ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS   **9758/2**

JC2 Prelim Paper 2 (100 marks)   **16 Sept 2024**

                                                                          3 hours

Additional Material(s):   List of Formulae (MF26)

CANDIDATE NAME [   ]

CLASS [   ][   ] / [   ][   ]

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.

Please write clearly and use capital letters.

Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.

Do not tear out any part of this booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
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10	
11	
Total	

This document consists of 22 printed pages and 6 blank pages.

[Turn Over

Section A: Pure Mathematics [40 marks]

1   By using the substitution $x = \cot\theta,$ for $0 < \theta < \dfrac{\pi}{4}$, find $\displaystyle\int \dfrac{1}{x^2\sqrt{1+x^2}}\,dx.$ [4]

2   **(a)(i)**   Find $\displaystyle\int \dfrac{9u-8}{4+9u^2}\,du$.   [3]
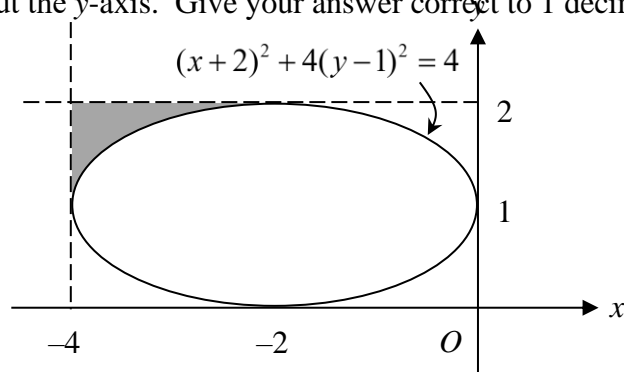
   (ii)   The curve C is given by the parametric equations

$$x = u^2 + u + 1,\ \ y = \dfrac{9u}{4+9u^2}, \text{where } u \geq 0.$$

   Find the exact area bounded by C, the x-axis and the line $x = 3$.   [4]

   (b)   Find the volume of the solid formed when the shaded region bounded by the

   lines $x = -4$, $y = 2$ and the ellipse $(x+2)^2 + 4(y-1)^2 = 4$ is rotated through

 2π radians about the y-axis. Give your answer correct to 1 decimal place.

[3]

3   With respect to the origin O, the points A, B, C, D and E have coordinates

   $A(2,3,4)$, $B(6,5,7)$, $C(8,9,6)$, $D(4,7,3)$ and $E(5,6,10)$.

   (a)   Show that the cartesian equation of the surface containing the points A, B

   and E is $x - 5y + 2z = -5$.   [2]

   A line passes through the point D and the midpoint M of the line segment EC.

   (b)   Find the vector equation of the line DM.   [3]

   (c)   Find the exact coordinates of the foot of the perpendicular from the point M

   to the surface found in part **(a)**.   [3]

   (d)   Hence find the exact shortest distance from the point M to the surface found

   in part **(a)**.   [2]

   (e)   Verify the line DM intersects the surface found in part **(a)** at the point P with

   coordinates $(9,8,13)$. Hence find the vector equation of the reflection of the

   line DM about this slant surface.   [4]

4 A popular toy company is designing a new water play feature for children. The toy consists of a cylindrical water container that will hold water for various playful activities. This cylindrical container is designed to be inscribed within a fixed, rigid hemispherical shell made of durable plastic of negligible thickness.

r cm $2a$ cm
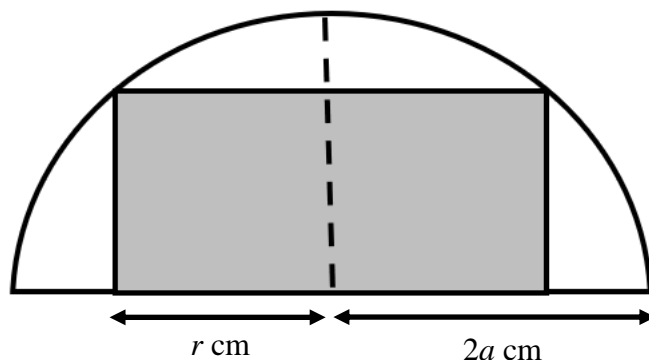
The shaded region in the diagram above shows the cross-sectional view of the upright cylindrical container that is inscribed in a hemisphere with fixed radius $2a$ cm.

(a) If the radius of the cylindrical water container is r cm, show that the volume V cm^3 of the water container is given by $V = \pi r^2 \sqrt{4a^2 - r^2}$.

[1]

The unique feature of this toy is that the height of the cylindrical container is adjustable, allowing it to expand or contract while always touching the inner surface of the hemisphere.

(b) Water is pumped into the container at a rate of $100\pi \, \text{cm}^3\text{s}^{-1}$ while the adjustment is taking place. If $a = 50$, find the exact rate of change of the radius of the container at the instant when the height of the water container is 80 cm.

[5]

(c) Using differentiation, find in terms of a, the value of r which gives a maximum value of V. Justify that this value indeed gives a maximum V. Hence write down the exact maximum volume of the cylinder in terms of a.

[4]

(d) Sketch the graph showing the volume of the cylinder as the radius of the cylinder varies.

[2]

[Turn Over

Section B: Probability and Statistics [60 marks]

5   An amateur music composer is arranging a sequence of four musical notes followed by three beats. There are 7 possible notes (labeled A to G) and 5 possible beats (labeled 1 to 5). The order of the notes and beats is important in the composition. Find the probability that a randomly chosen sequence has

    (i) the third beat being a higher number than the second beat,   [1]

    (ii) exactly two notes the same or exactly two beats the same, but not both.   [3]

6   Tetrahedral dice have four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled, and the numbers faced down on the two dice are recorded. The random variable T is defined as the score on the red die multiplied by the score on the blue die.

