1}{2} = 30^\circ \therefore \theta = 210^\circ, 330^\circ$$

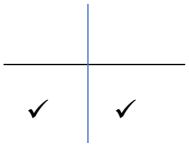
4a. $10\sin x + 9 = -2\,\text{cosec}\,x$ for $0^o \le x \le 360^\circ$

$$10\sin x + 9 = -\frac{2}{\sin x}$$

$$10\sin^2 x + 9\sin x + 2 = 0$$

$$(5\sin x + 2)(2\sin x + 1) = 0$$

$$\sin x = -\frac{2}{5} \quad \text{or} \quad \sin x = -\frac{1}{2}$$

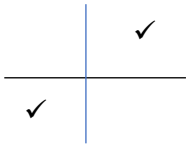


$$\alpha = \sin^{-1}\frac{2}{5} = 23.578^\circ \qquad \alpha = \sin^{-1}\frac{1}{2} = 30^\circ$$

$\therefore x = 203.6^\circ,\quad 210^\circ,\quad 336.4^\circ,\quad 330^\circ$

4b. $\sin\left(2\theta + \frac{\pi}{4}\right) = \cos\left(2\theta + \frac{\pi}{4}\right)$ for $\frac{\pi}{4} \le 2\theta + \frac{\pi}{4} \le \frac{9\pi}{4}$.

$$\tan\left(2\theta + \frac{\pi}{4}\right) = 1$$

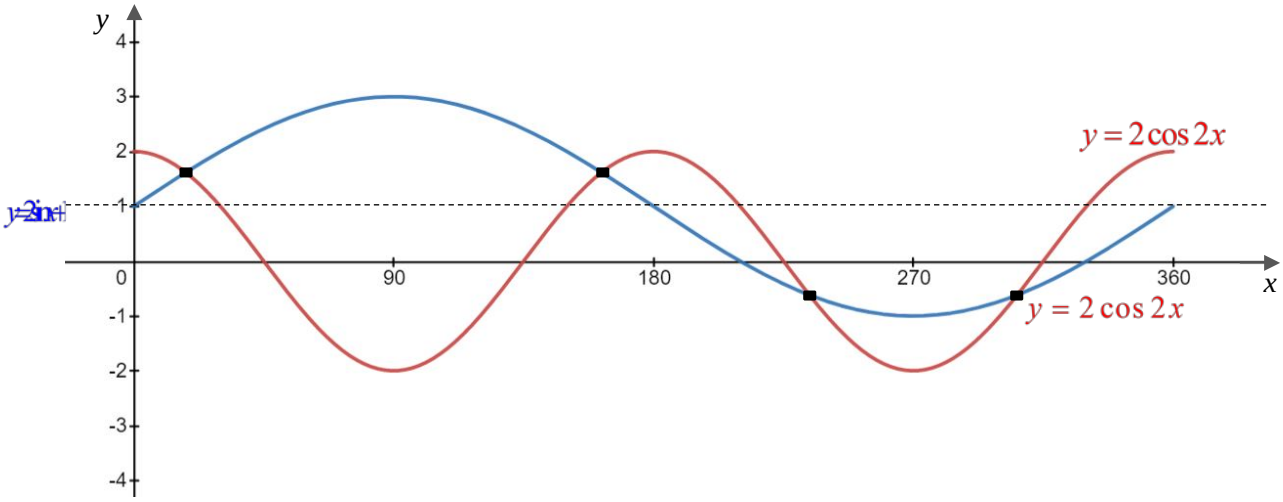


$$\alpha = \tan^{-1}1 = \frac{\pi}{4} \text{ rad}$$

$$\therefore 2\theta + \frac{\pi}{4} = \frac{\pi}{4},\quad \frac{5\pi}{4},\quad \frac{9\pi}{4}$$

$$\therefore \theta = 0,\quad \frac{\pi}{2},\quad \pi$$

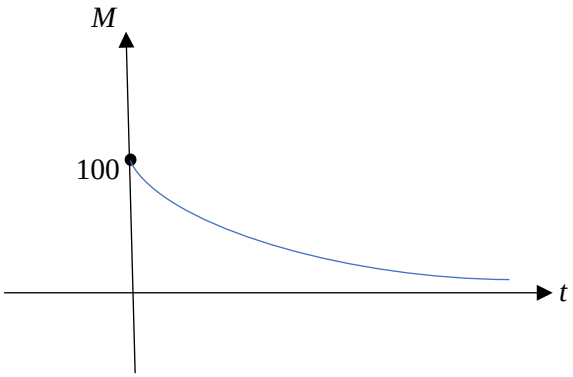
4c.



