2022 C1 Block Test Revision Package Solutions
Chapter 1 Sequences and Series

Qn	Solution	Comments
1(i)	**ACJC14/C1Mid-year/Q6** $\frac{9x^2+15x-2}{9x^2+15x+4}=1-\frac{6}{9x^2+15x+4}$. $\therefore A=1$. $\frac{6}{9x^2+15x+4}=\frac{B}{3x+1}+\frac{C}{3x+4}$. By comparing numerators, $6=B(3x+4)+C(3x+1)$ When $x=-\frac{4}{3}$, $C=2$ When $x=-\frac{1}{3}$, $B=-2$ $\therefore \frac{9x^2+15x-2}{9x^2+15x+4}=1-\frac{2}{3x+1}+\frac{2}{3x+4}$.	Identify improper fraction and do long division 1 $9x^2+15x+4\overline{)9x^2+15x-2}$ $-\left(9x^2+15x+4\right)$ -6 Good Practice to check your answer before continuing the rest of the parts
(ii)	$\sum_{r=1}^{n}\frac{9r^2+15r-2}{9r^2+15r+4}=\sum_{r=1}^{n}\left(1-\frac{2}{3r+1}+\frac{2}{3r+4}\right)$ $=\left\{1-\frac{2}{4}+\cancel{\frac{2}{7}}\right.$ $+1-\cancel{\frac{2}{7}}+\cancel{\frac{2}{10}}$ $\vdots$ $\left.+1-\cancel{\frac{2}{3n+1}}+\frac{2}{3n+4}\right\}$ $=n-\frac{1}{2}+\frac{2}{3n+4}$.	**Remember to put your brackets** for $\sum_{r=1}^{n}\left(1-\frac{2}{3r+1}+\frac{2}{3r+4}\right)$ Show the cancellations. Alternatively use $\sum_{r=1}^{n}\left(1-\frac{2}{3r+1}+\frac{2}{3r+4}\right)$ $=\sum_{r=1}^{n}(1)+\sum_{r=1}^{n}\left(\frac{2}{3r+4}-\frac{2}{3r+1}\right)$
(iii)	As $n\to\infty$, $S_n=n-\frac{1}{2}+\frac{2}{3n+4}\to\infty$. Therefore the series in not convergent.	Note that sum of series is $n-\frac{1}{2}+\frac{2}{3n+4}$

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Qn	**Solution**	**Comments**
(iv)	"Replace r by $r-1$." $\sum_{r=0}^{n-2}\frac{9(r+1)^2+15r+13}{9(r+1)^2+15r+19}$ $=\sum_{r-1=0}^{r-1=n-2}\frac{9((r-1)+1)^2+15(r-1)+13}{9((r-1)+1)^2+15(r-1)+19}$ $=\sum_{r=1}^{n-1}\frac{9r^2+15r-2}{9r^2+15r+4}$ $=(n-1)-\frac{1}{2}+\frac{2}{3(n-1)+4}$ $=n-\frac{3}{2}+\frac{2}{3n+1}$	The approach for such question is to use to expression in (ii) $\sum_{r=1}^{n}\frac{9r^2+15r-2}{9r^2+15r+4}=n-\frac{1}{2}+\frac{2}{3n+4}$
2(i)	**AJC14/C1Mid-year/Q7** $\frac{2}{x(x+2)}$ $=\frac{x+2-x}{x(x+2)}$ $=\frac{x+2}{x(x+2)}-\frac{x}{x(x+2)}$ $=\frac{1}{x}-\frac{1}{x+2}$ (shown)	Since this is a show question, please show all steps or use Partial Fractions. $\frac{2}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}$ $2=A\left(x+2\right)+Bx$ When $x=0$, $A=1$ When $x=-2$, $B=-1$
(ii)	If N is even, $\sum_{n=1}^{N}\mathrm{f}(n)=\sum_{n=1}^{N}\left[(-1)^n\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$	The question asks for N is even. $\sum_{n=1}^{N}\left[(-1)^n\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$

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	$=\left[\begin{array}{l} -\frac{1}{1}+\cancel{\frac{1}{3}} \\ +\frac{1}{2}-\cancel{\frac{1}{4}} \\ -\cancel{\frac{1}{3}}+\cancel{\frac{1}{5}} \\ +\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}} \\ -\cancel{\frac{1}{5}}+\cancel{\frac{1}{7}} \\ +\ldots \\ -\cancel{\frac{1}{N-1}}+\frac{1}{N+1} \\ +\cancel{\frac{1}{N}}-\frac{1}{N+2} \end{array}\right]$ $=\left(-1+\frac{1}{2}+\frac{1}{N+1}-\frac{1}{N+2}\right)$ $=\frac{1}{(N+1)(N+2)}-\frac{1}{2}$	$=\left\{\begin{array}{ll} (-1) & \left[\frac{1}{1}\quad -\cancel{\frac{1}{3}}\right] \\ + & \left[\frac{1}{2}\quad -\cancel{\frac{1}{4}}\right] \\ (-1) & \left[\cancel{\frac{1}{3}}\quad -\cancel{\frac{1}{5}}\right] \\ + & \left[\cancel{\frac{1}{4}}\quad -\cancel{\frac{1}{6}}\right] \\ & \vdots \\ (-1)^{N-3} & \left[\cancel{\frac{1}{N-3}}\quad -\cancel{\frac{1}{N-1}}\right] \\ (-1)^{N-2} & \left[\cancel{\frac{1}{N-2}}\quad -\cancel{\frac{1}{N}}\right] \\ (-1)^{N-1} & \left[\cancel{\frac{1}{N-1}}\quad -\frac{1}{N+1}\right] \\ (-1)^{N} & \left[\cancel{\frac{1}{N}}\quad -\frac{1}{N+2}\right] \end{array}\right.$ $=-\frac{1}{2}+(-1)^{N-1}\left(\frac{-1}{N+1}\right)+(-1)^{N}\left(\frac{-1}{N+2}\right)$ $=-\frac{1}{2}+(-1)^{N}\left(\frac{1}{N+1}\right)-(-1)^{N}\left(\frac{1}{N+2}\right)$ $=-\frac{1}{2}+(-1)^{N}\left[\left(\frac{1}{N+1}\right)-\left(\frac{1}{N+2}\right)\right]$ $=-\frac{1}{2}+\frac{(-1)^{N}}{(N+1)(N+2)}$ for all $N\in\mathbb{Z}^{+}$ Then let $N\to\infty$, $-\frac{1}{2}+\frac{(-1)^{N}}{(n+1)(n+2)}\to-\frac{1}{2}$

