Candidate Name	Class	Register Number	Calculator Model No

CHANGKAT CHANGI SECONDARY SCHOOL
End of Year Examination 2023

Subject	:	**Mathematics**
Level	:	**Sec 2 Express**
Paper	:	**4052/01**
Date	:	**3 October 2023**
Duration	:	**1 hour 15 minutes**
Setter	:	**Ms Wong SY**

READ THESE INSTURCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [   ] at the end of each question or part question.
The total of the marks for this paper is 50.

For Examiners' Use	Marks
Marks	/ 50
Personal Target	Actual Grade
Parent's / Guardian's Signature	

This document consists of **11** printed pages, including the cover page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P(1 + \frac{r}{100})^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi r l$$

$$\text{Curved surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2}$$

Answer **all** the questions

1 **(a)** Simplify $3(2x-1)+5$.

Answer .... [1]

(b) Factorise x^2y-3x.

Answer .... [1]

---

2 **(a)** Express 441 as a product of its prime factors.

Answer $441 = $ ……………….................... [2]

(b) Using your answer to part **(a)**, explain why 441 is a perfect square.

Answer .... [1]

....

....

(c) The number $441k$ is a perfect cube.

Find the smallest positive integer value of k.

Answer .... [1]

(d) Find two numbers, both smaller than 150, that have a lowest common multiple of 441 and a highest common factor of 21.

Answer .... , ... [2]

[Turn Over

3 The stem-and-leaf diagram shows the masses, in grams, of 15 packets of various brands of instant noodles.

7	2 2 5 5 8 8 8
8	4 5 5
9	8
10	0
11	0 5
12	5

For these masses, find

| Key | 7 | 2 | represents 72 grams |

(a) the range,

Answer .... *grams* [1]

(b) the mean,

Answer ....*grams* [2]

(c) the median,

Answer ....*grams* [1]

(d) the mode.

Answer ....*grams* [1]

4 Kate, Ting and Yen shared $450.
The ratio of the amount that Kate receives to the total amount that Ting and Yen receive is 5 : 4. Ting receives $40 less than Yen.
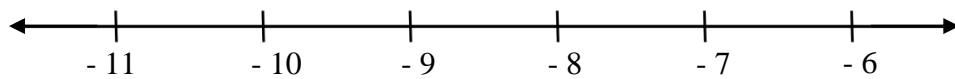Calculate the difference between the largest and smallest share.

Answer $................................ [3]

5 (a) Solve $-5x - 1 > -x + 27$.

Answer .... [2]

(b) Represent the solution in **(a)** on the number line provided below. [1]

$$-11 \qquad -10 \qquad -9 \qquad -8 \qquad -7 \qquad -6$$

(c) Write down the largest possible integer that satisfy the solution in **(a).**

Answer .... [1]

6 *n* is a positive integer.

Show that, for all *n*, $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 12. [2]

Answer

7 Factorise completely $6xy - 3x + 2y - 1$.

Answer .... [2]

8 Solve $\dfrac{y+1}{2} - \dfrac{5y}{4} = 1$.

Answer y = ... [3]

[Turn Over

9 $s = \dfrac{v^2 - u^2}{2a}$

    (a) Calculate the value of s when $v = 3$, $u = -2$ and $a = 1$.

                                                    Answer .... [1]

    (b) Make v the subject of the formula.

                                                    Answer .... [2]

10 (a) Factorise completely $2x^2 + 5x - 12$.

                                                    Answer .... [2]
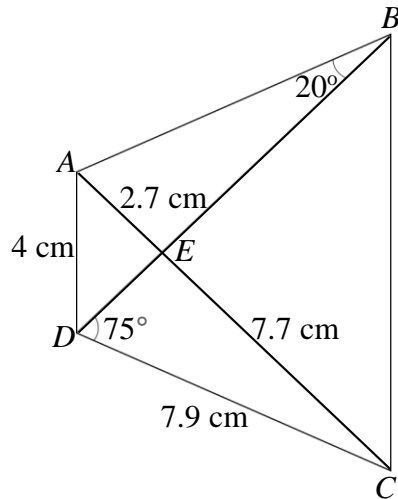
    (b) **Hence,** factorise completely $2(3a - 1)^2 + 5(3a - 1) - 12$.
           Write your answer as simply as possible.

