Subject TG	Class	Index Number

Candidate Name ________________________________

ST ANDREW'S SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS 4052/02

Paper 2

TUESDAY 15 AUGUST 2023 **2 hours 15 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, subject TG, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\left(\frac{x^8}{16y^{12}}\right)^{-\frac{1}{4}}$.

Answer ..... [2]

(b) Simplify $\frac{15ax+10bx-6ay-4by}{9a^2-4b^2}$.

Answer ..... [3]

(c) $a=\frac{1-b}{3+b}+3$

Express b in terms of a.

Answer b = ... [3]

(d) Solve the equation $x^2 - 7x + 9 = 0$ by completing the square.
Leave your answers correct to 3 decimal places.

Answer x = ... or ..... [3]

2 **(a)** Ruth bought a refrigerator selling at $2300.
She chose to pay for the item by monthly instalments over 2 years.
A simple interest of 1.5% per annum was charged on the price of the refrigerator.

Calculate the amount Ruth needed to pay each month.
Give your answer correct to the nearest cent.

Answer $ ..... [3]

(b) Ruth changed Singapore dollars ($) $800 into Malaysian Ringgit (RM) for her holiday trip.
The exchange rate was $1 = RM 3.16.
She spent RM 2250 and then changed the remaining money back into Singapore dollars upon return from her trip.

How many dollars did she receive on her return?
Give your answer correct to the nearest dollar.

Answer $ ..... [3]

(c) Ruth invested $\$P$ in an account.
The account had a compound interest rate of 1.8% per annum compounded quarterly.
After 2 years, the money had earned **total interest** of \$675.68.

Calculate the value of P.

Answer P = ... [3]

3 **(a)** **(i)** The sum of the first n terms of a sequence is given by $pn^2 + qn$.

The sum of the first 2 terms is 26.
The sum of the first 4 terms is 76.

Form and solve the two simultaneous equations to find the value of p and the value of q.

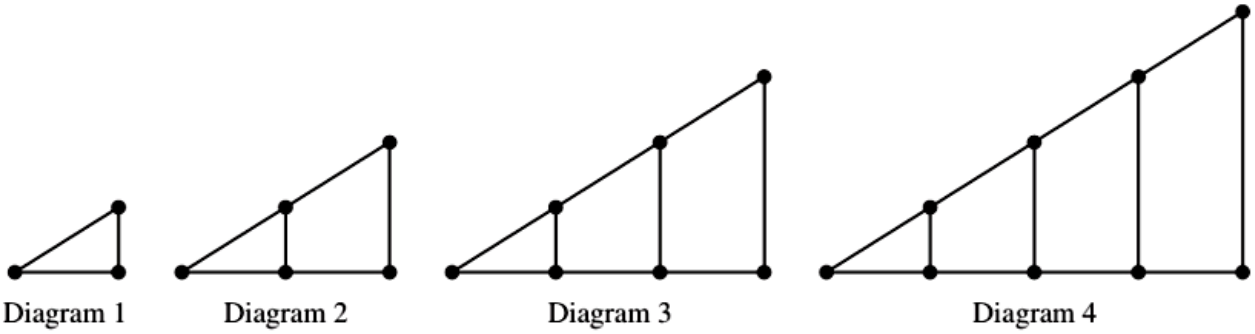
Answer p = ...

q = ... [3]

(ii) Hence, find the 90th term of this sequence.

Answer ..... [2]

(b) The first four diagrams in a sequence are shown below.



(i) In Diagram 2, there are 5 dots and 6 lines.

Find the number of dots and the number of lines in Diagram 8.

Answer Dots = ...

Lines = ... [1]

(ii) Find an expression, in terms of n, for the number of **dots** in Diagram n.