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	If N is odd $\sum_{n=1}^{N} \mathrm{f}(n) = \sum_{n=1}^{N}\left[(-1)^{n}\left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$ $= \left[\begin{array}{l} -\frac{1}{1}+\frac{1}{3} \\ +\frac{1}{2}-\frac{1}{4} \\ +\ldots \\ +\frac{1}{N-1}-\frac{1}{N+1} \\ -\frac{1}{N}+\frac{1}{N+2} \end{array}\right]$ $=\left(-1+\frac{1}{2}-\frac{1}{N+1}+\frac{1}{N+2}\right)$ $=-\frac{1}{(N+1)(N+2)}-\frac{1}{2}$	
(iii)	In general, we have $\sum_{n=1}^{M} \mathrm{f}(n) = \frac{(-1)^{M}}{(M+1)(M+2)}-\frac{1}{2},\ M \in \mathbb{Z}^{+}$ When $M \to \infty$, $\sum_{n=1}^{\infty} \mathrm{f}(n) = \lim_{M\to\infty}\left[\frac{(-1)^{M}}{(M+1)(M+2)}-\frac{1}{2}\right] = -\frac{1}{2}$	
3 (i)	**JJC13/C2Mid-year/Q4(b)** $u_n - u_{n+1} = \frac{1}{n!}-\frac{1}{(n+1)!}$ $=\frac{(n+1)-1}{(n+1)!}$ $=\frac{n}{(n+1)!}$ (Shown)	$(n+1)!=(n+1)n!$

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(ii)	$\sum_{n=1}^{N} \frac{n}{(n+1)!} = \sum_{n=1}^{N} (u_n - u_{n+1})$ $= u_1 - \cancel{u_2}$ $+ \cancel{u_2} - \cancel{u_3}$ $+ \cancel{u_3} - \cancel{u_4}$ $+ \dots$ $+ \cancel{u_N} - u_{N+1}$ $= u_1 - u_{N+1}$ $= 1 - \frac{1}{(N+1)!}$	Need not write out the u_n terms for $n = 1, \dots, N$. It is alright to just write u_1, u_2, etc.
(iii)	$\sum_{n=1}^{N} \frac{n}{(n+1)!} = 1 - \frac{1}{(N+1)!}$ $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{N}{(N+1)!} = 1 - \frac{1}{(N+1)!}$ $\frac{2}{3!} + \frac{3}{4!} + \dots + \frac{N}{(N+1)!} = 1 - \frac{1}{2} - \frac{1}{(N+1)!}$ $\frac{2}{3!} + \frac{3}{4!} + \dots + \frac{N}{(N+1)!} = \frac{1}{2} - \frac{1}{(N+1)!}$ Since $\frac{1}{(N+1)!} > 0$, $-\frac{1}{(N+1)!} < 0$ and $\frac{1}{2} - \frac{1}{(N+1)!} < \frac{1}{2}$ $\frac{2}{3!} + \frac{3}{4!} + \dots + \frac{N}{(N+1)!} = \frac{1}{2} - \frac{1}{(N+1)!} < \frac{1}{2}$ (shown)	DO NOT use $\lim_{N \to \infty} \frac{1}{(N+1)!} = 0$ to explain

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4(i)	**CJC14/C1Mid-year/Q11** $u_r - u_{r+1} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!} = \frac{r+2-1}{(r+2)!} = \frac{r+1}{(r+2)!}$ $\sum_{r=1}^{N} \frac{r+1}{2(r+2)!} = \frac{1}{2}\sum_{r=1}^{N} \frac{r+1}{(r+2)!}$ $= \frac{1}{2}\sum_{r=1}^{N}\left(u_r - u_{r+1}\right)$ $= \frac{1}{2}[u_1 - \cancel{u_2}$ $+\cancel{u_2} - \cancel{u_3}$ $\dots$ $+\cancel{u_{N-1}} - \cancel{u_N}$ $+\cancel{u_N} - u_{N+1}]$ $= \frac{1}{2}[u_1 - u_{N+1}] = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(N+2)!}\right] = \frac{1}{4} - \frac{1}{2(N+2)!}$	
4(ii)	From (i) $\sum_{\mathrm{r}=1}^{\mathrm{N}} \frac{\mathrm{r}+1}{2(\mathrm{r}+2)!} = \frac{1}{4} - \frac{1}{2(\mathrm{N}+2)!}$, we have $\frac{1}{2(\mathrm{N}+2)!} > 0 \Rightarrow -\frac{1}{2(\mathrm{N}+2)!} < 0 \Rightarrow \frac{1}{4} - \frac{1}{2(\mathrm{N}+2)!} < \frac{1}{4}$ For the lower bound, we have $N \geq 1 \Rightarrow N+2 \geq 3 \Rightarrow 2(N+2)! \geq 12$ $\frac{1}{-2\left(N+2\right)!} \geq -\frac{1}{12} \Rightarrow \frac{1}{4} - \frac{1}{2(N+2)!} \geq \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ $\therefore \frac{1}{6} \leq \frac{1}{4} - \frac{1}{2(\mathrm{N}+2)!} < \frac{1}{4}$	
4(iii)	As $N \to \infty, \frac{-1}{2(N+2)!} \to 0$ $\therefore \sum_{r=0}^{\infty} \frac{r+1}{2(r+2)!} = \frac{1}{4} + \sum_{r=1}^{\infty} \frac{r+1}{2(r+2)!} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ The series converges to $\frac{1}{2}$.	Note the change in the lower limit from 1 to 0. When $r = 0$, $\frac{r+1}{2(r+2)!} = \frac{1}{4}$.

