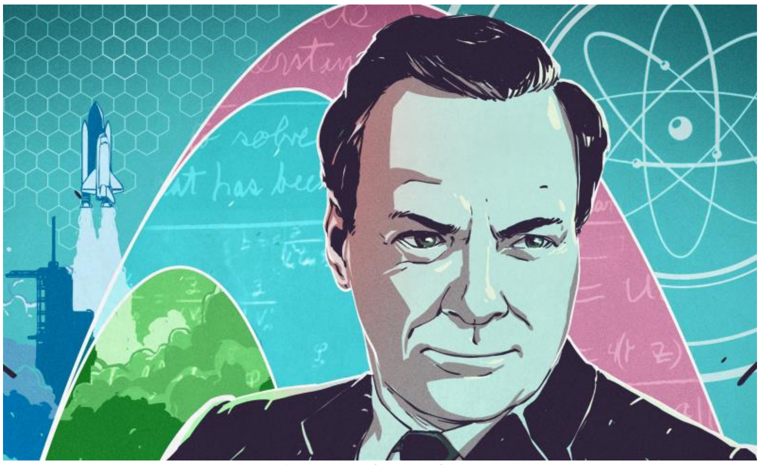
H2 Topic 05 – Work, Energy and Power



Richard Feynman in 1961 said, *"there is a fact, or if you wish, a law, governing all natural phenomena that are known to date. There is no known exception to this law—it is exact so far as we know. The law is called the* ***conservation of energy****. It states that there is a certain quantity, which we call* ***energy*** *that does not change in manifold changes which nature undergoes. That* ***is a most abstract idea, because it is a mathematical principle****; it says that there is a numerical quantity which does not change when something happens.* ***It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number and when we finish watching nature go through her tricks*** *and calculate the number again, it is the same."*

Content

- Work
- Energy conversion and conservation
- Efficiency
- Potential energy and kinetic energy
- Power

Learning Outcomes

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force
(b) calculate the work done in a number of situations including the work done by a gas* which is expanding against a constant external pressure: $W = p\Delta V$
(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation
(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems
(e) derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_k = ½mv^2$
(f) recall and use the equation $E_k = ½mv^2$
(g) distinguish between gravitational potential energy, electric potential energy^ and elastic potential energy
(h) deduce that the elastic potential energy in a deformed material is related to the area under the force extension graph
(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems
(j) derive, from the definition of work done by a force, the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface
(k) recall and use the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface
(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force

* Not required for 8867 H1 Physics, will be revised in greater details in H2 Topic 9: First Law of Thermodynamics

^ Will be dealt with in H2 Topic 13: Electric Fields

5.0 Introduction

Sometimes, trying to analyse Physics scenarios using forces can be very complicated – there may be many forces or there are many interactions between various forces. Using the relationships between work, energy and power can be an alternative to deciphering the Physics.

5.1 Work

In Physics, work has a very specific meaning:

> **Work done by a force** is the product of the force and the displacement in the direction of the force.

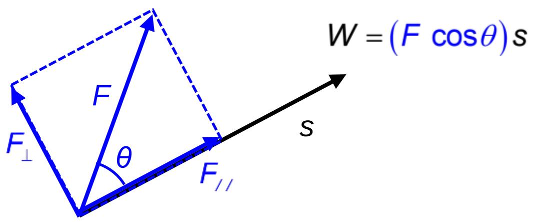
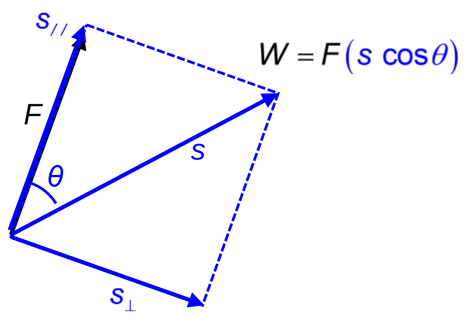
As both force *F* and displacement *s* are vector quantities, the angle *θ* helps to manage the information concerning direction.

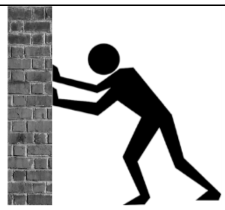
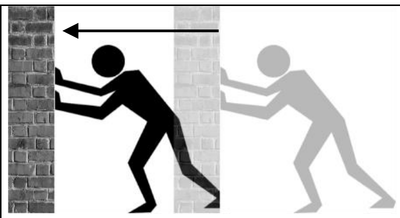
$$\boxed{W_{\text{by force}} = Fs\cos\theta}$$

Mathematically, the operation is known as the dot product or scalar product of 2 vectors:

$$W = F.s$$

We can interpret the above either of two ways:

the product of • the displacement and • the *component of force* $F_{//} = F\cos\theta$ along the same direction as the displacement  	the product of • the force and • the *component of displacement* $s_{//} = s\cos\theta$ along the same direction as the force 

