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CANDIDATE NAME

CLASS

ADMISSION NUMBER

2024 End-of-Year Examination
Pre-University 1

MATHEMATICS **9758/01**

Paper 1 **14 October 2024**

QUESTION PAPER **2 hours 15 minutes**

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

1 The sum, S_n, of the first n terms of the sequence $u_1,\ u_2,\ u_3,\ \ldots$ is given by

$$S_n = n^2 + kn,$$

where k is a non-zero constant.

(i) Find u_n in terms of n and k. [2]

(ii) Hence determine if the sequence is an arithmetic progression. [2]

2 **(i)** Without using a calculator, solve the inequality

$$\frac{x^2 - 3x + 2}{x + 4} \geq 2 .$$ [4]

(ii) Hence solve the inequality

$$\frac{x^2 + 3x + 2}{4 - x} \geq 2 .$$ [2]

3 **(i)** A curve C has equation $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. The curve C intersects the y-axis at the point (0, 7) and passes through the points $(-2, -11)$, $(1,\ 1)$ and $(3,\ 19)$. Find the values of a, b, c and d. [4]

(ii) C first undergoes a reflection about the y-axis and subsequently a translation of 1 unit in the positive y-direction. Find the equation of the resulting curve. [2]

4 Referred to the origin O, the points A, B and C have position vectors **a**, **b** and **c**. C lies on AB such that $AC : AB = 1 : 3$.

(i) Find **c** in terms of **a** and **b**. [2]

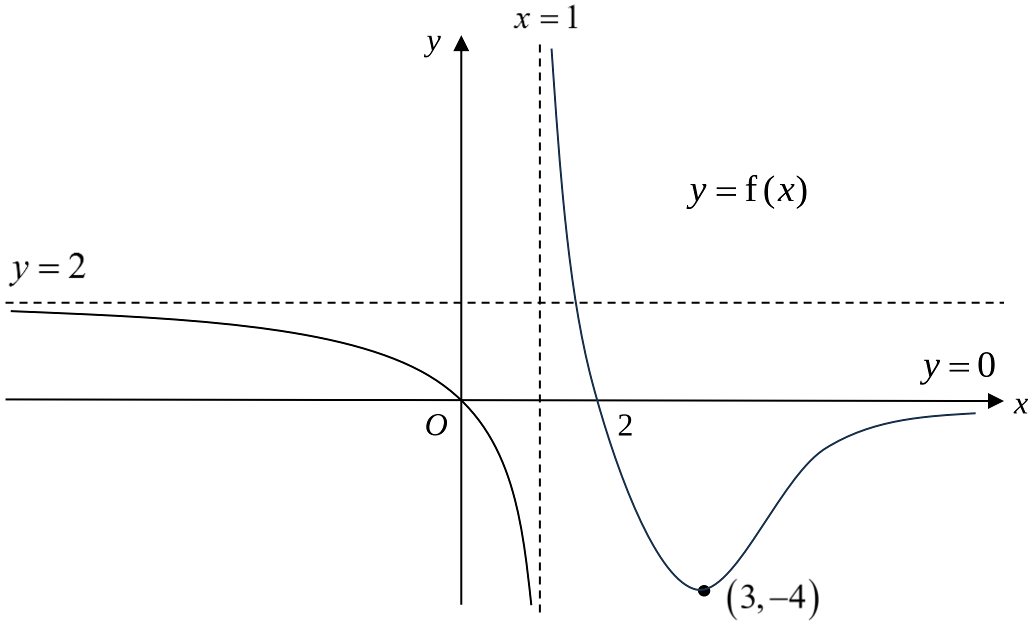
(ii) Show that the area of triangle OAC is given by $k\left|\mathbf{a} \times \mathbf{b}\right|$, where k is a constant to be determined. [2]

It is now given that **a** and **b** are unit vectors.

(iii) Using the result in part **(i)**, find the value of $(2\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}$. [3]

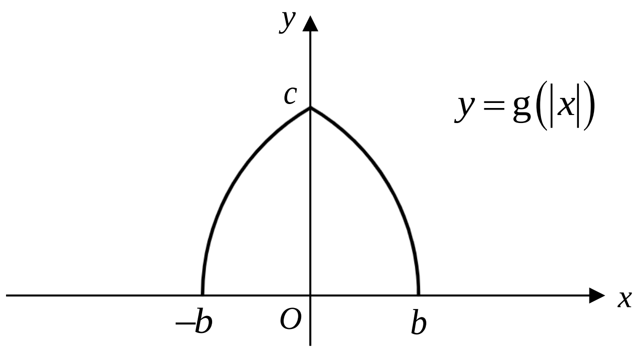
5 **(a)** Describe a sequence of 2 transformations that transforms the curve C with equation $\frac{x^2}{9}+y^2=1$ onto the curve D with equation $(x-2)^2+y^2=1$. [2]

(b) The diagram shows the curve with equation $y=\mathrm{f}(x)$. The curve passes through the origin and the point $(2,\ 0)$ and has a minimum point at $(3,-4)$. The asymptotes of the curve are the lines $x=1$, $y=0$ and $y=2$.