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4(iv)	$\sum_{r=6}^{N}\frac{r}{2(r+1)!}=\sum_{s+1=6}^{s+1=N}\frac{s+1}{2(s+2)!}$ (replace r with $s+1$) $=\sum_{s=5}^{N-1}\frac{s+1}{2(s+2)!}$ $=\sum_{s=1}^{N-1}\frac{s+1}{2(s+2)!}-\sum_{s=1}^{4}\frac{s+1}{2(s+2)!}$ $=\frac{1}{4}-\frac{1}{2(N+1)!}-\left[\frac{1}{4}-\frac{1}{2(6!)}\right]=\frac{1}{1440}-\frac{1}{2(N+1)!}$	Must remember to change the upper limit of the summation too Note that the lower limit is now 5 and not 1
5(i) **(ii)**	**HCI14/C1Mid-year/Q7** $\frac{r^2+5r+8}{r(r+1)(r+2)}=\frac{A}{r}+\frac{B}{r+1}+\frac{C}{r+2}$ $\therefore r^2+5r+8=A(r+1)(r+2)+Br(r+2)+Cr(r+1)$ Sub $r=0$, $8=2A\Rightarrow A=4$ Sub $r=-1$, $4=-B\Rightarrow B=-4$ Sub $r=-2$, $2=2C\Rightarrow C=1$ $\therefore\frac{r^2+5r+8}{r(r+1)(r+2)}=\frac{4}{r}-\frac{4}{r+1}+\frac{1}{r+2}$ $\sum_{r=1}^{n}\frac{r^2+5r+8}{r(r+1)(r+2)}\frac{1}{2^{r+2}}=\sum_{r=1}^{n}\left(\frac{4}{r}-\frac{4}{r+1}+\frac{1}{r+2}\right)\frac{1}{2^{r+2}}$ $=\sum_{r=1}^{n}\left(\frac{1}{2^r(r)}-\frac{2}{2^{r+1}(r+1)}+\frac{1}{2^{r+2}(r+2)}\right)$ $=\frac{1}{2^1(1)}\quad-\frac{2}{2^2(2)}\quad+\cancel{\frac{1}{2^3(3)}}$ $+\frac{1}{2^2(2)}\quad-\cancel{\frac{2}{2^3(3)}}\quad+\cancel{\frac{1}{2^4(4)}}$ $+\cancel{\frac{1}{2^3(3)}}\quad-\frac{2}{2^4(4)}\quad+\cancel{\frac{1}{2^5(5)}}$ $+\ldots$ $+\cancel{\frac{1}{2^{n-1}(n-1)}}-\cancel{\frac{2}{2^n(n)}}\quad+\frac{1}{2^{n+1}(n+1)}$ $+\cancel{\frac{1}{2^n(n)}}\quad-\cancel{\frac{2}{2^{n+1}(n+1)}}+\frac{1}{2^{n+2}(n+2)}$	Check that the numerators add up to zero

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	$= \frac{1}{2} - \frac{2}{8} + \frac{1}{8} + \frac{1}{2^{n+1}(n+1)} - \frac{2}{2^{n+1}(n+1)} + \frac{1}{2^{n+2}(n+2)}$ $= \frac{3}{8} - \frac{1}{2^{n+1}(n+1)} + \frac{1}{2^{n+2}(n+2)}$	
(iii)	METHOD 1 For $r \in \mathbb{Z}^+$ $\frac{r^2+5r+8}{(r+1)(r+2)} = \frac{r^2+5r+8}{r^2+3r+2} = 1 + \frac{2r+6}{(r+1)(r+2)} > 1$ i.e. $1 < \frac{r^2+5r+8}{(r+1)(r+2)}$ we have $\frac{1}{r2^{r+2}} < \frac{r^2+5r+8}{r(r+1)(r+2)} \frac{1}{2^{r+2}}$ for any $r > 0$ $\therefore \sum_{r=1}^{n} \frac{1}{r2^{r+2}} < \sum_{r=1}^{n} \frac{r^2+5r+8}{r(r+1)(r+2)} \frac{1}{2^{r+2}}$ $= \frac{3}{8} - \frac{n+3}{2^{n+2}(n+1)(n+2)} < \frac{3}{8}$ Since $\frac{n+3}{2^{n+2}(n+1)(n+2)} > 0$	
	METHOD 2 $\sum_{r=1}^{n} \frac{1}{r2^{r+2}} = \sum_{r=1}^{n} \frac{(r+1)(r+2)}{r(r+1)(r+2)2^{r+2}}$ $= \sum_{r=1}^{n} \frac{r^2+3r+2}{r(r+1)(r+2)2^{r+2}} < \sum_{r=1}^{n} \frac{r^2+5r+8}{r(r+1)(r+2)2^{r+2}}$ $= \frac{3}{8} - \frac{n+3}{2^{n+2}(n+1)(n+2)} < \frac{3}{8}$ Since $\frac{n+3}{2^{n+2}(n+1)(n+2)} > 0$	

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6(i)	**NJC14/C1Mid-year/Q6** $\cos\left[(n+1)\theta\right]-\cos\left[(n-1)\theta\right]=-2\sin\left(\frac{2n\theta}{2}\right)\sin\left(\frac{2\theta}{2}\right)$ $=-2\sin(n\theta)\sin\theta$	Apply Factor Formula with the help of MF26
(ii)	$\sum_{n=1}^{N}\sin(n\theta)$ $=-\frac{1}{2\sin\theta}\sum_{n=1}^{N}\left[\cos(n+1)\theta-\cos(n-1)\theta\right]$ $=-\frac{1}{2\sin\theta}\left(\begin{array}{ll} \cos 2\theta & -\cos 0 \\ +\cos 3\theta & -\cos\theta \\ +\cos 4\theta & -\cos 2\theta \\ & \vdots \\ +\cos(N-1)\theta & -\cos(N-3)\theta \\ +\cos(N)\theta & -\cos(N-2)\theta \\ +\cos(N+1)\theta & -\cos(N-1)\theta \end{array}\right)$ $=-\frac{1}{2\sin\theta}\left(\cos\left[(N+1)\theta\right]+\cos\left[N\theta\right]-\cos\theta-1\right)$	
(iii)	$\sin\frac{\pi}{6}+\sin\frac{\pi}{3}+\sin\frac{\pi}{2}+\cdots+\sin\frac{29\pi}{6}$ $=\sum_{n=1}^{29}\sin\left(n\frac{\pi}{6}\right)$ $=-\frac{1}{2\sin\frac{\pi}{6}}\left(\cos\left[(29+1)\frac{\pi}{6}\right]+\cos\left[\frac{29\pi}{6}\right]-\cos\frac{\pi}{6}-1\right)$ $=-\left(-1-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}-1\right)$ $=\sqrt{3}+2$	
7(i)	$\frac{6r+18}{(r-1)r(r+2)}=\frac{A}{r-1}+\frac{B}{r}+\frac{C}{r+2}$ $6r+18=Ar(r+2)+B(r-1)(r+2)+C(r-1)r$	