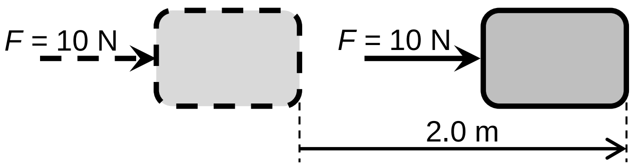
		
☑ force exerted	☑ force exerted	☑ force exerted
☑ weights displaced by force	☒ wall not displaced by force	☑ wall displaced by force
☑ work is done by force of man on weight	☒ no work done by force of man on wall	☑ work is done by force of man on wall

The S.I. unit for work done is *joule* (J). 1 J is defined as the work done when a force of 1 N moves its point of application through a distance of 1 m in the direction of the force.

Example 1

Calculate the work done by force F below where the body moves 2.0 m to the right:

(a)



Solution

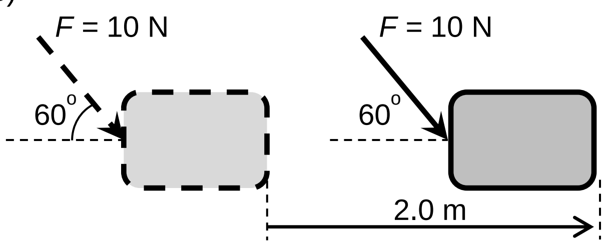
force is same direction with displacement

$$\theta = 0$$

$$W_F = Fs\cos\theta = Fs$$

$$= (10)(2.0) = 20 \text{ J}$$

(b)

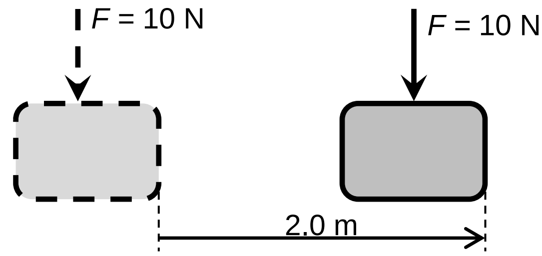


$$\theta = 60°$$

$$W_F = Fs\cos\theta = Fs\cos(60°)$$

$$= (10)(2.0)(\cos(60°)) = 10 \text{ J}$$

(c)



force is perpendicular to displacement

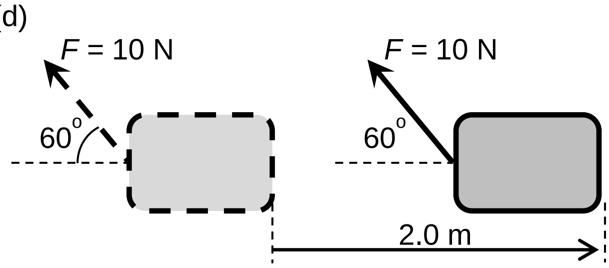
$$\theta = 90°$$

$$W_F = Fs\cos\theta = Fs\cos(90°)$$

$$= 0$$

no work done

(d)

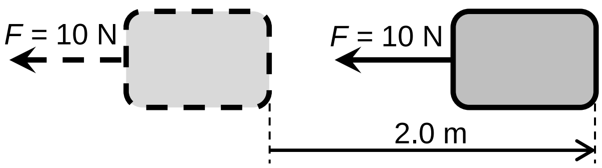


$$\theta = 120°$$

$$W_F = Fs\cos\theta = Fs\cos(120°)$$

$$= (10)(2.0)(\cos(120°)) = -10 \text{ J}$$

(e)



$$\theta = 180°$$

$$W_F = Fs\cos\theta = Fs\cos(180°)$$

$$= (10)(2.0)(\cos(180°)) = -20 \text{ J}$$

Notes: Work is a *process* that causes *energy transfer* to or from a body.

Positive work done on a body result in mechanical energy being transferred to the body, and hence the body gains energy.

Negative work done in (d) and (e) show mechanical energy being transferred away from the body. So, if the body was already moving to the right with constant speed (therefore constant kinetic energy) and then a leftward braking force is applied across 2.0 m of displacement - the body will lose kinetic energy.

5.1.1 Work Done by a Varying Force

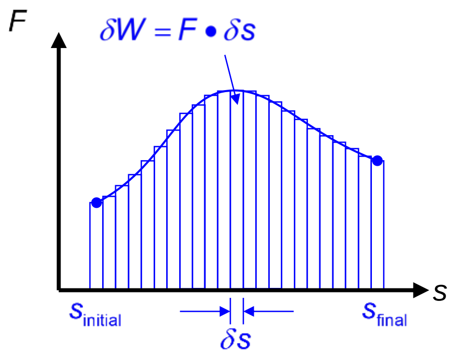
In general:

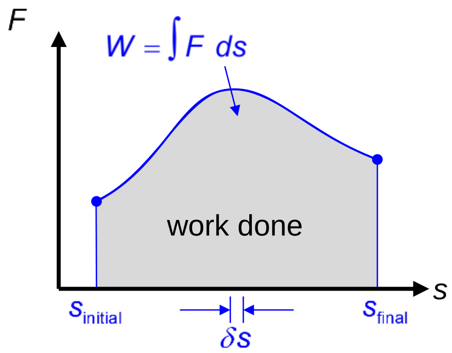
The area under a force-displacement graph is the work done by the force.