                                                    Answer .... [2]

[Turn Over

11 In the quadrilateral *ABCD*, the diagonals *AC* and *BD* intersect at *E*. Triangles *ABE* and *DCE* are congruent, *AD* = 4 cm, *CD* = 7.9 cm, *AE* = 2.7 cm, *CE* = 7.7 cm, angle *ABD* = 20° and angle *BDC* = 75°.

(a)   Find the length of *AB*.

Answer .... cm   [1]

(b)   Calculate angle *CED*.

Answer .... °   [1]

(c)   State a triangle that is congruent to triangle *ABD*.

Answer triangle ....   [1]

12 The force, F, between two objects is inversely proportional to the **square** of the distance, r between them. When the force is 10 N, the distance between the two objects is 2 cm. Find the

(a) Find the

(i) formula for F in terms of r.

Answer .... [2]

(ii) F when $r = 5$ cm

Answer ....N [1]

(b) The distance between the two objects is reduced to 20%.
Find the percentage increase of the force between the two objects.

Answer ....% [2]

13 A bag contains red counters, yellow counters and green counters.

The probability that the counter is red is $\frac{1}{3}$.

The probability that the counter is yellow is $\frac{7}{15}$.

(a) A counter is chosen at random from the bag.

Find the probability that the counter is

(i) white,

Answer .... [1]

(ii) green.

Answer .... [1]

(b) Another bag contains 7 blue counters and 11 pink counters.

More pink counters are added to the bag.

The probability of choosing a pink counter from this bag at random is now $\frac{2}{3}$.

Find the total number of the pink counters in the bag.

Answer .... [2]

[Turn Over

14 A map of Indonesia has a scale of 1 : 2 500 000.

(a) The distance between Singapore and Bali is 1660 km.
Calculate the distance, in centimetres, between Singapore and Bali on the map.

Answer…………………. cm   [2]

(b) The area of Riau Islands is 8 270 km^2.
Calculate the area, in square centimetres, of Riau Islands on the map.

Answer………………… cm^2 [2]

End of Paper

CHANGKAT CHANGI SECONDARY SCHOOL
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Answer **all** the questions

1 **(a)** Simplify $9xy + (x-y)^2$.

Answer .... [2]

(b) Express $\dfrac{5x}{2(x+y)} - \dfrac{4}{(x-y)}$ as a single fraction in its simplest form.

Answer .... [2]

(c) Given that $(x-y)^2 = 58$ and $\dfrac{1}{2}xy = 12$, find the value of $3x^2 + 3y^2$.

Answer .... [2]

[Turn Over

2 The diagram below shows part of a regular polygon.

How many sides does the polygon have?

B $\angle$140°

A $\angle$140°
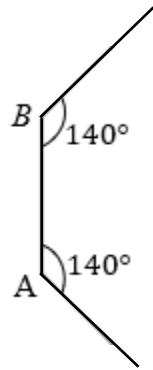
Answer ....sides [2]

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3 The diagram shows a right-angled triangle *ABC*, where *AB* = 5 cm, *BC* = 12 cm and *AC* = 13 cm.

A

13 cm

5 cm

B   12 cm   C
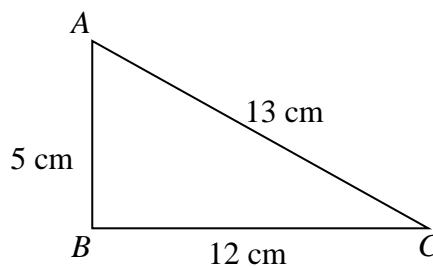
(a) Find $\cos \angle ACB$, giving your answers in fraction.