Sketch the graph of $y=\frac{1}{\mathrm{f}(x)}$, indicating the equations of any asymptotes and the coordinates of any points where it crosses the axes and of any turning points, where applicable. [3]

(c) The graph of $y=\mathrm{g}(x)$ is a semi-circle with its centre at the point $(a,\ 0)$, where $a<0$. The diagram shows the graph of $y=\mathrm{g}(|x|)$, which intersects the axes at the points $(-b,0)$, $(b,0)$ and $(0,c)$, where $a\neq -b$.



Sketch the graph of $y=\mathrm{g}(x)$, labelling the coordinates of the centre and the radius of the semi-circle, in terms of a and/or b. [3]

6 The curve C has equation $y=\dfrac{x^2-bx+b}{x-a}$, where a and b are constants. C has a vertical asymptote at $x=1$ and a turning point at $(0,-2)$.

(i) Determine the values of a and b. [2]

Use the values of a and b found in part **(i)** for the following parts.

(ii) Sketch C, stating the coordinates of any points of intersection with the axes and of any turning points and the equations of the asymptotes. [4]

(iii) By adding a suitable curve to your diagram in part **(ii)**, solve the inequality

$$\frac{x^2-bx+b}{x-a}-\ln(5-x)\geq 2.$$ [3]

7 The functions f and g are defined by

$$\begin{aligned} &\mathrm{f}:x\mapsto 5-(x-2)^2, &&\text{for } x\in\mathbb{R},\ x\geq 0,\\ &\mathrm{g}:x\mapsto \mathrm{e}^x+3, &&\text{for } x\in\mathbb{R}. \end{aligned}$$

(i) Show that the composite function fg exists. [2]

(ii) Find an expression for $\mathrm{fg}(x)$ and state its domain. [2]

(iii) Find the range of fg. [2]

(iv) Show that f^{-1} does not exist. [1]

(v) If the domain of f is restricted to $[k,\infty)$, find the smallest value of k such that f^{-1} exists. [1]

For the rest of the question, the domain of f is now restricted to $[k,\infty)$, with the value of k found in part **(v)**.

(vi) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

8 An arithmetic series has first term a and common difference d, where a is non-zero. The 8th, 4th and 2nd term of the arithmetic series are the first three consecutive terms of a geometric series respectively. It is also given that the terms of the arithmetic series are increasing.

(i) Show that $a = d$. [4]

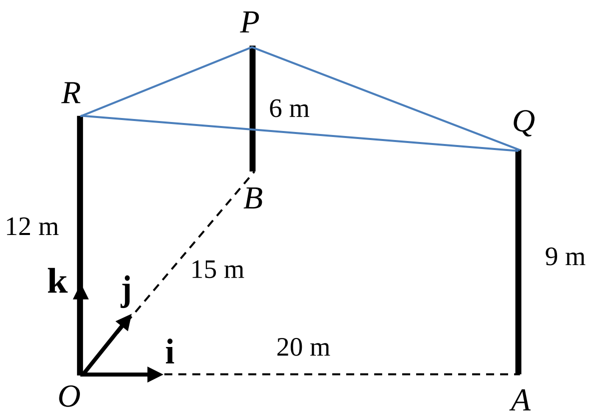
It is given that the sum of the first 20 terms of the arithmetic series is 630.

(ii) Find the value of a. [2]

(iii) Hence find the exact sum of the first 10 terms of the geometric series. [3]

(iv) Give a reason why the geometric series converges and find the sum to infinity of this series. [2]

9 To construct a special tentage, 3 steel poles OR, AQ and BP are secured to the horizontal ground such that each of the poles are perpendicular to the ground. The point O on the ground is taken as the origin and the unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OR respectively, with units in metres. It is given that $OR = 12$, $AQ = 9$, $BP = 6$, $OA = 20$ and $OB = 15$ (see diagram).



(i) Find $\overrightarrow{PR}$ and $\overrightarrow{PQ}$. [2]

(ii) The canvas roof PQR, which is of negligible thickness, is part of the plane p. Using your answers in part **(i)**, show that the equation of p is given by

$$\mathbf{r} \cdot \begin{pmatrix} 3 \\ 8 \\ 20 \end{pmatrix} = 240\,.$$ [2]

(iii) Find the acute angle between the canvas roof and the plane *OAB*. [2]

(iv) A cable is to be installed close to the tentage. This cable is part of the line *L* with vector equation

$$\mathbf{r} = \begin{pmatrix} -2 \\ 8 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} 20 \\ -13 \\ -4 \end{pmatrix}, \text{ where } \lambda \in \mathbb{R} .$$

Show that this cable is not parallel to the edge *PR* of the roof and does not meet the edge *PR*. [4]

(v) A lamp is hung vertically downwards from the canvas roof using a cable such that it is at the point *M* with coordinates $\left(\frac{20}{3},\ 5,\ 8\right)$.

To secure the lamp, a second cable is connected from the lamp to the point *N* on the roof. It is assumed that the cables are laid in straight lines and have negligible thickness.

For the shortest length to be used for the second cable, show that

$$\overrightarrow{ON} = \begin{pmatrix} s \\ \frac{2525}{473} \\ t \end{pmatrix},$$

where *s* and *t* are constants to be found. [3]

End of Paper