If a force is constant, then

- area under the F-s graph is a simple rectangle
- the area can be found by "length × height"
- which reduces to the equation given in 5.1.

We regard each small rectangular strip as having width δs over which a force F acts:





5.1.2 Work Done by an Expanding Gas

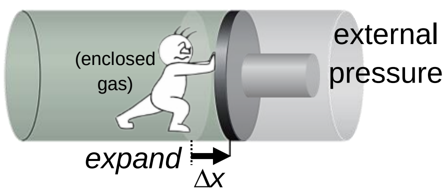
Consider a gas enclosed in a cylinder of cross-sectional area A fitted with a light, frictionless piston. When the gas expands against an external pressure, it does work against external pressure.

Work done <u>by</u> gas, $W_{by} = p\Delta V$ $= p(V_{final} - V_{initial})$

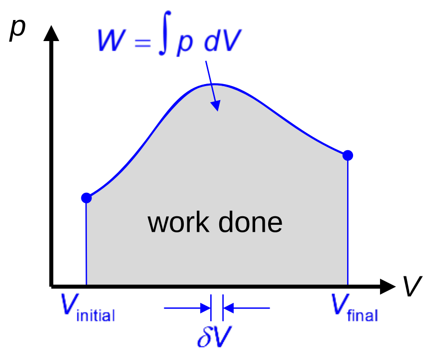
p : external pressure (Pa)
$V_{initial}$: initial volume (m^{-3})
V_{final} : final volume (m^{-3})

We imagine the gas "exerting" a force against external pressure as the piston is displaced "outwards"

$$
\begin{aligned}
W_{by} &= Fs\cos\theta \\
&= (F)(\Delta x) \\
&= (pA)(\Delta x) \\
&= (p)(A\Delta x) \\
&= p\Delta V
\end{aligned}
$$



The equation above assumes that the external pressure remains constant. In the event that pressure is not constant, we can apply the same principle as Section 5.1.1, and the work done by the expanding gas is given by the **area under the pressure-volume graph**.



5.2 Energy

When dealing with problems involving the motion of a body, we typically consider the mechanical energy, which includes kinetic energy and potential energy.

5.2.1 (Translational) Kinetic Energy

Kinetic energy is
the energy possessed by a mass due to its speed/motion.

$$E_K = \frac{1}{2}mv^2$$

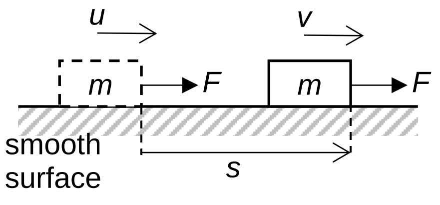
E_K : kinetic energy (J)
m : mass (kg)
v : velocity (m s^{-1})

For H2 Physics, we concentrate on translational motion and the associated kinetic energy. H2 Physics does not equip us with sufficient tools to analyse rotational motion, such as the rotational kinetic energy associated with a cylinder rolling down a slope.

Students who are interested can read up about moment of inertia, angular momentum, and the Newton's 2nd Law equivalent for rotation. You may also wish to consider offering H3 Physics.

Derivation

Consider a <u>mass *m* that is displaced by a displacement *s*</u> by a <u>constant net force *F*</u> on a <u>smooth, horizontal surface</u>. It has an initial velocity *u* and final velocity *v*:



Mass experiences constant acceleration due to constant force

$$v^2 = u^2 + 2as \rightarrow s = \frac{v^2 - u^2}{2a}$$

Work done on mass by force *F* is

$$\begin{aligned} W &= Fs = (ma)s \\ &= (ma)\left(\frac{v^2 - u^2}{2a}\right) \\ &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \end{aligned}$$

Since work done by force *F* results in the object's kinetic energy (only, not potential energy) thus $W = \Delta E_K = E_{K,\text{ final}} - E_{K,\text{ initial}}$

If mass is initially at rest (u = 0) then $E_{K,initial} = 0$ and kinetic energy of the object, $E_K = \frac{1}{2}mv^2$

5.2.2 Gravitational Potential Energy

Gravitational potential energy is the energy of a mass due to its position in a gravitational field.

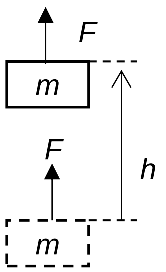
Near Earth's surface, ***change*** in gravitational potential energy is

$$\Delta E_P = mgh$$

E_p : potential energy (J)
m : mass (kg)
H : change in height (m)

The above equation $E_P = mgh$ <u>does not provide an absolute value for the gravitational potential energy</u>. The zero reference for height h can be chosen and is often taken to be at Earth's surface.