Answer .... [1]

(b) Hence, or otherwise calculate $\angle ACB$.

Answer .... [1]

4 The diagram below shows a sequence of figures made out of sticks.

Figure 4

Figure 3

Figure 2

Figure 1

5 sticks   8 sticks   11 sticks   ? sticks
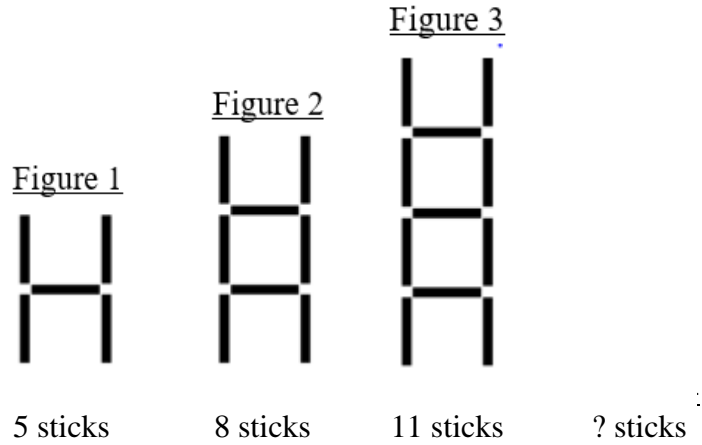
(a) Draw Figure 4 in the space above. [1]

(b) Find the number of sticks in Figure 5.

Answer .... [1]

(c) It is given that $T_n = 3n + 2$. Is it possible to form a figure with 2023 sticks in this sequence? Show your working and explain your answer clearly.
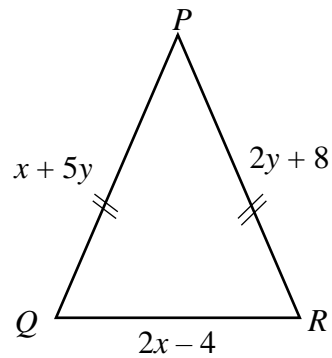
Answer [2]

Since *n* is ……...

....

....

....

5 In the diagram, *PQR* is an isosceles triangle.

$PQ = (x + 5y)$ cm, $PR = (2y + 8)$ cm and $QR = (2x - 4)$ cm.

(a) Given that $PQ = PR$, show that $x + 3y = 8$. [1]

Answer

(b) Given that the perimeter of triangle *PQR* is 26 cm, show that $3x + 7y = 22$. [2]

Answer

(c) Solve the simultaneous equations.

$$x + 3y = 8$$
$$3x + 7y = 22$$

Answer x =................................. [3]

y =.................................

(d) Hence, find the length of the shortest side of triangle *PQR*.

Answer................................. cm. [1]

6 Tennis balls are spherical shaped and covered in fibrous felt. According to the International Tennis Federation (IFD), an acceptable tennis ball can have a diameter ranging from 6.54 cm to 6.86 cm.

(a) **(i)** A spherical ball has a volume of 165 cm^3. Show that the radius is 3.40 cm.

Answer

[2]

(ii) Hence, explain if it is possible for the ball in part **(a)(i)** to be an acceptable tennis ball.
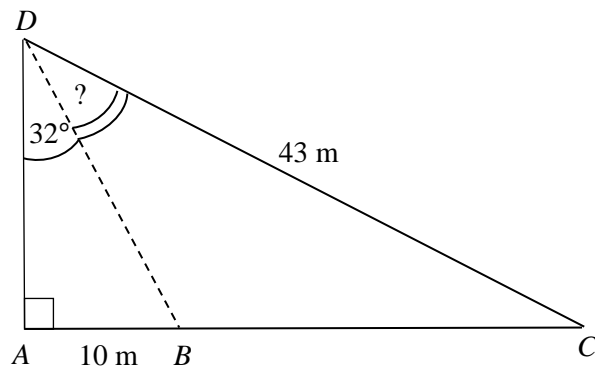
[1]

Answer …….…………………………………………………….............…

…………………………………………………………………………………..