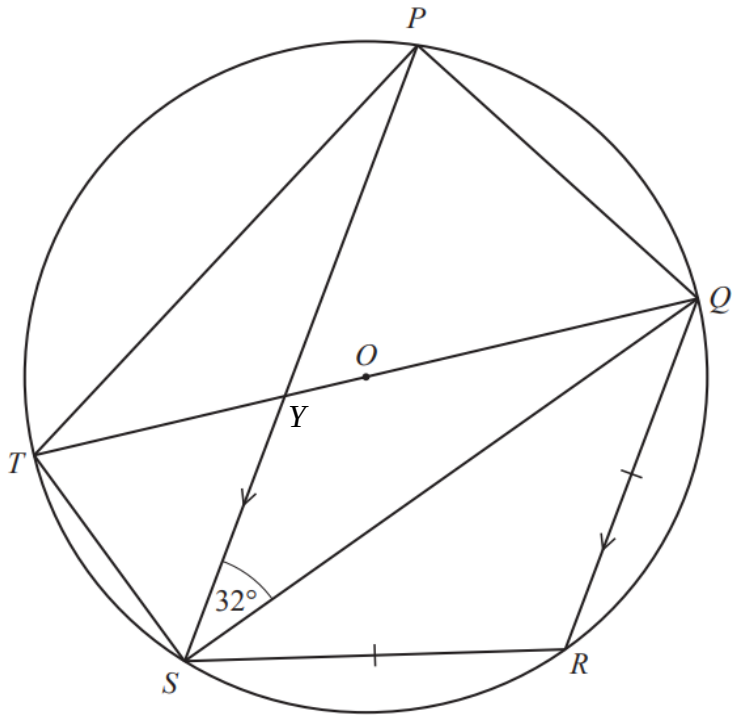
Answer ..... [1]

(iii) Explain, with calculations, whether it is possible to have a diagram that is made up of 97 dots and 150 lines in this sequence.

.....

..... [3]

4 **(a)**



NOT TO SCALE

P, *Q*, *R*, *S* and *T* are points on the circumference of a circle, centre *O*.
Angle *PSQ* = 32° and *O* lies on *TQ*.
PS intersects *TQ* at *Y*, *PS* is parallel to *QR* and *QR* = *RS*.

(i) Prove that triangle *TYS* is similar to triangle *PYQ*.
Give a reason for each statement you make.

[2]

(ii) Find angle *TPS*.
Give a reason for each step of your working.

Answer Angle *TPS* = ... [4]

(b)

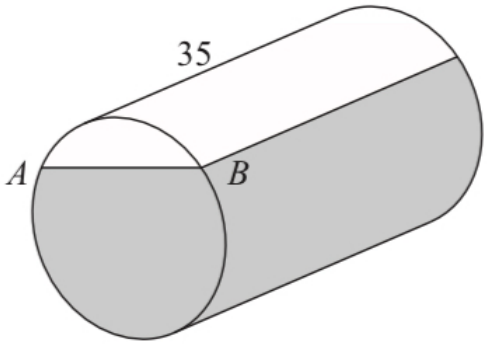


Diagram 1

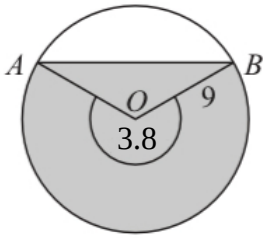


Diagram 2

Diagram 1 shows a solid cylinder, made of wood, floating in water.
The shaded region represents part of the surface of the cylinder that is in contact with water.

Diagram 2 shows the circular cross-section of one end of the cylinder with centre O.
The water level reaches the points A and B on the circumference.
Reflex angle $AOB = 3.8$ radians.
The cylinder has radius 9 cm and length 35 cm.