Derivation
Consider a <u>mass inside a</u> <u>uniform gravitational field of field strength g</u> that is <u>displaced upwards by h</u> due to a <u>constant net force F at constant speed</u>:



Mass has constant speed so no net upward force
magnitude of $F = W$

Work done on mass by force F is
$$W = Fs = Wh$$
$$= mgh$$

Work done by F changes the body's gravitational potential energy
$$W = \Delta E_P = mgh$$

Note: The constant speed ensures that the kinetic energy remains constant; and that the work done by the force does not transfer energy to or from the body's kinetic energy. The work done in this case affects purely the gravitational potential energy only.

5.2.3 Elastic Potential Energy

Elastic potential energy is the ability to do work by the object when it is deformed.

For a spring that obeys Hooke's law:

$$\text{elastic } E_P = \frac{1}{2}kx^2$$

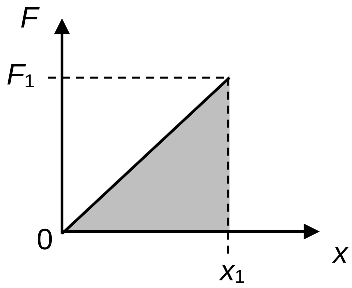
Deformation above can mean change in shape, being compressed, being stretched or being strained.

Recall that "elastic" in Physics conveys a meaning of being able to return to original state. So in a spring, elastic deformation implies that the spring returns to original length when the external force is removed.

E_p : potential energy (J)
k : elastic spring constant (N m^{-1})
x : length object is stretched or compressed (m s^{-1})

Derivation

Consider a spring being stretched by an extension of x_1 due to work done by force F:



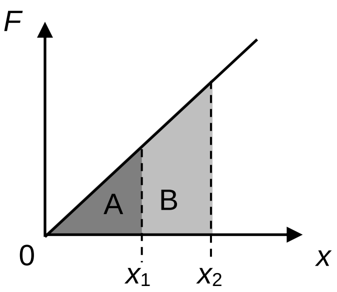
area under force-displacement graph gives work done

$$\begin{aligned}\text{area } &= \frac{1}{2}F_1x_1 \\ &= \frac{1}{2}(kx_1)\,x_1 \\ &= \frac{1}{2}kx_1^2\end{aligned}$$

Note: The “elastic” concept here is that we can recover all of the stored energy in the stretched spring and that the quantity of energy is equal to the initial input of energy through work done.

Example 2

A force F is applied to an extended spring such that its extension increases from x_1 to x_2.



Which shaded area(s) represent the

(a) energy stored in spring when it has extension of x_2?

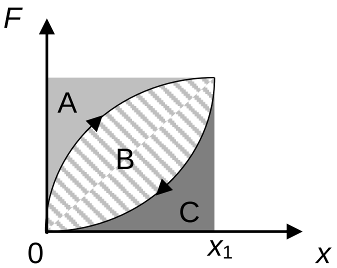
A + B

(b) increase in elastic potential energy when its extension increases from x_1 to x_2?

B

Example 3

The diagram shows an external force F stretching a material by an extension of x_1 where the upper path is taken. The force is then removed and the lower path is taken.



(a) Which shaded area(s) represent the

(i) work done by the external force in stretching the material?

B+C

(ii) elastic potential energy stored in the spring?

C (=recoverable energy)

(b) What does area B represent?

energy dissipated (e.g. heat) in material after undergoing the whole process of stretching and removal of the external force.

5.3 Principle of Conservation of Energy

Energy cannot be created or destroyed – it can only be converted from one form to another. This implies that the total energy of an isolated system remains constant. There can be two ways of looking at energy changes in systems:

Perspective 1:

total energy of **initial** state of system	net work done on system → process	total energy of **final** state of system

$$E_{\text{K, initial}} + E_{\text{P, initial}} \quad + W_{\text{on system}} \quad = \quad E_{\text{K, final}} + E_{\text{P, final}}$$

In word equations, Perspective 1 would be written as:

Total Initial Energy + Work done on system = Total Final Energy

Total Initial Energy – Work done against resistive forces = Total Final Energy

Perspective 2:

initial state		final state
initial kinetic energy	net work done on system	**final** kinetic energy
initial gravitational potential energy	→ process	**final** gravitational potential energy
... (other type(s) of energy)		... (other type(s) of energy)

⇓

final kinetic energy		**initial** kinetic energy		
final gravitational potential energy	minus	**initial** gravitational potential energy	equal	net work done on system
... (other type(s) of energy)		... (other type(s) of energy)		

In word equations, Perspective 2 would be more intuitive when written as:

Decrease in [e.g. E_P or E_K of X] + Work done on system = Increase in [e.g. E_P or E_K of Y]

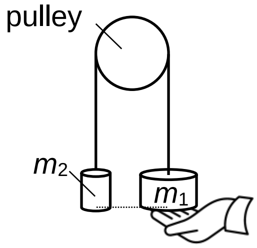
Decrease in [e.g. E_P or E_K of X] = Increase in [e.g. E_P or E_K of Y] + Work done by system
(e.g. against resistive forces)

> It would also be helpful to consider cases where there is **no net external work done** on the system. In this case, the word equations can be simplified as:
>
> Decrease in energies (E_p or E_K) = Increase in energies (E_P or E_K)

These perspectives are not "hard and fast" rules. Most problems can be successfully solved so long as you properly account for energy changes within the system.