(b) When manufacturing a tennis ball, a man-made fibre called felt is used to cover the ball. Felt cost $1.20 per square metres. The factory manufactures 700 tennis balls in a day. Find the daily cost of producing tennis balls.

Answer $.......................... [2]

7 In the diagram below, it is given that point *AB* is 10 m and $\angle ADB = 32°$.

(a) **(i)** Calculate the length of *AD*.
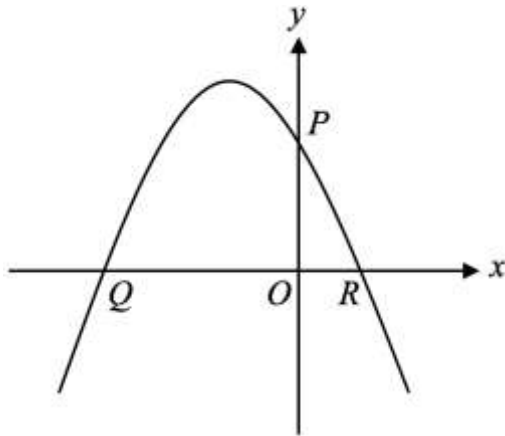
Answer......................... m   [2]

(ii) Find the length of *BD*.

Answer......................... m   [2]

(b) Given that *CD* = 43 m, calculate $\angle BDC$.

Answer............................^{o} [2]

[Turn Over

8 The graph of $y = (x+4)(1-x)$ cuts the x-axis at the points Q and R.

(a) Solve $(x+4)(1-x)=0$ and state the value of the two x-intercepts.

Answer $x =$ .... , ... [2]

(b) The graph cuts the y-axis at $P(0, y)$.

Write down the coordinates of P.

Answer $P($............... , ...$)$ [1]

(c) Is (-3 , 5) a point on the curve? Explain.

Answer ………………………………………………………………………… [2]

……………………………………………………………………………………

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9 **Answer the whole of this question on the graph paper, provided on the next page**

The variables x and y.are connected by the equation $y = \dfrac{1}{3}x(x+7)$.

Some corresponding values of x and y are given in the table below.

x	- 8	- 6	- 5	- 4	- 2	- 1	0	1	2
y	2.4	- 1.8	- 3	a	- 3	- 1.8	0	2.4	5.4

(a) Calculate the value of a.

Answer a = ... [1]

(b) Using a scale of 1 cm to represent 1 unit on both axes, draw the graph of [3]

$y = \dfrac{1}{3}x(x+7)$ for $-8 \le x \le 2$.

(c) State the minimum value of y.

Answer .... [1]

(d) From the graph, find the

    (i) value of y when $x = -2.6$,

Answer .... [1]

    (ii) the values of x when $y = -2$,

Answer .... , ... [1]

(e) Use the graph to solve $\dfrac{1}{3}x(x+7) = 0$.