    (i) Find the probability distribution of T.   [3]

    (ii) Find $E(T)$ and show that $\text{Var}(T) = \dfrac{115}{16}$. Show your workings clearly.   [2]

    (iii) Evaluate $P\big(|T - 2\mu| > \sigma\big)$, where $\mu = E(T)$ and $\sigma^2 = \text{Var}(T)$.   [2]

7   The masses, in grams, of the packets of semolina flour follow the distribution $N(225, 25^2)$ and the masses, in grams, of the packets of millet flour follow the distribution $N(\mu, \sigma^2)$.

    (a) Find the probability that 4 times the mass of a packet of semolina flour is between 0.85 kilograms and 1.05 kilograms.   [2]

    (b) Let M be the mean mass of 3 packets of semolina flour and 2 packets of millet flour. Given that $P(M < 125) = P(M > 265) = 0.02$, show that the value of μ is 150. Hence, by finding an equation involving σ, find the value of σ.   [5]

8   An office team of 10 people includes 7 men and 3 women named Anne, Beth, and Cathie. For an upcoming fire drill exercise, 5 individuals will be chosen, each assigned a unique role, to carry out the drill. Determine the number of possible ways to select 5 people from this group of 10

   (i)   to conduct the fire drill,   [1]

   (ii) such that at most 1 woman is selected to conduct the fire drill.   [2]

   (iii) After the fire drill exercise, the 10 people are to hold a discussion at a round table with 10 identical seats. Determine the number of ways in which Beth is seated between Anne and Cathie.   [1]

   (iv) A group photo of the 10 people, arranged in two rows of five, was taken after the discussion. Determine the number of ways in which Beth is not standing beside Anne or Cathie.   [4]

9   A bakery produces batches of cookies. On average, the proportion of flawed cookies produced is p, where $0 < p < 1$. The cookies are packed in boxes of 20. The number of flawed cookies in a box of cookies is denoted by C.

   (a) State, in context, one assumption needed for the number of flawed cookies in a box to be well-modelled by a binomial distribution.   [1]

   (b) Given that P($C = 0$ or 1) $= 0.15$, write down an equation for the value of p, and find this value numerically.   [2]

   For **(c)** and **(d)**, take $p = 0.08$.

   (c) Ten boxes of cookies are randomly chosen. As part of the bakery's quality control process, a box of cookies will be accepted if it contains fewer than 4 flawed cookies, otherwise it will be rejected. Find the probability there are at least 2 but no more than 5 rejected boxes.   [4]

   (d) A random sample of 15 boxes of cookies is taken and 3 of the boxes are found to be rejected. Find the probability that the third rejected box occurs on the fifteenth box.   [3]

[Turn Over

10 **(a)** Observations of 8 pairs of values (u, g), representing the hours of internet usage per week (u) and academic performance (g) in terms of Grade Point Average (GPA), are shown in the table below.

Internet usage (u)	4.0	6.0	8.0	a	12.0	16.0	18.0	20.0
GPA (g)	3.7	3.5	3.4	3.2	3.0	2.7	2.6	2.5

It is known that the equation of the linear regression line of g on u is $g = -0.0765u + 3.99$, find the value of a correct to 1 decimal place.

[2]

(b) A researcher is studying the relationship between the battery life (y, in hours) of a new smartphone model and the screen brightness setting (x, in %). The following data was collected from the tests conducted at different brightness levels.

Screen Brightness (x)	10	20	30	40	50	60	70
Battery life (y)	48.2	47.4	45.5	37.3	35.6	31.1	24.3

(i) Draw a scatter diagram for these values.

[2]

(ii) One of the values of y appears to be incorrect. Circle this point on your diagram and label it P.

[1]

(iii) Explain why a linear model $y = a + bx$ is not a suitable model.

[1]

(iv) It is thought that the battery life (y) can be modelled by one of the formulae after removing the point P.

(A) $y = a + bx^2$,

(B) $y = a + b\ln x$,

where a and b are non-zero constants.

Find, correct to 4 decimal places, the product moment correlation coefficient between y and x^2 as well as y and $\ln x$.

Explain clearly which model is a better model for this set of data.

For the case identified, find the equation of a suitable regression line.

[3]

(v) Using the regression line found in **(iv)**, estimate the battery life when the screen brightness is set to 80%.

[1]

(vi) Comment on the reliability of your answer in part **(v)**.

[1]

11 **(a)** The leaves of a particular plant species have an average length of 12 cm with a standard deviation of 3.5 cm. If a random sample of 100 leaves is selected, estimate the probability that their total length is at least 1138 cm.

[2]

(b) An operator of a public workspace at location A claims that users of its one-seater pods spend an average of 131 minutes using the facilities. To test this claim, a random sample of 64 users was observed, revealing a mean usage time of 127 minutes with a standard deviation of 16.4 minutes.

(i) Test at 3% level of significance whether the workspace operator's claim is overstated. You should state the hypotheses and define any symbols you use.

[5]

(ii) Explain the meaning of '3% level of significance' in the context of the question.

[1]

(iii) The workspace operator at location B claims that the mean time spent by users of its one-seater pods is 140 minutes, with a known population standard deviation of 20.1 minutes. A new sample of 15 pod users is taken, and the sample mean usage time, $\bar{w}$, is reported. A hypothesis test is conducted at a 5% significance level, and the operator's claim is not rejected.

State two necessary assumptions for the test and determine the range of values that $\bar{w}$ can take. Give your answer correct to one decimal place.

[5]

End of paper