Example 4

Two cylinders of different masses $m_1 > m_2$ are connected by a light inextensible string passing over a fixed smooth pulley. Which of the following options correctly describes the changes in energies of the both masses after they are released from rest?



	E_P	E_K
A	increase	increase
B	increase	decrease
C	decrease	increase
D	decrease	decrease

C. We apply perspective 2 and note that this is a scenario where there is no work done on system:

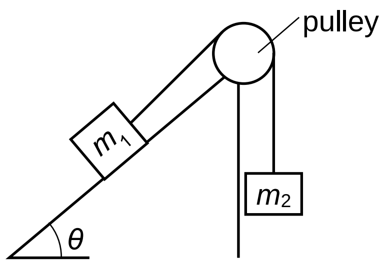
Mass m_1 accelerates downwards while m_2 accelerates upwards with the same magnitude.

$$\text{gain in } E_K = \text{loss in } E_P$$

We can also regard the net effect as the centre of gravity between m_1 and m_2 accelerating down.

Example 5

Two blocks each of equal mass are connected by a light inextensible string passing over a smooth fixed pulley. When released from an initial state of rest, m_2 falls vertically down while m_1 moves up the smooth plane inclined at $\theta = 30°$ to the horizontal. Calculate the speed of the masses when m_1 moves 2.0 m up the plane.



Solution

$$\sin\theta = \frac{\Delta h}{2}$$

$$\Delta h = 2\sin(30°) = 1.0 \text{ m}$$

m_1 gained 1.0 m in height

By Principle of Conservation of Energy

$$\begin{matrix}\text{gain in} \\ E_K \text{ of } m_1\end{matrix} + \begin{matrix}\text{gain in} \\ E_K \text{ of } m_2\end{matrix} + \begin{matrix}\text{gain in} \\ E_P \text{ of } m_1\end{matrix} = \begin{matrix}\text{loss in} \\ E_P \text{ of } m_2\end{matrix}$$

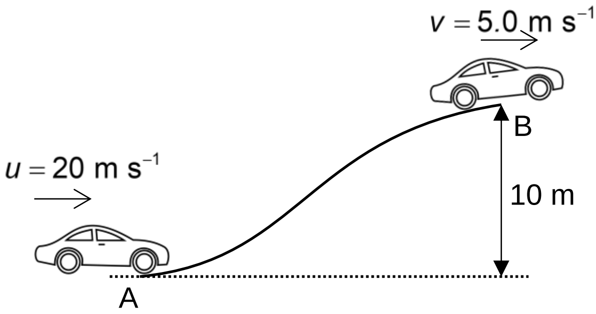
$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + m_1 g\Delta h = m_2 g h_{\text{drop}}$$

$$v = \sqrt{\frac{2mg\left(h_{\text{drop}} - \Delta h\right)}{2m}} = \sqrt{g\left(h_{\text{drop}} - \Delta h\right)}$$

$$= \sqrt{(9.81)(2.0 - 1.0)} = 3.1 \text{ m s}^{-1}$$

Example 6

A car of mass 2000 kg reaches point A, the foot of a hill, with initial speed of $u = 20\text{ m s}^{-1}$. The engine is cut and the car coasts up the hill. The car reaches point B, which is 10 m above the elevation of point A with a final speed of $v = 5.0\text{ m s}^{-1}$. The distance between A and B is 40 m. Calculate the average resistive force acting on the car when it coasts.



Solution

(Perspective 1) by Principle of Conservation of Energy

$$E_{\text{K, initial}} + E_{\text{P, initial}} + \left(-W_{\text{resistive, on car}}\right) = E_{\text{K, final}} + E_{\text{P, final}}$$
$$E_{\text{K, initial}} + E_{\text{P, initial}} = E_{\text{K, final}} + E_{\text{P, final}} + W_{\text{against resistive force}}$$

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgh + F_{\text{resistive}}s$$

$$F_{\text{resistive}} = \frac{m\left(u^2 - v^2 - 2gh\right)}{2s} = \frac{(2000)\left(20^2 - 5.0^2 - 2(9.81)(10)\right)}{2(40)}$$
$$= 4470\text{ N}$$