Answer .... , ... [1]
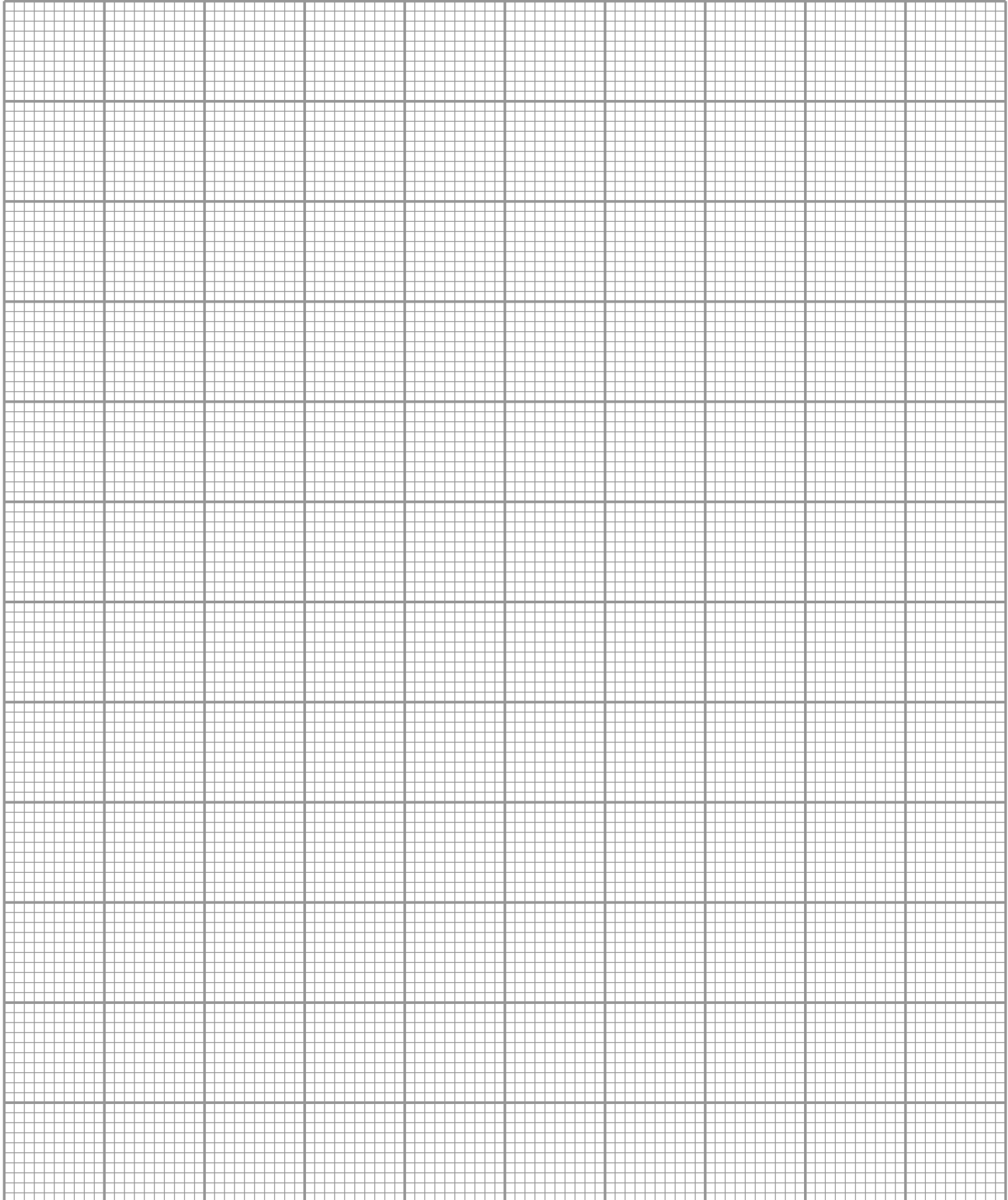
10 The Tan family plans to visit an amusement park when they travel to Taiwan. They
    found two Singapore travel agencies, ATAS Travel and HAN Travel, promoting a one-
    day trip to this amusement park.

Promotion Flyer from ATAS Travel

	Theme Park Pass		Ocean Park Pass		Theme Park + Ocean Park package	
	Adult	Child (3-11 years old)	Adult	Child (3-11 years old)	Adult	Child (3-11 years old)
Rates ($) per person	48	45	42	39	62	54

1 Day Getaway at Amusement Park in Taiwan (2 -way transport provided at no additional cost)

Peak Period: Saturdays, Sundays and Singapore Public Holidays

Child: Age 3-11 years old

Promotion Flyer from HAN Travel

1 Day Getaway at Amusement Park in Taiwan (transport is excluded)

Theme Park Pass	Season	Adult (12 years old and above)	Child (3-11 years old)
	Non-peak	$41	$34
	Peak	$43	$36
COMBO package	Season	Adult (12 years old and above)	Child (3-11 years old)
Theme Park Pass & Ocean Park Pass	Non-peak	$55	$48
	Peak	$59	$52
Ocean Park Pass	Season	Adult (12 years old and above)	Child (3-11 years old)
	Non-peak	$37	$30
	Peak	$40	$33

