

CATHOLIC JUNIOR COLLEGE General Certificate of Education Advanced Level Higher 2 JC2 Preliminary Examination

MATHEMATICS

Paper 2

9740/02

29 AUGUST 2012 3 hours

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in. Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER. Place this cover sheet in front and fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Name:						Class:								
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Marks														
Total	4	10	10	8	8	8	8	8	6	4	6	11	9	100

This document consists of 6 printed pages.



Catholic Junior College

Section A: Pure Mathematics [40 marks]

1 Let f(x) be a cubic polynomial where f(2) = 6 and f(-1) = 15. Also, f(x) takes on stationary [4] values at x = -2 and x = 1. Find f(x).

2 A line,
$$l_1$$
, and a plane p_1 , have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \lambda \in \Re \text{ and } \mathbf{r} \cdot \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = 4$

respectively.

- Find the intersection point of l_1 and p_1 . (i)
- (ii) Find an equation, in scalar product form, for the plane p_2 that contains l_1 and the point [3] B(0, -1, -2).
- Find an equation for the line of intersection, l_2 , between p_1 and p_2 . (iii)

...

(iv) A plane,
$$p_3$$
, has an equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} = 3$. What is the relationship between a and b
such that p_3 is parallel to l_2 ? [2]

such that p_3 is parallel to l_2 ?

3 The function f is defined as follows.

$$f: x \to \frac{-1}{x^2 + 1}$$
 for $x \in \mathfrak{R}$.

(i) Sketch the graph of y = f(x). [1] If the domain of f is restricted to $x \le k$, state with a reason the greatest value of k for (ii) which the function $f^{-1}(x)$ exists. [2]

With the restricted domain in (ii), find an expression for $f^{-1}(x)$ and the real value of (iii) a such that $f^{-1}(a) = f(a)$. [4]

The function g is defined as follows.

$$g: x \to x+3$$
 for $x \in \Re$.

Find a restriction on the domain of g such that $f^{-1}g$ exists, using the function $f^{-1}(x)$ (iv) found in part (iii). [3]

4 (i) Find
$$\int x \sin^2 x \, dx$$
.

The region R is bounded by the curve $y = \sqrt{x} \sin x$, the lines x = 0 and $x = \pi$, and (ii) the x-axis. Find the volume of the solid of revolution formed when R is rotated [2] through 4 right angles about the *x*-axis.

[Turn over

[4]

[3]

[2]

- (iii) Hence calculate the volume of the solid of revolution formed when S is rotated through 4 right angles about the x-axis, where S is the region bounded by the curve $y = \sqrt{x} \sin x$, the lines $x = \pi$ and $y = \sqrt{\pi}$, and the y-axis. [2]
- 5 A family of curves is defined by the differential equation

$$xy\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^2.$$

By substituting y = ux, where u is a function of x, find an expression for the family of

curves, expressing it in the form $y^2 = f(x)$.

Given that one of the members from the family of curves passes through (1, 2), find an expression for the particular solution curve and state the values of *x* when y = 3.

Section B: Statistics [60 marks]

6 In the Student Council of Charisma Junior College, there are a total of 31 members, comprising 22 members in the Student Services (SS) Wing and 9 members in the Charismatic Influence (CI) Wing. At the college's morning assembly, a total of 6 members are on duty, carried out in pairs.

The three different duties every morning consist of the Daily Inspirations, the Flag Raising and the Recital of the national anthem and pledge. All duties can be carried out by any student councillor except the Daily Inspirations component, which must be carried out by members of the CI Wing. Find

(i)	the total number of possible duty groupings,	[2]
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- (ii) the total number of duty groupings where Jill, a member of the CI Wing, is on duty, [2]
- (iii) the total number of duty groupings where Jack, a member of the SS Wing, is on duty. [2]
- (iv) Hence comment on whether it is fair to the Student Council members if the duty groupings in part (i) were picked at random, justifying your answer. [2]
- 7 In Sunny Island Republic, all male citizens are required to undergo a National Physical Fitness Assessment (NPFA) Test before enlisting for National Service in the army. Based on historical data, 75% of males who attempt the NPFA Test will pass the test and they are awarded either a Bronze, Silver or Gold standard. The performance at the NPFA Test is known to influence the chances of these National Servicemen being promoted to the rank of an Officer.

For male citizens who fail the NPFA Test, they are required to undergo a Physical Assessment Test (PAT) at the end of their basic training. It is known that those who pass the PAT will have a chance to be promoted to the rank of an Officer. The information is summarized in the tree diagram below.

