CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

MATHEMATICS   **4052/01**
Paper 1   **19 August 2024**
                                                    2 hours 15 mins

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11	12
Marks												
Question	13	14	15	16	17	18	19	20	21	22	23	24
Marks												

Table of Penalties		**Qn. No.**		
Presentation	−1			
Accuracy/ Units	−1		**Parent's/ Guardian's Signature**	**90**

This question paper consists of 20 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

3

Answer **all** the questions.

1   Calculate $\dfrac{1.75(-2.03)^3}{-3.85^2 \div \sqrt[3]{135.7}}$.

(a)   Write down the first six digits on your calculator display.

Answer ……………………… [1]

(b)   Write your answer in part **(a)** correct to 3 decimal places.

Answer ……………………… [1]

2   The following items are to be packed into gift bags for distribution to an old folks' home.

84 packets of cereals. 126 cans of baked beans. 168 packets of biscuits.

The items are to be packed such that the contents in each gift bag are the same and there are no leftovers. Find

(a)   the maximum number of gift bags required,

Answer ………………………... [2]

(b)   the list of contents in each gift bag.
Answer

…………………………………………………………………………………………………

………………………………………………………………………………………………… [1]

3 In a concert, $\frac{1}{3}$ of the audience were under the age of 20, 25% were over 50 and $\frac{2}{3}$ of those over 50 were females.

(a) Find the fraction of the whole audience that were between 20 and 50 inclusive.

Answer ………………………... [1]

(b) 100 of the audience were men above 50. Find the audience size.

Answer ………………………... [2]

4 y is inversely proportional to x^2.

If x is decreased by 60%, find the percentage increase in y.

Answer ………………………..% [3]

5 Finance company *A* pays investors compound interest at 1.2% per annum, compounded every month.
Finance company *B* pays investors simple interest at 1.5% per annum.
Mr Tan intends to invest a certain sum for 5 years. He thinks he should invest in finance company *A*.
Do you agree? Explain.
Answer

…………………………………………………………………………………………………….

………………………………………………………………………………………………. [3]

6 **(a)** A pair of shoes costs $\$(3x^2 - 7)$ and a bag costs $\$(2x^2 - 3x + 5)$.

Find the total cost of three such pairs of shoes and five of the bags, in terms of *x*.

Answer $ …………………………………………... [2]

(b) Factorise $p^3 - 2p^2 - 4p + 8$ completely.

Answer …………………………………………... [2]

7 **(a)** Solve the inequalities $\dfrac{3}{2}x - 2 < 3x - 8 \leq \dfrac{2x + 71}{3}$.

Answer ……………………... [2]

(b) $\zeta = \{\text{positive integers less than } 17\}$

$A = \{x : 3 \leq x < 15\}$

$B = \{\text{factors of } 20\}$

(i) List down the elements in the set $A \cap B$.

Answer …………….………………… [1]

(ii) Find $n(A \cup B)'$

Answer …………………… [1]

8 P is the point $(-6, -4)$ and Q is the point (x, y). The gradient of the line PQ is $\dfrac{2}{3}$.

Find the ratio of $x : y$ in its simplest form.

Answer …………… : …………… [2]

9 A map is drawn to a scale of 1 : 25 000.

   (a) The distance between two points on the map is 27.5 centimetres. Find the actual distance, in kilometres, between the two points.

   Answer …..…………………... km   [1]

   (b) A farm has an area of 36 square centimetres on this map.
   Find the area of the farm, in square centimetres, on a second map which is drawn to a scale of 1 : 120 000.

   Answer …..…………………... cm²   [3]

10 The first four terms in a sequence of numbers are given below.

$$T_1 = 3^2 + 2 = 11$$
$$T_2 = 4^2 + 5 = 21$$
$$T_3 = 5^2 + 8 = 33$$
$$T_4 = 6^2 + 11 = 47$$

(a) Find T_5.