**All tourists must take the 2-way transport provided with an additional charge of $8 per person.

Peak Season:
11 – 19 March, 27 May – 25 June, 2 – 11 September, 18 November – 31 December

10   The Tan family consists of Mr and Mrs Tan, their 12-year old daughter and 5-year old son. They prefer to visit the Theme park on any day during the first week in December 2023.

    (a)   Calculate the cost if they choose ATAS Travel on a non-peak day.

Answer $ .... [2]

    (b)   Suggest which travel agency The Tan family should choose.
          Justify the suggestion you make and show your calculations clearly.
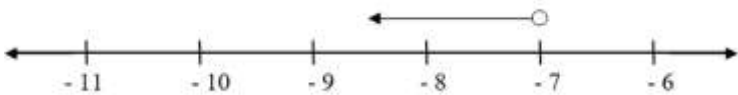
Answer The Tan family should choose .... travel agency   [3]

because..

....

End of Paper

1	**(a)**	$6x + 2$
	(b)	$x(xy - 3)$
2	**(a)**	$\begin{array}{r|r} 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ 7 & 7 \\ \hline & 1 \end{array}$ $441 = 3^2 \times 7^2$
	(b)	441 is a perfect square because the product of its prime factors have even powers 2 and can be square rooted exactly.
	(c)	$k = 21$
	(d)	63 , 147
3	**(a)**	53 grams
	(b)	$\dfrac{2(72) + 2(75) + 3(78) + 84 + 2(85) + 98 + 100 + 110 + 115 + 125}{15}$ $= \dfrac{1330}{15}$ $= 88\dfrac{2}{3} \text{ grams}$
	(c)	85 grams
	(d)	78 grams
4		$\begin{aligned} 9u &\to \$450 \\ 1u &\to \$50 \\ 5u &\to \$250 \\ 4u &\to \$200 \end{aligned}$ $\text{Kate} : \text{Ting} : \text{Yen}$ $250 : \ 80 \ : 120$ $Difference = 250 - 80$ $\qquad\qquad\ = \$170$
5	**(a)**	$-4x > 28$ $x < -7$
	(b)	
	(c)	-8
6		$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$ $= 9n^2 + 6n + 1 - 9n^2 + 6n - 1$ $= 12n$ There is a factor of 12, $\therefore (3n + 1)^2 - (3n - 1)^2$ is a multiple of 12.
7		$= 3x(2y - 1) + 1(2y - 1)$ $= (3x + 1)(2y - 1)$ or $(2y - 1)(3x + 1)$
8		$\dfrac{2y + 2 - 5y}{4} = 1$ $-3y + 2 = 4$

		$y = -\dfrac{2}{3}$
9	**(a)**	2.5
	(b)	$2as = v^2 - u^2$ $v^2 = 2as + u^2$ $v = \pm\sqrt{2as + u^2}$
10	**(a)**	$\begin{array}{cc\|c} x & +4 & +8x \\ 2x & -3 & -3x \\ \hline 2x^2 & -12 & +5x \end{array}$ $= (x + 4)(2x - 3)$
	(b)	$(3a - 1 + 4)(2(3a - 1) - 3))$ $= (3a + 3)(6a - 2 - 3)$ $= 3(a + 1)(6a - 5)$

