## Gravitation Tutorial 7B Discussion Question Suggested Solutions

**D1**

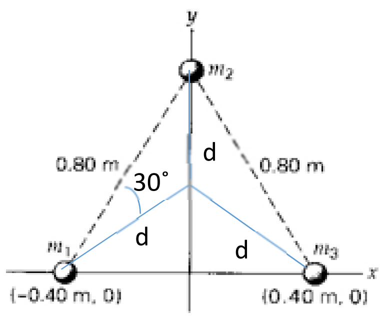
**(a)** Total gravitational potential energy of the configuration, $U$ = Work done by the external force in assembling the system

$$U = -\frac{Gm_1m_2}{0.80} + \left(-\frac{Gm_1m_3}{0.80}\right) + \left(-\frac{Gm_2m_3}{0.80}\right)$$

$$= 3\times\left(-\frac{G(1.0)(1.0)}{0.80}\right)$$

$$= -2.50\times10^{-10}\text{ J}$$

**(b)**




d cos30˚ = 0.80/2 → d = 0.4619

$$\phi_{centre} = 3\times\left(-\frac{G(1.0)}{0.4619}\right) = -4.332\times10^{-10}\text{ J kg}^{-1}$$

$$WD = \Delta U$$

$$= m\left(\phi_{centre} - \phi_{\infty}\right)$$

$$= (1.0)\times\left(-4.332\times10^{-10} - 0\right)$$

$$= -4.33\times10^{-10}\text{ J}$$

Gravitational force is **attractive** in nature and the gravitational **potential is set to be zero at infinity**. To move the 4th mass from infinity to the centre of the system, **the force exerted on the mass by the external agent will be in opposite direction to the displacement of the mass**. Thus negative work is done by the external force (agent).

**D2 2020 P1 Q12**

**A**

Increase in gravitational potential energy = $-\frac{Gm_1m_2}{(2r)} - \left(-\frac{Gm_1m_2}{r}\right) = \frac{Gm_1m_2}{2r}$

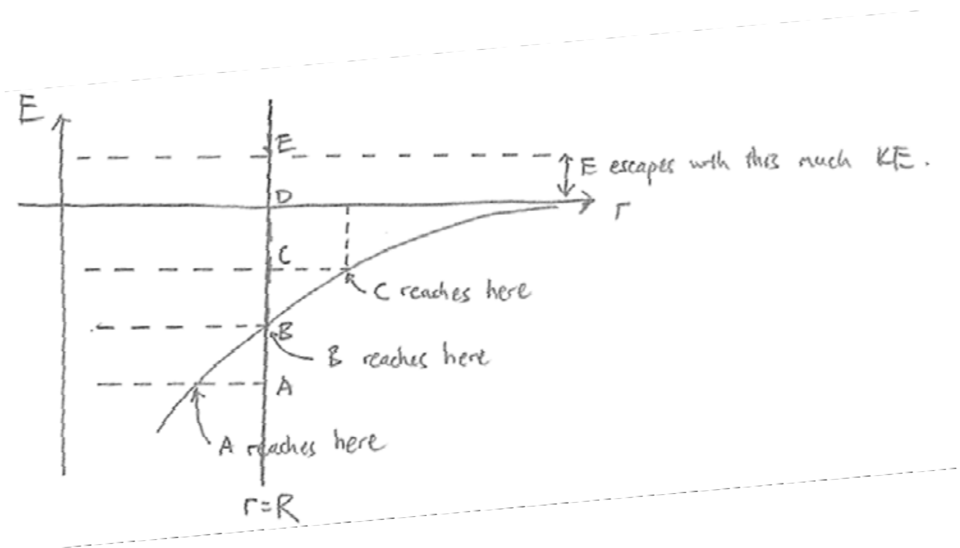
**D3**

**(a)** It represents the gravitational force on the body on Earth's surface. $F_g = -\frac{dE_p}{dr}$ . The negative sign indicates that the gravitational force is in the direction of decreasing potential energy.

**(b)** B. This amount of total energy allows the mass to just reach r = R. At r = R, TE = GPE and KE = 0.

**(c)** C. The mass will go beyond r = R, comes to rest at r where the PE = TE and then turns back towards Earth – and that's why it is "falling towards the Earth".