(Perspective 2) by Principle of Conservation of Energy

Loss in E_K = Gain in E_P + Work Done against resistive force

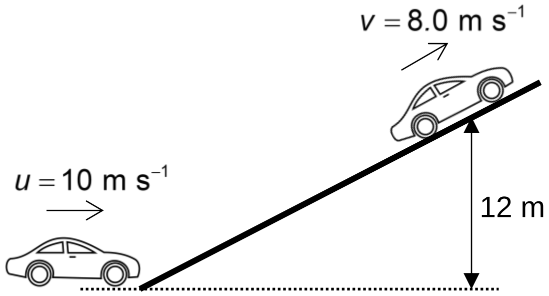
$$\frac{1}{2}m\left(u^2 - v^2\right) = mgh + Fs$$

$$F = \frac{-1}{s}\left(\frac{1}{2}m\left(v^2 - u^2\right) + mgh\right)$$
$$= \frac{m\left(u^2 - v^2 - 2gh\right)}{2s} = \frac{(2000)\left(20^2 - 5^2 - 2(9.81)(10)\right)}{2(40)}$$
$$= 4470\text{ N}$$

Note: There is no "hard and fast" rule on which perspective to use. For this example, we deliberately kept all terms positive. Both perspectives are equally valid as long as you keep track of the energy changes properly.

Example 7

A car of mass 2000 kg goes up a slope, with initial speed of $u = 10 \text{ m s}^{-1}$. The engine continues to provide a constant driving force as the car goes up the hill. The car reaches the top of the slope, which is 12 m vertically above point where it enters the slope, with a final speed of $v = 8.0 \text{ m s}^{-1}$. The distance the car traversed along the slope is 50 m.



Calculate the driving force acting on the car as it goes up the hill.

Solution

(Perspective 1) by Principle of Conservation of Energy

$E_{\text{K, initial}} + E_{\text{P, initial}} + W_{\text{driving, on car}} = E_{\text{K, final}} + E_{\text{P, final}}$

$$\frac{1}{2}mu^2 + F_{driving}s = \frac{1}{2}mv^2 + mgh$$

$$F_{driving} = \frac{m\left(v^2 + 2gh - u^2\right)}{2s} = \frac{(2000)\left(8.0^2 + 2(9.81)(12) - 10^2\right)}{2(50)}$$
$$= 3990 \text{ N}$$

(Perspective 2) by Principle of Conservation of Energy

Loss in E_K + Work Done by driving force = Gain in E_P

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 + F_{driving}s = mgh$$

$$F_{driving} = \frac{m\left(2gh + v^2 - u^2\right)}{2s} = \frac{(2000)\left(2(9.81)(12) + 8.0^2 - 10^2\right)}{2(50)}$$
$$= 3990 \text{ N}$$

Note: An assumption was made in arriving at the answer. Can you spot it?

5.4 Power

Power is the work done per unit time.	$P = \frac{W}{t}$	P : power (W) W : work done (J) t : time (s)

Power is a scalar quantity. The S.I. unit for power is *watt* (W). 1 W is the rate when 1 J of work is done per 1 s.

Power can also be viewed as the
- (i) rate of work done or
- (ii) rate of transfer of energy, or
- (iii) rate of conversion of energy.

Sometimes, power can be expressed as the product of a force F and velocity v in the direction of the force:

$$\boxed{P = Fv}$$

Derivation

When power is defined as rate of work done, then

$$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

$$= \frac{F.\mathrm{d}s}{\mathrm{d}t} \quad \text{where d}s \text{ is the displacement in the direction of } F$$

$$= Fv \quad \text{where } v \text{ is the velocity in the direction of } F$$

Average power over a time interval is equal to the total work done or the energy transferred / converted divided by the time interval:

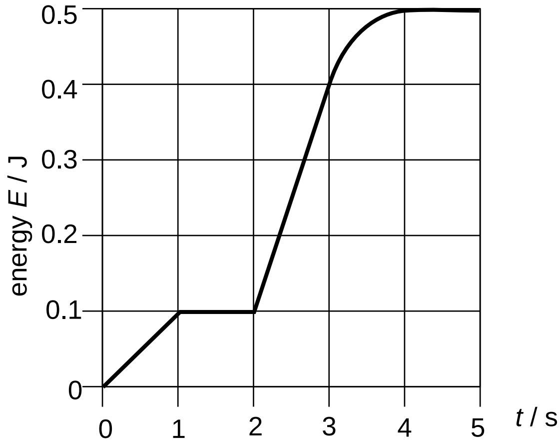
$$\langle P \rangle = \frac{\text{twork done OR energy transferred/converted}}{\text{time taken}}$$

$$\boxed{= \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t}}$$

$\langle P \rangle$: average power (W)
W : work done (J)
ΔE : energy transferred / converted (J)
Δt : time interval (s)

Example 8

A bicycle dynamo attaches to the wheel and converts kinetic energy into electrical energy. A particular dynamo starts operating at time t = 0 s. In the first 5.0 s, the total energy transformed by the dynamo increases as shown in the graph.