11	(a)	7.9 cm	B1
	(b)	85°	B1
	(c)	DCA	B1
12	(a)(i)	$k = 40$	B1
	(a)(ii)	$F = \dfrac{40}{r^2}$	M1 A1
	(b)	$F = \dfrac{40}{5^2}$ $= 1\frac{3}{5}$	 B1 (Accept 1.6 N)
	(c)	new $r = 0.2r$ $F = \dfrac{40}{r^2}$ new $F = \dfrac{40}{(0.2\,r)^2}$ $= \dfrac{40}{(0.04)r^2}$ $= \dfrac{(25)40}{r^2}$ $= 25F$ % increase $= \dfrac{25F - F}{F} \times 100\%$ $= 2400\%$	M1 M1 (Accept any correct method) A1
13	(a)(i)	0	B1
	(a)(ii)	$\dfrac{1}{5}$	B1
	(b)	$\dfrac{2}{3} = \dfrac{11 + x}{7 + 11 + x}$ $x = 3$ $11 + 3 = 14$	M1 A1
14	(a)	$1660 \div 25$ $= 66.4\ cm$	M1 A1

	(b)	1 cm² : 625 km²
		  ? : 8270 km²
		
		Area on map = 8270 ÷ 625
		          = 13.232 cm²
			M1
			
			A1

CHANGKAT CHANGI SECONDARY SCHOOL

End of Year Examination 2023 Solution
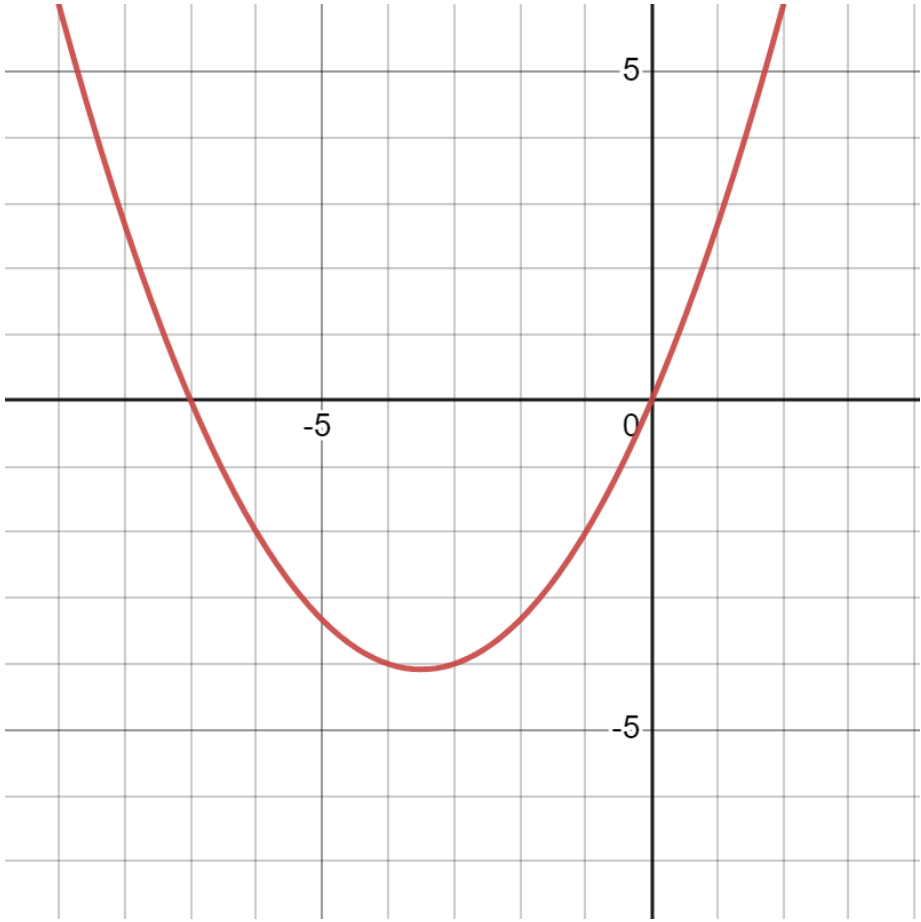
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Duration	:	**1 Hour 15 Minutes**
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General:
1. Deduct 1 mark for each instance of premature approximation.
2. Deduct a maximum of 1 mark per question for not giving non-exact numerical answer to 3 s.f. unless otherwise stated.
3. Deduct a maximum of 1 mark for missing/wrong units for one question.
4. Do not award marks for work done in pencil or ink other than blue or black; if entire script is done in that way, award 50% of total marks attained.