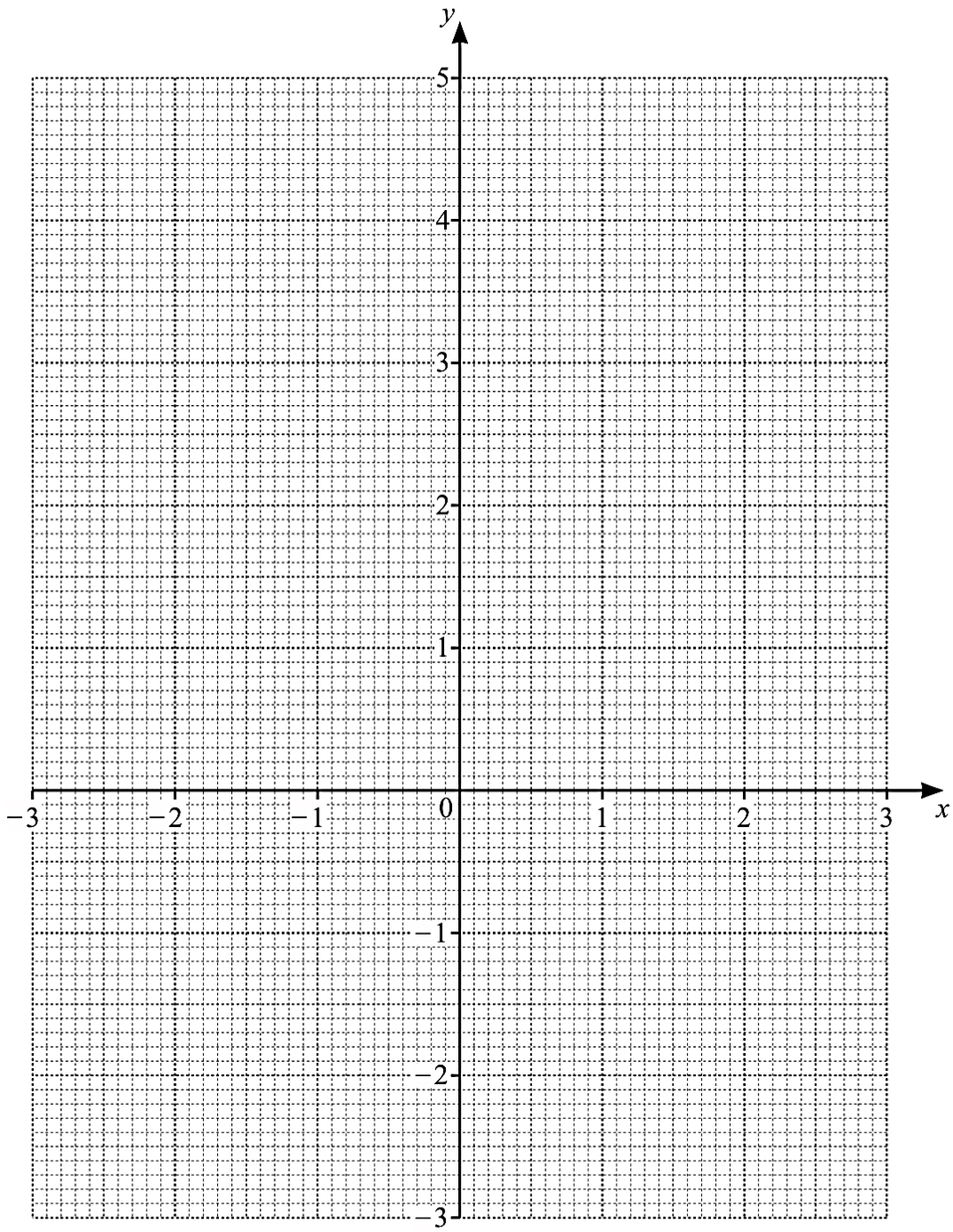
Calculate the **total** surface area of the cylinder that is in contact with water.

Answer ..... cm^2 [4]

5 The table shows some values for $y = \frac{x^3}{4} - x + 1$.

x	–3	–2	–1	0	1	2	3
y	–2.75	1	1.75	1	0.25	1	4.75

(a) On the grid, draw the graph of $y = \frac{x^3}{4} - x + 1$ for $-3 \leq x \leq 3$.



[3]

(b) By drawing a tangent, estimate the gradient of the graph of $y = \frac{x^3}{4} - x + 1$ at (2, 1).

Answer ..... [2]

(c) The equation $\frac{x^3}{4} - x + 1 = k$ has exactly two solutions.

Use your graph to find one possible value of k.

Answer k = ... [1]

(d) **(i)** On the same grid, draw the line $3y = x + 3$ for $-3 \leq x \leq 3$.

[2]

(ii) The x-coordinates of the points where this line intersects the graph of $y = \frac{x^3}{4} - x + 1$ are the solutions of the equation $3x^3 = px$.

Find the value of p.

Answer p = ... [2]

6 **(a)** A class of 15 boys and 15 girls took a music test.
The marks are shown in the stem-and-leaf diagram.

Boys		**Girls**
9 5 2 0	5	2 2 2 3 6 9
8 5 5 4 3 2 1 1 1	6	1 2 2 4 4 6 8
2	7	1
	8	
x	9	8

Key: 0|5 means 50 Key : 5|2 means 52

(i) The range of the boys' marks is 47.

Find the value of *x*.

Answer x = ... [1]

(ii) Find the upper quartile for the girls.

Answer ..... marks [1]

(iii) Explain why the mean is not a good representation of the marks scored by each group of boys and girls.