**(d)** D and E. For D, the mass will just reach infinity. For E, the mass will reach infinity where GPE = 0 and still have some KE.




**D4 2021 P3 Q2**

(a) Gravitational force is attractive.
In displacing a mass from infinity towards M (at constant speed), an external force opposite in direction to the displacement needs to be applied. Work done by this external force is negative [OR positive work needs to be done by an external force to move a mass away from M, implying that the potential gets larger as one moves away from M.]
Infinity is assigned a potential-value of zero. Hence the potential at any other point is negative.

(b) (i) $\phi = -\frac{GM}{r} = -\frac{(6.67\times10^{-11})(6.2\times10^{23})}{\left(\frac{6.8\times10^{6}}{2}\right)} = -1.2\times10^{7}\ \text{J kg}^{-1}$

(ii) To travel to infinity, the total energy must be greater than or equal to zero
At surface, GPE + KE = $(-1.22 \times 10^7)(2.8) + \frac{1}{2}(2.8)(3800)^2 = -1.4 \times 10^7$ J
Hence, it does not escape, it returns to the planet.

OR
Initial KE of rock = $\frac{1}{2}(2.8)(3800)^2 = 2.0 \times 10^7$ J
To reach infinity, GPE to be gained = $m(\phi_f - \phi_i) = 2.8\left(0-\left(-1.22\times10^{7}\right)\right) = 3.4\times10^{7}$ J
Not enough KE to escape.

OR (calculate the escape speed)
Loss in KE = Gain in GPE
$KE_i - KE_f = GPE_f - GPE_i$

$$\frac{1}{2}(2.8)v^2 - 0 = 0 - (2.8)\left(-1.22\times10^7\right)$$

$$v = 4.9\times10^3 \text{ m s}^{-1}$$

$3.8 \times 10^3$ m s$^{-1}$ is not enough to escape.

**D5**

Total energy of the space station in orbit = KE + GPE

= ½ GPE of the space station at the orbital radius

= $-\dfrac{GMm}{2r}$ [see example 10 in lecture notes]

Energy required

$$= -\frac{GMm}{2r_f} - \left(-\frac{GMm}{2r_i}\right) = \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{6.67\times10^{-11}\left(6.0\times10^{24}\right)(450000)}{2}\left(\frac{1}{6.378\times10^6 + 415\times10^3} - \frac{1}{6.378\times10^6 + 20200\times10^3}\right)$$

= 9.9 x $10^{12}$ J

**D6**

**(a)(i)** The gravitational force provides the centripetal force on the satellite.

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$v^2 = \frac{GM}{R}$$

**(ii)** $E_k = \dfrac{1}{2}mv^2$

$$E_k = \frac{1}{2}m\frac{GM}{R} = \frac{1}{2}\frac{GMm}{R}$$

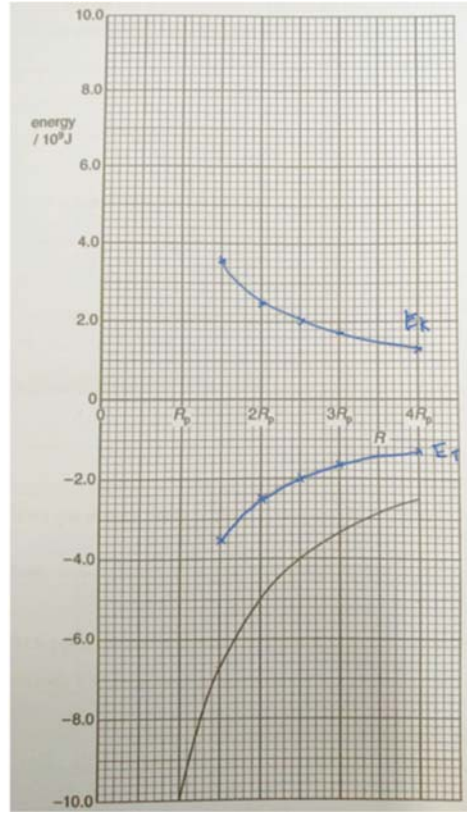
**(b)(i)** $E_p = -\dfrac{GMm}{R}$

[like D4(a)] Gravitational force is **attractive** in nature and the **potential is set to be zero at infinity**. To move a mass from infinity to a point in the field (of the source mass), **the force exerted on the mass by the external agent will be in opposite direction to the displacement of the mass**. Thus negative work is done by the external force (agent).