$$\frac{1}{2} - \cos 2x = -\sin x \quad \Rightarrow \quad 2\cos 2x = 2\sin x + 1$$

$\therefore$ 4 solutions

5a.



5b. $M = 100e^{-0.0004279(20000)}$

$= 0.0192$ g

5c. $50 = 100e^{-0.0004279t}$

$$\frac{1}{2} = e^{-0.0004279t}$$

$$\ln\frac{1}{2} = -0.0004279t$$

$$\therefore t = \frac{\ln\frac{1}{2}}{-0.0004279} \simeq 1619.88$$

$\simeq 1620$

$\therefore$ Half-life of radium is 1620 years.

6a. $y = 3x^2$ --------(1)

$y = px - 3.$ --------(2)

Solving (1) & (2) simultaneously:

$3x^2 = px - 3$ or $3x^2 > px - 3$

$3x^2 - px + 3 = 0$ $3x^2 - px + 3 > 0$

For curve to be above the line: $(-p)^2 - 4(3)(3) < 0$

$p^2 - 36 < 0$

$(p+6)(p-6) < 0$

$\therefore -6 < p < 6$

6b. $-\frac{1}{2} < x < \frac{2}{3} \Rightarrow (2x+1)(3x-2) < 0$

$6x^2 - x - 2 < 0$

Comparing corresponding terms:

$\therefore p = 6, \quad q = -1$

7a. $m_{PR} = \frac{2-(-1)}{0-6} = -\frac{1}{2}$ $\Rightarrow$ Equation for *PR*: $y = -\frac{1}{2}x + 2$

$\Rightarrow m_{QS} = 2$ $\Rightarrow$ Equation for *QS*: $y - (-1) = 2(x-1)$

$\therefore y = 2x - 3$

7b.

Coordinate of *M*

$y = -\frac{1}{2}x + 2$ ------(1)

$y = 2x - 3$ -------(2)

$-\frac{1}{2}x + 2 = 2x - 3$

$\frac{5}{2}x = 5$

$\therefore x = 2, \quad y = 1$

$\therefore M(2,1)$

Coordinate of *P*

Using midpoint *M* of *PR*:

$\frac{x+6}{2} = 2 \quad ; \quad \frac{y-1}{2} = 1$

$\therefore x = -2, \quad y = 3$

$P(-2,3)$

Coordinate of *S*

Both diagonals intersect at *M*:

$1 + x = 6 - 2 \; ; -1 + y = -1 + 3$

$\therefore x = 3, \quad y = 3$

$\therefore S(3,3)$

8a. **Table of values**

x	1	2	3	4	5
$\frac{y+3}{x}$	5	9	13	17	21

Linearizing the given equation:

$y = ax^2 + bx - 3$

$y + 3 = ax^2 + bx$

$\frac{y+3}{x} = ax + b$

$Y = mX + c$

$\Rightarrow m \equiv a \quad ; \quad c \equiv b$

b. **From the graph**:

- Gradient: $m \simeq \frac{23-7}{5.5-1.5} \simeq 4.0 \quad \Rightarrow \therefore a \simeq 4.0$
- $\frac{y+3}{x}$- intercept: c $\simeq 1.0 \quad \Rightarrow \therefore b \simeq 1.0$

c. $ax^2 + (b-20)x = 0$

$ax^2 + bx - 3 = 20x - 3$

$\Rightarrow y = 20x - 3$

$\Rightarrow \frac{y+3}{x} = 20$

$\Rightarrow$ Insert the horizontal line $\frac{y+3}{x} = 20$, or $Y = 20$ then read off the corresponding value of x from the point of intersection.

8a. Graph of $\frac{y+3}{x}$ against x