.....

..... [1]

(iv) Each student was given another 1 mark due to marking error.

Explain how the standard deviation of the marks will change after the increase of marks.

.....

..... [1]

(b) A wind ensemble is made up of a woodwind section and a brass section.
This table shows information about a wind ensemble that comprises 20 members.

	Woodwind section	Brass section
Number of members who can play the piano	5	2
Number of members who cannot play the piano	7	6

(i) One member is chosen at random from the wind ensemble.

Find, as a fraction in its lowest terms, the probability that this member **cannot** play the piano.

Answer ..... [1]

(ii) Two members are chosen at random from the wind ensemble.

Find, as a fraction in its lowest terms, the probability that

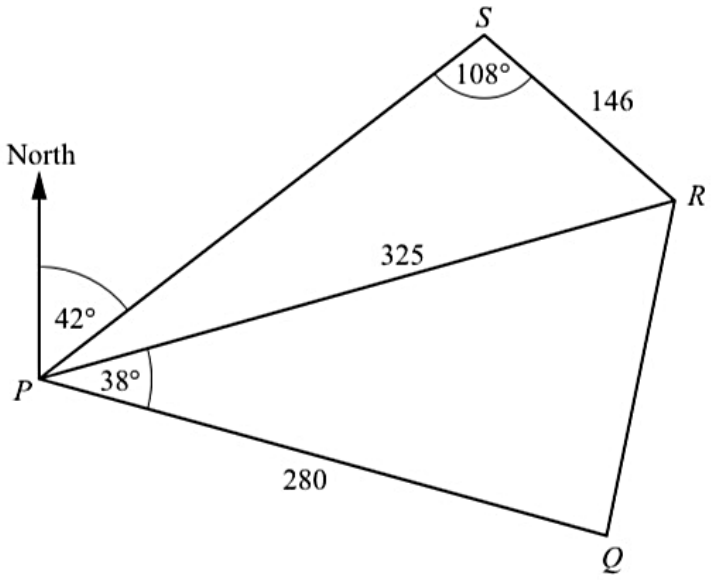
(a) the first member chosen can play the piano and the second member chosen cannot play the piano,

Answer ..... [2]

(b) at least one member is in the brass section.

Answer ..... [2]

7



NOT TO SCALE

A field is in the shape of a quadrilateral *PQRS*.
A path crosses the field from *P* to *R*.
PQ = 280 m, *RS* = 146 m and *PR* = 325 m.
S is on a bearing of 042° from *P*, angle *PSR* = 108° and angle *RPQ* = 38°.

(a) Calculate the bearing of *R* from *P*.

Answer ..... [3]

(b) Show that *QR* = 201.5 m, correct to one decimal place.

Answer

[3]

(c) A building stands vertically at Q.
The angle of elevation of the top of the building when viewed from R is 25°.

Calculate

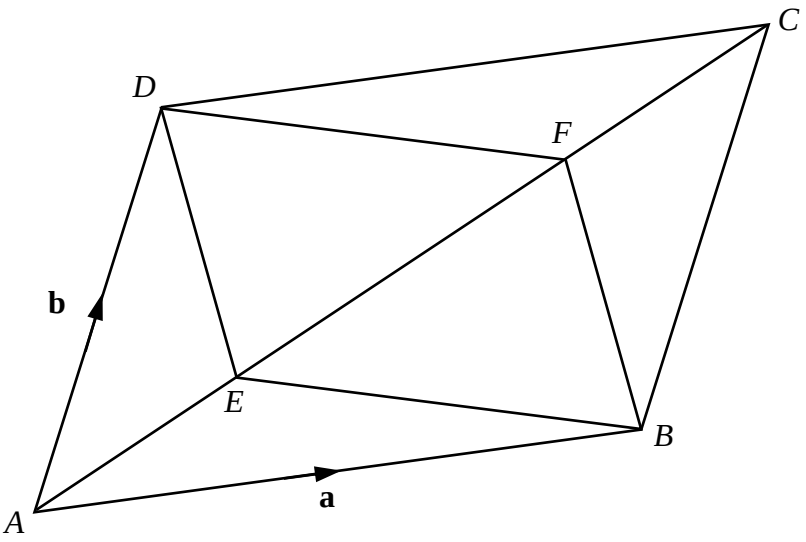
(i) the height of the building,

Answer ..... m [2]

(ii) the greatest possible angle of depression of a point on PR when viewed from the top of the building.