[5]

[3]



(i)	It is known that the probability that a randomly chosen serviceman is an Officer is	
	0.60. Find the value of a.	[2]
(ii)	Find the probability that a randomly chosen serviceman failed his NPFA Test given	
	that he is an Officer.	[2]
(iii)	Hence find the probability that a randomly chosen Officer passed his NPFA Test.	[1]
(iv)	Find the probability that a randomly chosen serviceman who is a Non-officer had	
	failed his NPFA Test.	[3]

- 8 On average, the probability that John receives at least one Short Message (SMS) on his mobile phone in any given 30-minutes period is 0.95. A school day consists of 14 such periods of 30 minutes each. The number of periods in which John receives at least one SMS is the random variable *X*.
 - (i) State, in the context of this question, the assumptions needed to model X by a binomial distribution.
 (ii) Explain why one of the assumptions stated in part (i) may not hold in this context.
 (iii) Find the se assumptions in part (i) hold.
 (iii) Find the probability that in a school day, there are at most 10 periods in which John receives at least one SMS.
 (iv) Using an appropriate approximation, find the most likely number of periods in a
 - week of 5 school days in which John does not receive any SMS. State the approximate distribution clearly. [3]

- **9** In a school canteen, the number of people joining the noodle stall queue in a period of 30 seconds is a random variable with the distribution Po(1.5).
 - (i) Find the probability that, in a period of 5 minutes, at least 10 people join the queue. [1]
 - (ii) The number of people leaving the same queue in a period of 30 seconds is a random variable with distribution Po(1.9). In a period of 5 minutes, find the probability that at least 10 people left the queue. (You may assume that there are more than 10 people in the queue.)
 - (iii) Using appropriate approximations, find the probability that a queue of 20 people will be left with 5 people at the end of 10 minutes. [4]
- Susan wants to set up a food stall in a particular food centre at Hougang and intends to interview 50 people on their food preference and average food expenditure.
 (i) Identify an appropriate sampling method and justify your choice.
 - (ii) Describe how Susan can carry out the chosen method in (i). [2]
- 11 A sample of *n* observations was taken from a population of mean μ and variance 10. Find the least sample size required such that the probability of the sample mean lying between $\mu - 0.5$ and $\mu + 0.5$ is more than 0.89. State an assumption that you have to make in order to proceed with this calculation.
- 12 A bakery claims that the mass of a cake, on average, is 600 grams each. A random sample of 10 cakes is examined and the mass, x grams, of each cake is determined. It is found that

$$\sum (x-500) = 800$$
 and $\sum (x-500)^2 = 65764$.

- (i) Find the unbiased estimates of the population mean and variance of x. [2]
- (ii) It is found that there is significant evidence that the mass of the cakes differs from 600 grams. Stating a necessary assumption, find an inequality satisfied by the significance level of the test. [4]
- (iii) On a Sunday, the bakery baked 100 cakes, with a sample mean of *m* grams and standard deviation of 33 grams. The hypotheses are as follows:

$$H_0: \mu = 600$$

$H_1: \mu < 600$

Another test is carried out at the 5% significance level and it is found that there is
insufficient evidence that the bakery overstated the mean mass of the cakes.[4]Find the least value of m.[4]Giving a reason, state whether it is necessary to make any assumptions about the
distribution of the masses of cakes.[1]

[2]

[6]

13 An experiment was conducted with a ramp placed on top of a table. A ball is then released from the ramp, at a height H mm above the table. When the ball leaves the ramp's edge, it travels in a parabolic motion to the floor. The horizontal distance travelled by the ball before it lands, D mm, is then measured and recorded.



Observations at different heights are captured in the following table:

Н	100	200	300	450	600	800	1000
D	253	337	395	451	495	534	573

The value of the product moment correlation coefficient, r, for this data is 0.962, correct to 3 significant figures.

(i) Sketch a scatter diagram for the above data. [1]
(ii) Using the diagram in (i), explain why it is insufficient to rely on the value of *r* to indicate a linear relationship between any two variables. [2]

It is thought that the horizontal distance *D* can be modelled by

$$D = a + bH^2$$
 or $D = a + b\ln H$

- (iii) Explain which of the above models would provide a better estimate of *D* for a given value of *H*.
- (iv) Using the model chosen in (iii) and an appropriate regression line, find the value of H when D = 400 mm. [2]
- (v) Comment on the reliability of the estimate of H when D = 700 mm, as compared to the estimate obtained in (iv). [2]

$$-$$
 THE END $-$