For the first 5.0s, calculate the

(a) maximum instantaneous power, and

(b) average power

Solution

(a) Gradient of graph gives rate of energy E converted, or the rate of work done by the dynamo. In other words, gradient gives the instantaneous power.

Max instantaneous power is where gradient is steepest, which happens between t = 2.0 s and t = 3.0 s.

$$\begin{aligned} P_{\max} &= \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\max} \\ &= \frac{0.40-0.10}{3.0-2.0} \\ &= 0.30\ \text{W} \end{aligned}$$

(b)

$$\begin{aligned} \langle P \rangle &= \frac{\text{total work done}}{\text{time taken}} \\ &= \frac{\Delta E}{\Delta t} = \frac{E_{5.0}-E_{0.0}}{5.0-0.0} \\ &= \frac{0.50-0.0}{5.0} \\ &= 0.10\ \text{W} \end{aligned}$$

Note: since gradient is instantaneous power, you could also think of average power as the 'average gradient' in the time interval.

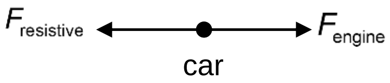
Example 9

A car of mass 1600 kg traveling at a constant speed of $25\ \text{m s}^{-1}$ on a level road meets 700 N of resistive force.

(a) Calculate the power developed by the engine.

(b) Calculate the additional power needed for the car to travel up a slope of 10° incline at the same speed.

Solution



(a) Constant speed so no net force. Magnitude of the force developed by the engine is the same as resistive forces.

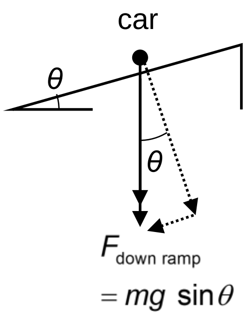
$$\text{magnitude of } F_{\text{engine}} = F_{\text{resistive}}$$

$$\begin{aligned} P_{\text{engine}} &= F_{\text{engine}} v \\ &= 700(25) \\ &= 17\ 500\ \text{W} \end{aligned}$$

(b) additional power needed = rate of increase of E_P

$$\begin{aligned} &= \frac{mgh}{t} = mgv_y = mg(v\ \sin\theta) \\ &= (1600)(9.81)\left[25\ \sin(10^\circ)\right] \\ &= 68\ 100\ \text{W} \end{aligned}$$

OR

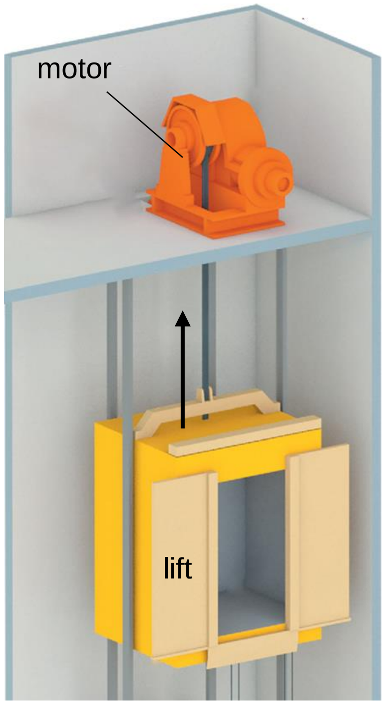


engine needs to provide additional force to balance out the force acting down the ramp

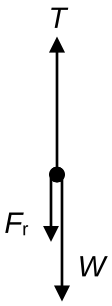
$$\begin{aligned} &P_{\text{additional}} \\ &= F_{\text{additional}} v \\ &= (mg\ \sin\theta) v \\ &= \left[(1600)(9.81)\ \sin(10^\circ)\right](25) \\ &= 68\ 100\ \text{W} \end{aligned}$$

Example 10

A lift of mass 1000 kg can carry a maximum load of 800 kg. During operation, a constant frictional force of 4000 N opposes upward motion. Calculate the minimum power that the motor needs to deliver such that a fully-loaded lift can ascend at a constant speed of $3.0\ \text{m s}^{-1}$.



Solution



constant speed
so lift is in translational equilibrium
during upwards motion.

no resultant force upwards so

$$\text{magnitude of } T = F_r + W$$
$$= F_r + m_{max} g$$

$$\text{magnitude of } T = \text{magnitude of } F_{motor}$$

$$P = F_{motor} v$$
$$= (4000 + (1000 + 800)(9.81))(3.0)$$
$$= 6.50 \times 10^4\ \text{W}$$

5.5 Efficiency

Efficiency

$$= \frac{\text{useful work done}}{\text{energy input}} \times 100\% = \frac{\text{useful power output}}{\text{power input}} \times 100\%$$

By the Principle of Conservation of Energy, the efficiency of devices cannot be more than 100%, and is almost always less than 100% in the real world as some energy will inevitably be dissipated in other forms alongside the desired output energy.

Example 11

A small motor is used to raise a weight of 2.0 N through a vertical height of 0.80 m. The efficiency of the motor is 20%. Calculate the minimum energy required by the motor to raise the weight.