Answer ..... [3]

8



NOT TO SCALE

In the diagram, *ABCD* is a parallelogram.
E and *F* are points on the diagonal *AC* such that $AE = FC$ and $FC : AC = 1 : 4$.
$\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{AE}$,

Answer ..... [1]

(ii) $\overrightarrow{ED}$,

Answer ..... [2]

(iii) $\overrightarrow{BF}$.

Answer ..... [2]

(b) What type of quadrilateral is *DFBE*?
Use your answers in part **(a)** to explain your answer.

.....

..... [2]

(c) Find the ratio, in its lowest terms,

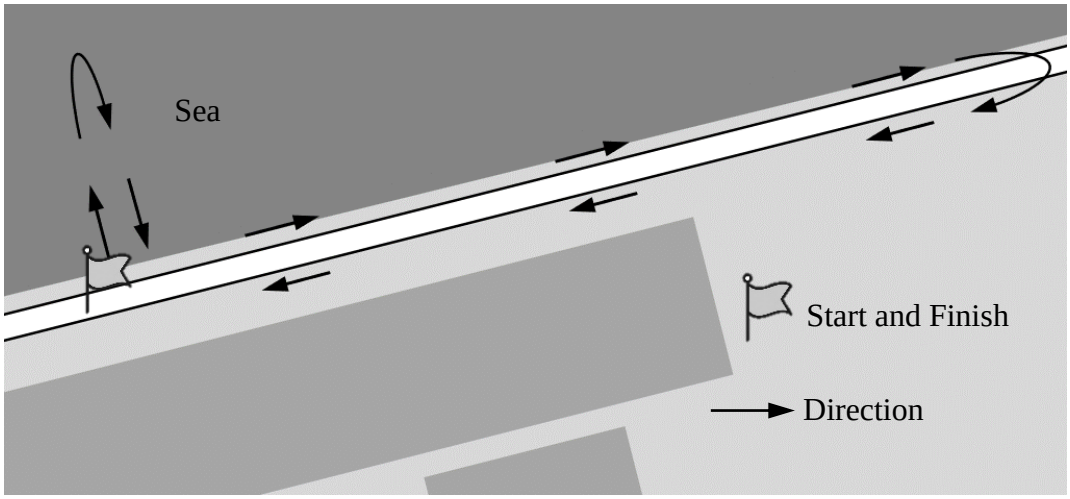
(i) area of quadrilateral *DFBE* : area of parallelogram *ABCD*,

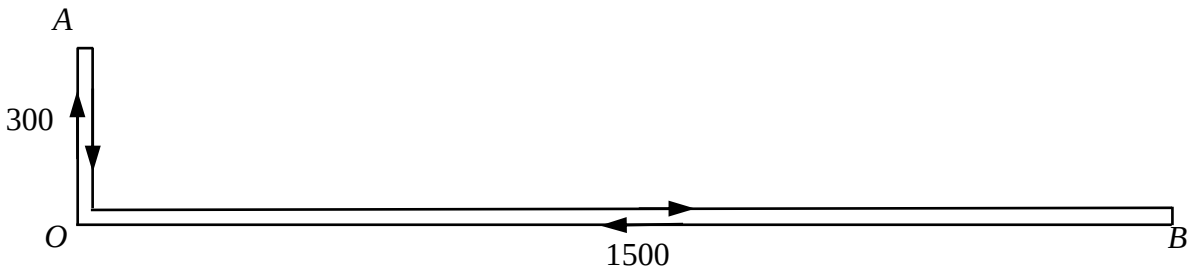
Answer ..... : ..... [1]

(ii) area of triangle *EDF* : area of quadrilateral *ABFD*.

Answer ..... : ..... [2]

9 In a biathlon, participants are required to swim 600 m and run 3000 m.
Route *A* for the race is shown in the diagram below.
The participants start to swim 300 m from *O* to *A* and then return to *O*.
After swimming, they run 1500 m along a straight road next to the coast to *B* and then return to *O* to complete the race, where angle *AOB* = 90°.
Assume that the distance which the participants make a turn is negligible.