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	$6r+18=A(r^2+2r)+B(r^2+r-2)+C(r^2-r)$ $6r+18=(A+B+C)r^2+(2A+B-C)r-2B$ $A+B+C=0$ $2A+B-C=6$ $-2B=18$ $A=8,\ B=-9,\ C=1$ $\therefore \frac{6r+18}{(r-1)r(r+2)}=\frac{8}{r-1}-\frac{9}{r}+\frac{1}{r+2}$	
(ii)	$\sum_{r=2}^{n}\frac{r+3}{(r-1)r(r+2)}=\frac{1}{6}\sum_{r=2}^{n}\frac{6r+18}{(r-1)r(r+2)}$ $=\frac{1}{6}\sum_{r=2}^{n}\left(\frac{8}{r-1}-\frac{9}{r}+\frac{1}{r+2}\right)$ $=\frac{1}{6}\Big[\frac{8}{1}-\frac{9}{2}+\frac{1}{4}$ $+\frac{8}{2}-\frac{9}{3}+\frac{1}{5}$ $+\frac{8}{3}-\frac{9}{4}+\frac{1}{6}$ $+\frac{8}{4}-\frac{9}{5}+\frac{1}{7}$ $+\quad\vdots$ $+\frac{8}{n-4}-\frac{9}{n-3}+\frac{1}{n-1}$ $+\frac{8}{n-3}-\frac{9}{n-2}+\frac{1}{n}$ $+\frac{8}{n-2}-\frac{9}{n-1}+\frac{1}{n+1}$ $+\frac{8}{n-1}-\frac{9}{n}+\frac{1}{n+2}\Big]$ $=\frac{1}{6}\left[8-\frac{9}{2}+\frac{8}{2}-\frac{9}{3}+\frac{8}{3}+\frac{1}{n}-\frac{9}{n}+\frac{1}{n+1}+\frac{1}{n+2}\right]$ $=\frac{1}{6}\left[\frac{43}{6}-\frac{8}{n}+\frac{1}{n+1}+\frac{1}{n+2}\right]$ $=\frac{43}{36}-\frac{4}{3n}+\frac{1}{6(n+1)}+\frac{1}{6(n+2)}$	

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(iii)	$\sum_{r=2}^{n}\frac{r+4}{r(r+1)(r+3)}$ $=\sum_{k-1=2}^{k-1=n}\frac{(k-1)+4}{(k-1)(k-1+1)(k-1+3)}$ (substitute $r=k-1$) $=\sum_{k=3}^{n+1}\frac{k+3}{(k-1)(k)(k+2)}$ $=\sum_{k=2}^{n+1}\frac{k+3}{(k-1)k(k+2)}-\frac{5}{(1)(2)(4)}$ $=\frac{43}{36}-\frac{4}{3(n+1)}+\frac{1}{6(n+2)}+\frac{1}{6(n+3)}-\frac{5}{8}$ $=\frac{41}{72}-\frac{4}{3(n+1)}+\frac{1}{6(n+2)}+\frac{1}{6(n+3)}$	
(iv)	$\sum_{r=2}^{\infty}\frac{r+4}{r(r+1)(r+3)}$ $=\lim_{n\to\infty}\left(\frac{41}{72}-\frac{4}{3n}+\frac{1}{6(n+1)}+\frac{1}{6(n+2)}\right)$ $=\frac{41}{72}$	
(v)	Since $(r+3)^3>r(r+1)(r+3)$ $\frac{1}{(r+3)^3}<\frac{1}{r(r+1)(r+3)}$ $\frac{r+4}{(r+3)^3}<\frac{r+4}{r(r+1)(r+3)}$ $\sum_{r=2}^{\infty}\frac{r+4}{(r+3)^3}<\sum_{r=2}^{\infty}\frac{r+4}{r(r+1)(r+3)}$ $\sum_{r=2}^{\infty}\frac{r+4}{(r+3)^3}<\sum_{r=2}^{\infty}\frac{r+4}{r(r+1)(r+3)}=\frac{41}{72}$ $\sum_{r=2}^{\infty}\frac{r+4}{(r+3)^3}<\frac{41}{72}$	

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8(i)	**RI19/C1Promo/Q3** $u_n = \tan(n+2)\tan(n+3)$ $\tan\left((n+3)-(n+2)\right) = \dfrac{\tan(n+3)-\tan(n+2)}{1+\tan(n+3)\tan(n+2)}$ $\tan(1) = \dfrac{\tan(n+3)-\tan(n+2)}{1+u_n}$ $(1+u_n)\tan(1) = \tan(n+3)-\tan(n+2)$ $u_n = \dfrac{\tan(n+3)-\tan(n+2)}{\tan 1} - 1 \text{ (shown)}$	
(ii)	$\sum_{r=2}^{n} u_r = \sum_{r=2}^{n}\left[\dfrac{\tan(r+3)-\tan(r+2)}{\tan 1} - 1\right]$ $= \dfrac{1}{\tan 1}\sum_{r=2}^{n}\left[\tan(r+3)-\tan(r+2)\right] - \sum_{r=2}^{n} 1$ $= \dfrac{1}{\tan 1}\Big[\ \cancel{\tan 5} - \tan 4$ $+\cancel{\tan 6} - \cancel{\tan 5}$ $+\cancel{\tan 7} - \cancel{\tan 6}$ $+\ldots$ $+\cancel{\tan(n+2)} - \cancel{\tan(n+1)}$ $+\tan(n+3) - \cancel{\tan(n+2)}\Big] - (n-1)$ $= \dfrac{\tan(n+3)-\tan 4}{\tan 1} + 1 - n$	
9	**RI19/C1Promo/Q2** $S_n = \dfrac{n}{2}\left[2a+(n-1)d\right] = 6600$ $n^2 d + 2an - nd - 13200 = 0$ - (1) $a + 20d = 91$ - (2) $a + 52d = 155$ - (3) Solving (2) and (3) using GC, $a = 51$ and $d = 2$.	