**(ii)** $\dfrac{E_p}{E_k} = -\dfrac{GMm}{R}\bigg/\dfrac{GMm}{2R} = -2$

**(iii)** $E_T = E_p + E_k = -\dfrac{GMm}{R} + \dfrac{GMm}{2R} = -\dfrac{GMm}{2R}$

**(c)(i), (ii)** See graphs on the right.




**(d)(i)** More negative, since total energy must decrease.

**(ii)1.** Radius decreases. Satellite does not have enough energy to stay at that altitude. Given the formula for total energy, radius must decrease.

**2.** Speed increases. Gravitational potential energy is converted into kinetic energy and work done against air resistance. According to the formula for kinetic energy, kinetic energy must increase.

**(iii)** At R = 4$R_p$,

$$E_k = 1.25 \times 10^9$$

$$\frac{1}{2}(1600)v_i^{\,2} = 1.25 \times 10^9$$

$$v_i = 1250 \text{ m s}^{-1}$$

At R = 2$R_p$,

$$E_k = 2.50 \times 10^9$$

$$\frac{1}{2}(1600)v_f^{\,2} = 2.50 \times 10^9$$

$$v_f = 1768 \text{ m s}^{-1}$$

Change in speed = 1768 – 1250 = 518 m s$^{-1}$

| $R$ | $E_p$ /$10^9$ J | $E_k$/$10^9$ J | $E_T$/$10^9$ J |
|---|---|---|---|
| 1.5 $R_P$ | -6.70 | 3.35 | -3.35 |
| 2 $R_P$ | -5.00 | 2.50 | -2.50 |
| 2.5 $R_P$ | -4.00 | 2.00 | -2.00 |
| 3 $R_P$ | -3.38 | 1.69 | -1.69 |
| 4 $R_P$ | -2.55 | 1.28 | -1.28 |

**D7**

**(a)** $V = -\frac{GM_E m}{r}$

**(b)** The potential gradient gives the magnitude of the gravitational field strength, the direction of which is towards decreasing potential, $g = -\frac{d\phi}{dr}$. The force on the tektite is given by the mass of the tektite multiplied by the potential gradient, and the direction of the force is towards decreasing potential.

**(c)** At P, the gradient is zero, implying $\Sigma$ g = 0

$$g_M = g_E$$
$$\frac{GM_M}{X^2} = \frac{GM_E}{Y^2}$$
$$\left(\frac{X}{Y}\right)^2 = \frac{M_M}{M_E}$$
$$\frac{X}{Y} = \sqrt{\frac{7.4 \times 10^{22}}{6.0 \times 10^{24}}} = 0.11$$

**(d)** The tektite needs to be given enough energy to reach P, beyond which the resultant gravitational field towards the Earth will accelerate the tektite towards the Earth.

By the principle of conservation of energy,

From moon's surface to point P,

Loss in KE = Gain in GPE

$$\frac{1}{2}mV_0^{\,2} = m\left(-1.3-\left(-3.9\right)\right)\times 10^6$$
$$V_0 = 2280 \text{ m s}^{-1}$$

**(e)** Tektite will reach Earth with a speed greater than 2280 m $s^{-1}$. Earth's surface is at a lower potential than moon's surface. This larger loss in GPE is converted into a larger gain in KE when the tektite hits Earth.

**D8**

**D**. Gravitational force = $-dE_p/dr$. Near the earth's surface, the (gravitational) force is constant as the gravitational field strength is constant.

**D9**

**B**. Uniform $g$ → magnitude of $g = \frac{\Delta\phi}{\Delta x}$

$$g = \frac{6.0}{10} = 0.60 \text{ N kg}^{-1}$$

$$mg\Delta h = 2.0(0.60)(2.5) = 3.0 \text{ J}$$