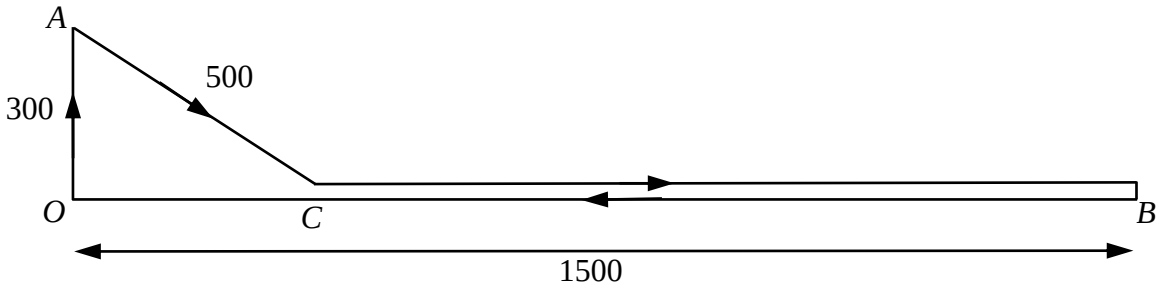
Route *A*

Peter can swim at an average speed of 2 m/s and run at an average speed of 3 m/s for the race.

(a) Calculate the total time taken by Peter for the race.
Give your answer in minutes and seconds.

Answer ..... minutes ..... seconds [2]

(b) In the following year, the organiser for the biathlon decides to change the route of the race. **Route *B*** for the race is shown in the diagram below.
The participants start to swim from *O* to *A* and then swim back 500 m to the coast from *A* to *C*, where *C* is a point on *OB*.
After swimming, they run from *C* to *B* and then run from *B* to *O* to complete the race.



Route *B*

To meet Peter's target timing of 19 minutes for **Route *B***, he must increase his average speeds for swimming and running by the same amount.

Do you agree that with this same amount of increase of average speeds for swimming and running, Peter's timing for **Route *A*** will improve by 15%?
Justify your decision and show your calculations clearly.

Answer

.....

..... [8]

Answer Key

1 **(a)** $\frac{2y^3}{x^2}$

(b) $\frac{5x-2y}{3a-2b}$

(c) $b=\frac{10-3a}{a-2}$ or $\frac{3a-10}{2-a}$

(d) $x = 1.697$ or 5.303

2 **(a)** \$98.71

(b) \$88

(c) $P = 18\,500$

3 **(a)** **(i)** $p = 3$, $q = 7$

(ii) 544

(b) **(i)** 17 dots, 24 lines

(ii) $2n+1$

(iii) It is not possible because Diagram 48 comprises 97 dots and 144 lines or Diagram 50 comprises 101 dots and 150 lines or Diagram numbers for 97 dots and for 150 lines are different.

4 **(a)** **(i)** Angle TYS = angle PYQ (vertically opposite angles)
Angle STY = angle QPY (Angles in the same segment)
Angle TSY = angle PQY (Angles in the same segment)
AAA or AA Property

(ii) 26°

(b) 1550 cm^2

5 **(b)** 1 to 3

(c) $k = 1.75$ or 0.25

(d) **(ii)** $p = 16$

6 **(a)** **(i)** $x = 7$

(ii) 66

(iii) The outlier value of 97 and 98 would skew the mean higher for the group of boys and girls respectively.

(iv) No change to standard deviation.

(b) **(i)** $\frac{13}{20}$

(ii) **(a)** $\frac{91}{380}$

(b) $\frac{62}{95}$

7 **(a)** 067.3°

(c) **(i)** 94.0 m

(ii) 28.6°

8 **(a)** **(i)** $\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(ii) $-\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(iii) $-\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

(b) *DFBE* is a parallelogram.
Since $\overrightarrow{ED} = \overrightarrow{BF}$, *ED* is parallel to *BF* and *ED* = *BF*.

(c) **(i)** 1 : 2

(ii) 1 : 3

9 **(a)** 21 minutes 40 seconds

(b) % improvement = 9.67% or
1174.2431815... > 1105

Since % improvement (ie 9.67%) is less than 15%, we cannot agree that this same amount of increase of average speeds improves Peter's timing for Route *A* by 15%.

or

Since the new total time taken for Route *A* (ie 1174.24 seconds) is greater than its original time taken (ie 1105 seconds), we cannot agree that this same amount of increase of average speeds improves Peter's timing for Route *A* by 15%.