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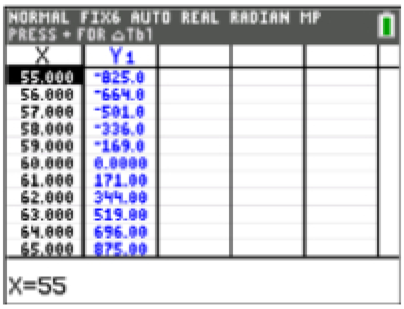
	Sub a and d into (1), $2n^2+2n(51)-2n-13200=0$ $n^2+50n-6600=0$ NORMAL FIX6 AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y1 55.000 ⁻825.0 56.000 ⁻664.0 57.000 ⁻501.0 58.000 ⁻336.0 59.000 ⁻169.0 60.000 0.0000 61.000 171.00 62.000 344.00 63.000 519.00 64.000 696.00 65.000 875.00 X=55 Using GC, $n = 60$ (–110 not accepted as $n > 0$).	Can use GC table since n is a positive integer
10(i)	**NYJC19/C1Promo/Q1** **Method 1** $S_n=\ln\left(2^n3^{n^2}\right)=n\ln 2+n^2\ln 3$ $u_n=S_n-S_{n-1}$ $=n\ln 2+n^2\ln 3-\left[(n-1)\ln 2+(n-1)^2\ln 3\right]$ $=\ln 2+\left[n^2-(n-1)^2\right]\ln 3$ $=\ln 2+(2n-1)\ln 3$ **Method 2** $u_n=S_n-S_{n-1}$ $=\ln\left(2^n3^{n^2}\right)-\ln\left(2^{n-1}3^{(n-1)^2}\right)$ $=\ln\left(\frac{2^n3^{n^2}}{2^{n-1}3^{(n-1)^2}}\right)$ $=\ln\left(2\times 3^{2n-1}\right)$ $=\ln 2+(2n-1)\ln 3$	Remember formula is $u_n=S_n-S_{n-1}$ and NOT $u_n=S_{n+1}-S_n$

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(ii)	Since $u_n - u_{n-1} = \ln 2 + (2n-1)\ln 3 - \left[\ln 2 + \left(2(n-1)-1\right)\ln 3\right]$ $= (2n-1)\ln 3 - (2n-3)\ln 3$ $= 2\ln 3$ is a constant, the sequence is AP.	It is NOT enough to show that $u_2 - u_1 =$ constant
11	**AJC14/C1Mid-year/Q11**	
(ai)	Since T_2, T_6 and T_9 are consecutive terms of a geometric progression, $\frac{T_9}{T_6} = \frac{T_6}{T_2}$ $\frac{a+8d}{a+5d} = \frac{a+5d}{a+d}$ $(a+8d)(a+d) = (a+5d)^2$ $a^2 + 9ad + 8d^2 = a^2 + 10ad + 25d^2$ $d(a+17d) = 0$ $a = -17d$ (since $d \neq 0$) Common ratio $= \frac{a+5d}{a+d} = \frac{-17d+5d}{-17d+d} = \frac{-12}{-16} = \frac{3}{4}$	
(aii)	$11+(n-1)(2) = 35 \Rightarrow n = 13$ $T_{11} + T_{13} + T_{15} + \ldots\ldots + T_{35} = 455$ $\frac{13}{2}\ a + 10d + a + 34d \ = 455$ $13(a+22d) = 455$ $-17d + 22d = 35$ $d = 7$ $\therefore\ a = -17(7) = -119$	
(bi)	Amount at the end of 1st year $= 27000(1.04)$	

	Amount at the end of 2nd year = 1.04[27000(1.04)+200] = 27000(1.04)2 + 200(1.04) Amount at the end of 3rd year = 1.04[27000(1.04)2 + 200(1.04) + 200] = 27000(1.04)3 + 200(1.04 + 1.042) $\vdots$ Amount in account under plan B at the end of n years $=27000(1.04)^n + 200(1.04 + 1.04^2 + \ldots + 1.04^{n-1})$ $=27000(1.04)^n+\frac{200\left[1.04\left(1.04^{n-1}-1\right)\right]}{1.04-1}$ $=27000(1.04)^n+5000\left(1.04^n-1.04\right)$ $=32000(1.04)^n-5200$	
(bii)	Total amount of interest under plan B at the end of n years $=32000(1.04)^n-5200-27000-200(n-1)$ $=32000(1.04)^n-200n-32000$	Note that this part is asking for total amount of interest. Hence (i) minus total amount invested.
(biii)	Total interest under plan A after n years = $1800n$ Total interest under plan B > Total interest under plan A $32000(1.04)^n-32000-200n > 1800n$ Let f(n) = $32000(1.04)^n-32000-2000n>0$ From GC, f(22) = –162.6 < 0, f(23) = 870.9 > 0 Least number of years = 23	

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12(a)	**DHS14/C1Mid-year/Q13** Method 1 $\frac{a}{1-r}=64$ --- (1) $\frac{a\left(1-r^5\right)}{1-r}=64-2=62$ ---(2) Substitute (1) into (2): $64\left(1-r^5\right)=62$ $r^5=\frac{1}{32}\Rightarrow r=\frac{1}{2}$ Substitute into (1): $a=32$ Method 2 $\frac{a}{1-r}=64$ --- (1) $\frac{ar^5}{1-r}=2$ ---(2) Substitute (1) into (2): $64r^5=2$ $r^5=\frac{1}{32}\Rightarrow r=\frac{1}{2}$ Substitute into (1): $a=32$	
(b)	Maximum distance travelled $=S_\infty=\frac{10}{1-0.7}=33.333<34$ $\therefore$ the motorcyle will not hit the obstacle.	
(ci)	Distance, in m, travelled in 25th second $=5+\left(25-1\right)0.5=17$	
c(ii)	Total distance travelled by car in first n seconds $=\frac{n}{2}\left(2\left(5\right)+\left(n-1\right)0.5\right)$ $=\frac{n}{2}\left(9.5+0.5n\right)$	