Solution

$$\text{efficiency } = \frac{\text{useful work done}}{\text{energy input}} \times 100\% = 20\%$$

$$\text{energy input} = \frac{100}{20}(\text{useful work done}) = 5(mgh)$$

$$= 5Wh = 5(2.0)(0.80)$$

$$= 8.0 \text{ J}$$

Example 12

Over a section of the Niagara Falls, water flows at a mass rate of 5.0×10^6 kg s^{-1} and falls 50 m.

(a) Calculate the power generated by the falling water.

(b) If a hydropower plant with 95% efficiency is situated here, how many 50 W bulbs can be lit?

Solution

(a)

$$\text{power } = \frac{\text{conversion of energy}}{\text{time taken}} = \frac{\Delta E_P}{\text{time taken}} = \frac{mgh}{t} = \frac{m}{t}gh$$

$$= (5.0 \times 10^6)(9.81)(50)$$

$$= 2.45 \times 10^9 \text{ W}$$

(b) let n be the number of bulbs

$$\text{efficiency } = \frac{\text{power output}}{\text{power input}} \times 100\% = 95\%$$

$$\text{power output} = \frac{95}{100}(\text{power input})$$

$$50n = \frac{95}{100}(2.45 \times 10^9)$$

$$n = \frac{95}{5000}(2.45 \times 10^9)$$

$$= 4.66 \times 10^7 \text{ bulbs}$$

Example 13

A car has a mass of 800 kg. It has an efficiency rated at 18%. The petrol that is pumped into its tank has a chemical potential energy value of 5.0×10^{7} J kg^{-1}. Calculate the mass of petrol burnt to accelerate the car from rest to 30 m s^{-1}.

Solution

$$\text{efficiency} = \frac{\text{useful work done}}{\text{energy input}}\times100\% = 18\%$$

$$\frac{\Delta E_K}{\text{energy input}} = \frac{18}{100}$$

$$\text{energy input} = \frac{100}{18}\left(\frac{1}{2}mv^2\right) = \frac{100}{18}\left(\frac{1}{2}(800)(30)^2\right)$$

$$= 2.0\times10^{6}\ \text{J}$$

$$\text{mass of petrol burnt} = \frac{2.0\times10^{6}}{5.0\times10^{7}} = 0.040\ \text{kg}$$

Note: The mass of petrol burnt is insignificant (5 orders of magnitude smaller) so we can take that the mass of the car is effectively constant during the acceleration.

Example 14

NASA's Perseverance Mars rover carries a multi-mission radioisotope thermoelectric generator (MMRTG) of mass 45 kg to supply electricity. A MMRTG is essentially a nuclear battery that reliably converts heat from the decay of plutonium-238 into electricity. A particular MMRTG is designed to provide 125 W of electrical power at the start of a mission, which falls linearly to 100 W after 14 years. Calculate the energy density (i.e. energy contained per unit mass) of the MMRTG in the 14-year lifespan. Compare the value with the energy density of a 3.7 V battery in a typical smartphone.

Solution

$$\text{total energy output in 14 y} = \langle P\rangle t = \left[\frac{1}{2}(125+100)\right]\left[14(365)(24)(60)^2\right]$$

$$= 4.97\times10^{10}\ \text{J}$$

$$\text{energy density in MMRTG} = \frac{\text{energy}}{\text{mass}} = \frac{4.97\times10^{10}}{45}$$

$$= 1.10\times10^{8}\ \text{J kg}^{-1}$$

$$\text{stored chemical potential energy} = QV \approx (4000\ \text{mAh})(3.7\ \text{V}) = \left(4000\times10^{-3}\right)\left(60^2\right)(3.7)$$

$$= 53\ 280\ \text{J}$$

$$\text{mass of smartphone} \approx 200\ \text{g}$$

$$\text{energy density in smartphone} \approx \frac{\text{energy}}{\text{mass}} = \frac{53\ 280}{200\times10^{-3}}$$

$$= 266\ 400\ \text{J kg}^{-1} = 2.66\times10^{5}\ \text{J kg}^{-1}$$

Energy density in a MMRTG is about 3 orders of magnitude greater than smartphone battery.

5.6 Ending Notes

Because energy is a scalar, you may find that on occasions, solving a mechanics problem using energetic considerations can be easier due to not dealing with directional information.

As a reminder, when faced with a question that you are uncertain about the approach for, a good "fall-back strategy" is to consider the following matrix *after you have decided on the system/object of interest*:

	magnitude	**Direction**
force	"which force?" "how strong?" "are they balanced?" "what is the resultant?"	"where is the force directed to?" "is the force being displaced?"
energy	"what type of energy?" "how much energy?"	"being converted from what type to what type?" "being transferred from who to who?" "is there energy loss / dissipation?" "is there motor / engine / driving force doing work?"

Use the space below to do up your own mind-map to summarise this topic.