1	(a)	$9xy + x^2 - 2xy + y^2$ $= 9xy + x^2 - 2xy + y^2$ $= 7xy + x^2 + y^2$
	(b)	$\dfrac{5x(x-y) - 8(x+y)}{2(x+y)(x-y)}$ $= \dfrac{5x^2 - 5xy - 8x - 8y}{2(x+y)(x-y)}$
	(c)	$x^2 + y^2 - 2(24) = 58$ $x^2 + y^2 = 106$ $3x^2 + 3y^2 = 3(106)$ $\qquad\qquad = 318$
2		$\dfrac{360}{40}$ $= 9$
3	(a)(i)	$\dfrac{12}{13}$
	(ii)	22.6°

4	(a)	 Figure 4	B1
	(b)	$T_5 = 3(5) + 2 = 17$	B1
	(c)	$2023 = 3n + 2$ $n = 673\dfrac{2}{3}$ Since n is not an integer, it is not possible to form a figure with 2023 sticks.	M1 A1
5	(a)	$x + 5y = 2y + 8$ (Shown) $x + 3y = 8$	A1
	(b)	$(x + 5y) + (2y + 8) + (2x - 4) = 26$ $3x + 7y + 4 = 26$ (Shown) $3x + 7y = 22$	M1 A1
	(c)	$9y - 7y = 24 - 22$ $2y = 2$ $y = 1$ $x = 8 - 3(1)$ $x = 5$	M1 A1 A1
	(d)	$2(5) - 4 = 6$ cm	B1
6	(a)(i)	$r^3 = \dfrac{165}{\dfrac{4}{3}\pi}$ $r = 3.40$ (Shown)	M1 A1
	(a)(ii)	$r = 3.4025$ $d = 3.4025 \times 2$ $= 6.805$ Since d is within the given range, it is an acceptable ball.	M1 A1

	(b)	Surface Area of 1 ball $= 4\pi r^2$
		                $= 4\pi(3.4)^2$
		                $= 145.48$ cm^2
		Surface Area of 700 balls $= 700 \times 145.48$
		                          $= 101836.6658$
		                          $= 101\,837$ cm^2
		                          $= 10.108$ m^2
		                          $= 10$ m^2
		Cost of producing 700 balls $= \$1.20 \times 10.108$
		                               $= \$12.13$ (2 d.p.)
			M1
			
			A1 (Also accept \$12.20)

7	(a)(i)	$tan32 = \dfrac{10}{AD}$	M1

		$AD = \dfrac{10}{tan32}$ $=16$ cm	A1
	(ii)	$BD = \sqrt{10^2 + 16^2}$ $= 18.9$ cm	M1 (accept any correct method) A1
	(b)	$cos\,\theta = \dfrac{16}{43}$ $\theta = 68.2°$ $\angle BDC = 68.2° - 32°$ $= 36.2°$	M1 A1
8	**(a)**	-4 , 1	B1
	(b)	$P(0,4)$	B1
	(c)	When $x = -3$, $y = 4 \neq 5$ No, $(-3,5)$ did not satisfy the equation $y = (x+4)(1-x)$.	M1 A1
9	**(a)**	-4	B1
	(b)	Correct scale Correct points plotted Smooth curve	A1 P1 C1

	(c)	–4.1 (±0.1)	B1
	(d)(i)	–3.8 (±0.1)	B1
	(d)(ii)	–7.8 (±0.1) , 0.8 (±0.1)	B1 (for both answers)
	(e)	–7 (±0.1) , 0 (±0.1)	B1 (for both answers)
10	**ATAS** Non peak: 3(48) + 45 = $189 **HAN** Peak : 3(43) + 36 + 8(4) = $197 The Tan family should choose HAN travel because it is cheaper.		M1. A1 A1 A1 A1