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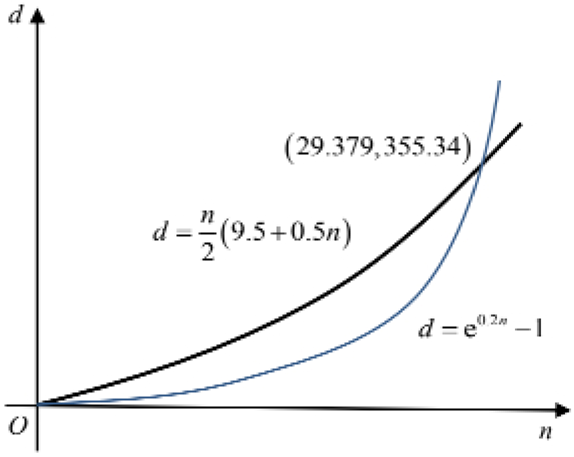
	For the van to overtake the car, $\mathrm{e}^{0.2n}-1>\frac{n}{2}(9.5+0.5n)$ Using the GC, $\therefore n>29.379$ the van will overtake the car after 30 seconds. 	It is difficult to solve this inequality algebraically, hence we use GC graph to help us. Take note that the question asks for COMPLETE seconds, so your answer must be rounded up to integer.
13(a)	**RI14/C1Mid-year/Q8** Let a and d be the first term and common difference of the arithmetic series. $u_{17}=a+16d=73;\quad u_{33}=a+32d=71$ Solving, $a=75,\ d=-0.125$	
	Method 1 $S_n=S_{n+1}\ \Rightarrow\ u_{n+1}=0$ Thus, $a+nd=0\Rightarrow 75-0.125n=0\Rightarrow n=600$ Method 2 $S_n=S_{n+1}\ \Rightarrow\ \frac{n}{2}\left[2a+(n-1)d\right]=\frac{n+1}{2}(2a+nd)$ $\frac{n}{2}\left[150-0.125(n-1)\right]=\frac{n+1}{2}(150-0.125n)$ $0.125n=75$ $n=600$	

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(b)	Let r be the common ratio of the GP. $u_6-u_5=u_5-u_1$ $ar^5-ar^4=ar^4-a$ $r^5-2r^4+1=0$ $r\approx -0.77480,\ 1,\ 1.9276$ Since the series is convergent (i.e. $\lvert r\rvert<1$), $r\approx -0.77480=-0.775$ (3s.f.)	
	Given: $S=\frac{a}{1-r}=10$ $\lvert S_m-S\rvert<0.001$ $\left\lvert\frac{a\left(1-r^m\right)}{1-r}-\frac{a}{1-r}\right\rvert<0.001$ $\lvert 10(1-r^m)-10\rvert<0.001$ $\left(0.77480\right)^m<0.0001$ $m>\frac{\ln 0.0001}{\ln 0.7748}\approx 36.098$ Least value of m is 37.	
14(a)	**SAJC14/C2Mid-yearP2/Q1** $S_n=9-\frac{5^n}{3^{n-2}}$ $u_n=S_n-S_{n-1}$ $=9-\frac{5^n}{3^{n-2}}-\left(9-\frac{5^{n-1}}{3^{n-3}}\right)$ $=\frac{5^{n-1}}{3^{n-3}}\left(1-\frac{5}{3}\right)$ $=-\frac{2\left(5^{n-1}\right)}{3^{n-2}}$ Consider $\frac{u_n}{u_{n-1}}$:	

	$\frac{u_n}{u_{n-1}} = \frac{-2\left(\frac{5^{n-1}}{3^{n-2}}\right)}{-2\left(\frac{5^{n-2}}{3^{n-3}}\right)} = \frac{5}{3}$ The ratio $\frac{u_n}{u_{n-1}}$ is a constant, therefore the sequence is a geometric progression with common ratio $\frac{5}{3}$.	Note that it is NOT enough to show that $\frac{u_2}{u_1}$ is a constant.
(bi)	Total number of elements in first n sets $= \underbrace{2+3+4+...+(n+1)}_{\text{A.P.: } a=2, d=1, l=(n+1), \text{ no. of terms}=n}$ $=\frac{n}{2}\left[2+(n+1)\right]$ $=\frac{n}{2}(n+3) \qquad (\text{shown})$	
(bii)	Consider the sequence without grouping: 1, 3, 5, 7, 9, 11, 13, 15, 17, … The first element of the set A_{n+1} is the $\left[\frac{n}{2}(n+3)+1\right]^{\text{th}}$ term in this sequence, which is an A.P. with first term 1 and common difference 2. First element of the set A_{n+1} $=1+\left[\frac{n}{2}(n+3)+1-1\right]2$ $=1+n(n+3)$ $=n^2+3n+1$	

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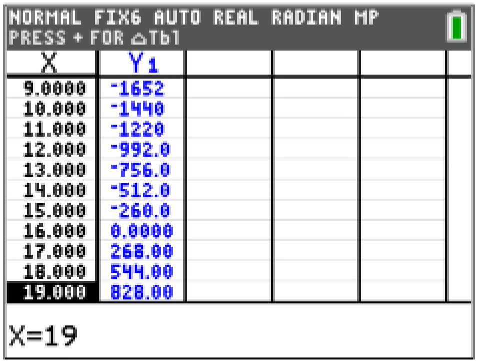
15(i)	**HCI14/C1Mid-year/Q8** Amount at the end of n months $=1000+\frac{n}{2}\left[2\times10+10\left(n-1\right)\right]=1000+5n^2+5n$	
(ii)	$1000+5n^2+5n>2000$ METHOD 1 $\Rightarrow n^2+n-200>0$ $\Rightarrow n<-14.7\text{(rej)}$ or $n>13.65$ Since $n>0$, least $n=14$ So by the end of the 14th month. METHOD 2 (Use table from GC) When $n=13$, LHS = 1910 When $n=14$, LHS = 2050 Hence least $n=14$ So by the end of the 14th month.	
(iii)	1st month, $1000\times1.06-10$ 2nd month, $\left(1000\times1.06-10\right)1.06-10$ $=1000\times1.06^2-\left(1.06+1\right)10$ 3rd month, $\left[1000\times1.06^2-\left(1.06+1\right)10\right]1.06-10$ $=1000\times1.06^3-\left(1.06^2+1.06+1\right)10$ $\therefore$ the amount by the end of the n^{th} month $=1000\times1.06^n-\left(1.06^{n-1}+...+1.06+1\right)10$ $=1000\times1.06^n-\frac{1.06^n-1}{1.06-1}10$ $=\left(\frac{2500}{3}\right)1.06^n+\frac{500}{3}$ (Shown)	
(iv)	When Account A exceeds Account B, $1000+5k^2+5k>\left(\frac{2500}{3}\right)1.06^k+\frac{500}{3}$ $5k^2+5k-\left(\frac{2500}{3}\right)1.06^k+\frac{2500}{3}>0$ From GC, When $k=14$, $\text{LHS}=-0.75<0$ When $k=15$, $\text{LHS}=36.2>0$ $\therefore\ k=15$	

16(a)	**(IB May12/MathSLP2/TZ1/Q4 modified)** $u_1 + 5d = 100$ --- (1) $u_1 + 9d = 124$ --- (2) Solve (1) and (2) simultaneously, $u_1 = 70,\ d = 6$ $S_{20} = \frac{20}{2}\left(2\times 70 + 6\left(20-1\right)\right)$ $= 2540$	
(b)	$\frac{n}{2}\left(2\times 70 + 4\left(n-1\right)\right) = 1600$ $4n^2 + 136n - 3200 = 0$  NORMAL FIX6 AUTO REAL RADIAN MP PRESS + FOR △Tbl X Y₁ 9.0000 ⁻1652 10.000 ⁻1440 11.000 ⁻1220 12.000 ⁻992.0 13.000 ⁻756.0 14.000 ⁻512.0 15.000 ⁻260.0 16.000 0.0000 17.000 268.00 18.000 544.00 19.000 828.00 X=19 Using GC $\therefore n = 16$	
(c)	Total number of people $= \frac{3^7 - 1}{3-1} = 1093$	
	$\frac{3^n - 1}{3-1} = 29524$ $3^n = 59049$ $n = \frac{\ln 59049}{\ln 3}$ $= 10$ Therefore the exact time is 12:45	

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17 **EJC 2018/BT/2**

(i) Duration of each session in third week $=100\times\left(1.1\right)^2=121$ minutes

Distance run $=\left(121\times 60\right)\times 3=21780$ metres

(ii) Let n be the number of weeks that Mr. Daya trains for. Then $100\times\left(1.1\right)^{n-1}\times 60\times 3\geq 42195$

Method 1

$$n-1\geq \log_{1.1}\left(\frac{42195}{18000}\right)$$

$$n-1\geq 8.938\ldots$$

$$n\geq 9.938\ldots$$

Method 2

From GC,

n	$18000\left(1.1\right)^{n-1}$
9	38585
10	42443

Hence, Mr. Daya trains for 10 weeks.

Total distance run during training

$$=2\times\frac{\left(100\times 60\times 3\right)\times\left(1.1^{10}-1\right)}{1.1-1}$$

$$=573747 \text{ m (to the nearest metre)}$$

18 **EJC 2018/BT/3**

(i) $u_2=t_3+t_4$

$$=\left(a+2d\right)+\left(a+3d\right)$$

$$=2a+5d$$

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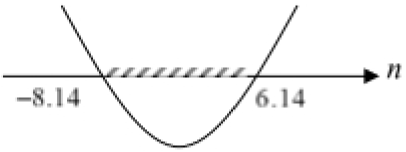
<table>
<tr>
<td>(ii)</td>
<td>$u_n = t_{2n-1} + t_{2n}$
$= \left(a + (2n-2)d\right) + \left(a + (2n-1)d\right)$
$= 2a + (4n-3)d$</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>$u_n - u_{n-1} = \left(2a + (4n-3)d\right) - \left(2a + (4n-7)d\right)$
$= 4d$ which is a <u>constant</u> independent of n ,
so the sequence is an arithmetic progression.
<u>OR:</u>
$u_n = 2a + (4n-3)d$
$= (2a+d) + (n-1)(4d)$
which forms an arithmetic progression with first term $(2a+d)$ and common difference $4d$.</td>
<td>Note the 2 different methods to show a sequence being arithmetic.</td>
</tr>
<tr>
<td>19(a)</td>
<td>SAJC 2018/BT/8
Total amount of drug, $S_n = \frac{n}{2}\left(2(3) + 2(n-1)\right) \le 50$
<u>Either</u>
<table>
<tr><td>n</td><td>S_n</td></tr>
<tr><td>6</td><td>48 (≤50)</td></tr>
<tr><td>7</td><td>63 (>50)</td></tr>
</table>
<u>Or</u>
$\frac{n}{2}(4+2n) \le 50$
$n^2 + 2n - 50 \le 0$
$-8.14 \le n \le 6.14$


Hence John can continue taking his medication until Day 6.</td>
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</table>

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<tr>
<td>(b)(i)</td>
<td>
<u>Either</u>
Amount of drug after 3 complete days
$=\left(30+[30+30(0.4)]0.4\right)0.4$
$=30(0.4)+30(0.4)^2+30(0.4)^3$
$=18.72 \text{ mg}$

<u>Or</u>
Amount of drug after
1 day: $(30)0.4=12$
2 days: $(30+12)0.4=16.8$
3 days: $(30+16.8)0.4=18.72\,\text{mg}$
</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>
<table>
<tr><th>n</th><th>Start</th><th>End</th></tr>
<tr><td>1</td><td>30</td><td>$30(0.4)$</td></tr>
<tr><td>2</td><td>30+30(0.4)</td><td>$30(0.4)+30(0.4)^2$</td></tr>
<tr><td>3</td><td>$30+30(0.4)+30(0.4)^2$</td><td>$30(0.4)+30(0.4)^2+30(0.4)^3$</td></tr>
<tr><td>⋮</td><td></td><td></td></tr>
<tr><td>n</td><td>$30+30(0.4)+\ldots+30(0.4)^{n-1}$</td><td>$30(0.4)+30(0.4)^2+\ldots+30(0.4)^n$</td></tr>
</table>

Total amount of drug after n days
$=30(0.4)+30(0.4)^2+...+30(0.4)^n$
$=\dfrac{30(0.4)\left(1-(0.4)^n\right)}{1-0.4}$
$=20\left(1-(0.4)^n\right)$
</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>
The drug levels at the end of each day form an increasing sequence.

In the long run (as $n\to\infty$), $20(1-(0.4)^n)\to 20$.

The drug level is highest at the start of the day, but still $<20+30$ i.e. <50.

Hence David can take the drug indefinitely.
</td>
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</tr>
</table>

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(iv)	Let r be the proportion of drug left in the body at the end of the day. Total amount of drug after 20 days $=30r+30r^2+...+30r^{20}$ $=\frac{30r\left(1-r^{20}\right)}{1-r}$ If 53 mg was found in the body $\frac{30r\left(1-r^{20}\right)}{1-r}=53$ Using GC, $r=0.6385$. Hence the percentage is left in his body at the end of each day is 63.9%	
20(i)	**VJC 2018/BT/7** $u_k=3r^{k-1}$ $\ln u_k=\ln(3r^{k-1})=\ln 3+(k-1)\ln r$ Consider $\ln u_k-\ln u_{k-1}=\left[\ln 3+(k-1)\ln r\right]-\left[\ln 3+(k-2)\ln r\right]$ $=\left(k-1-(k-2)\right)\ln r$ $=\ln r$ Since, r is a constant, ln r is also a constant. Hence, $\ln u_1,\ln u_2,\ln u_3,\ldots$ is an AP.	Using the difference of the first few consecutive terms to show that sequence is arithmetic is wrong, i.e. $u_2-u_1=\ln r$ $u_3-u_2=\ln r$ You are merely showing that the first 3 terms form an AP! Using $\ln u_k=\ln 3+(k-1)\ln r$ and stating that $a=\ln 3$ and $d=\ln r$ is not accepted as well as we are looking for the distinct nature of arithmetic sequences – **any two consecutive terms have a common difference.**
(ii)	$\sum_{k=1}^{30}\ln u_k=45$ $\frac{30}{2}\left(\ln 3+\ln\left(3r^{29}\right)\right)=45$ $\ln\left(9r^{29}\right)=3$ $9r^{29}=\mathrm{e}^3$ $r=\sqrt[29]{\frac{\mathrm{e}^3}{9}}=1.03$ (3 s.f.)	Algebraic errors such as $\ln\left(3r^{29}\right)=29\ln\left(3r\right)$ could be costly.

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<table>
<tr>
<td>(iii)</td>
<td>Consider $\frac{\frac{1}{u_n}}{\frac{1}{u_{n-1}}} = \frac{u_{n-1}}{u_n} = \frac{3r^{n-2}}{3r^{n-1}} = \frac{1}{r}$.

Since $\frac{1}{r}$ is a constant, the sequence is geometric.

$\frac{1}{r} = \frac{1}{\sqrt[29]{\frac{e^3}{9}}} = 0.973$

Since $-1 < \frac{1}{r} < 1$, hence, this geometric progression is convergent, and $S_\infty = \frac{1/3}{1-0.97270} = 12.2$ (3 s.f.)</td>
<td>When applying the formula to find the sum to infinity of a geometric series, ensure you are substituting the correct first term and common ratio.

Always use a 5 s.f. or a more accurate answer in your intermediate working.</td>
</tr>
<tr>
<td>21(i)</td>
<td>Amount at the end of June 2010

$= 150000\left(1+\frac{0.2}{100}\right)^6 = \$151\,809.02$</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Amount at the end of January 2010

$= (150000-1000)(1.002) = \$149\,298$</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td colspan="2">Taking Jan 2010 as the first month
<table>
<tr><th>Month</th><th>Beginning of month after withdrawal</th><th>Amount at end of month</th></tr>
<tr><td>1</td><td>$(150000-1000)$</td><td>$(150000-1000)(1.002)$</td></tr>
<tr><td>2</td><td>$(150000-1000)(1.002)-1000$</td><td>$\left[(150000-1000)(1.002)-1000\right](1.002)$
$=150000(1.002)^2-1000\left(1.002^2+1.002\right)$</td></tr>
<tr><td>3</td><td>$150000(1.002)^2-1000\left(1.002^2+1.002\right)$
-1000</td><td>$\begin{bmatrix}150000(1.002)^2-1000\left(1.002^2+1.002\right)\\-1000\end{bmatrix}(1.002)$
$=150000(1.002)^3-1000\left(1.002^3+1.002^2+1.002\right)$</td></tr>
<tr><td>…</td><td></td><td></td></tr>
<tr><td>n</td><td></td><td>$150000(1.002)^n-1000\left(1.002^n+\cdots+1.002^2+1.002\right)$</td></tr>
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	Amount at the end of the n-th month $=150000(1.002)^n-1000\left[(1.002)^n+\ldots+(1.002)^2+1.002\right]$ $=150000(1.002)^n-1000\left[\dfrac{1.002\left(1.002^n-1\right)}{1.002-1}\right]$ $=150000(1.002)^n-501000\left(1.002^n-1\right)$ $=501000-351000(1.002)^n$	
(iv)	$501000-351000(1.002)^n\le 0$ $(1.002)^n\ge\dfrac{501000}{351000}$ $n\ge\dfrac{\ln\left(501/351\right)}{\ln(1.002)}$ $n\ge 178.09$ Therefore account is depleted in 179th month which is November 2024.	
(v)	Amount for last withdrawal $=\ 501000-351000(1.002)^{178}=\ \$\,87.87$	