Answer ……………………... [1]

(b) Show that the nth term of the sequence, T_n, is given by $n^2 + 7n + 3$.   [1]

Answer

(c) T_n and T_{n+1} are consecutive terms in the sequence. Find and simplify an expression, in terms of n, for $T_{n+1} - T_n$.

Answer …………………... [2]

(d) Explain why two consecutive terms of the sequence cannot have a difference of 6.

Answer

………………………………………………………………………………………………

………………………………………………………………………………………… [2]

11 **(a)** The number of sides of regular polygon *A* is triple that of another regular polygon *B*.
The ratio of each interior angle of the polygon *A* to that of the polygon *B* is 4 : 3.
(i) Find the number of sides of the polygon *A*.

Answer …..………………... [2]

(ii) Hence, find the exterior angle of the polygon *B*.

Answer …..………………...° [1]

(b) Explain briefly why each interior angle of a polygon cannot be 110°.

……………………………………………………………………………………
…………………………………………………………………………………… [1]

12 Solve the simultaneous equations.

$$4x + \frac{37}{3} = \frac{1}{3}y$$

$$\frac{1}{3}x - 3 = -4y$$

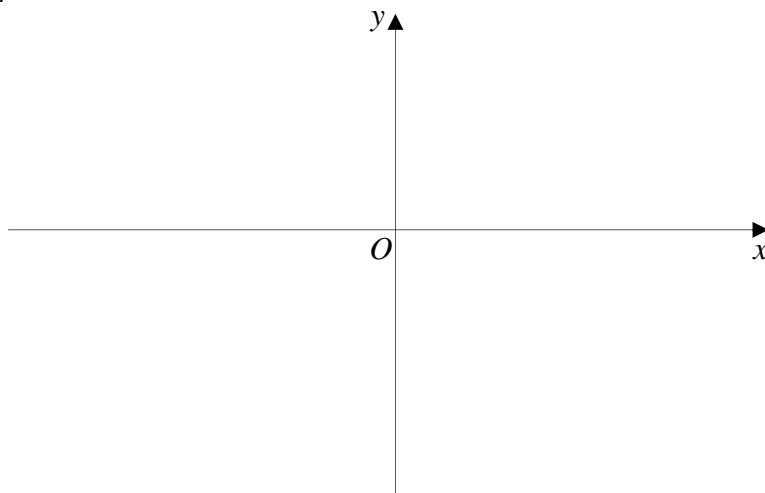*Answer* x = ……………… , y = …..………... [3]

13 **(a)** Express $y = -x^2 - 6x - 5$ in the form $y = a(x+b)^2 + c$.

Answer ………………………….... [1]

(b) Sketch the graph of $y = -x^2 - 6x - 5$ on the axes below.

Indicate the values where the graph crosses the axes and the maximum point on the graph.

Answer

[2]

14 Jane had some drinks with her friends at a café which gave a \$6 discount. All prices are subjected to 10% service charge and 9% Goods and Services Tax. Her final bill was \$83.93. Find the marked price of her meal.

Answer \$ ……………... [2]

15   At a fruit juice stall, a cup of orange juice costs $4.40, a cup of apple juice costs $4.80 and a cup of grape juice costs $6.20. On Saturday, there were 55 orders for orange juice, 41 orders for apple juice and 25 orders for grape juice. On Sunday, there were 42 orders for orange juice, x orders for apple juice and 20 orders for grape juice.

(a)   Write down a 2×3 matrix, **A**, representing the orders over Saturday and Sunday.

Answer **A** = ……………………...   [1]

(b)   Find, in terms of x, the matrix $\mathbf{P}=\mathbf{A}\begin{pmatrix} 4.40 \\ 4.80 \\ 6.20 \end{pmatrix}$.

Answer **P** = ……………………...   [1]

(c)   Explain clearly what each element in matrix **P** represents.

………………………………………………………………………………………………

………………………………………………………………………………………………

………………………………………………………………………………………………   [1]

(d)   If the total revenue on Sunday is about 10% less that the total revenue on Saturday, calculate

    (i)   the value of x,

Answer ……………………...   [2]

    (ii)   and the total revenue on the weekends using matrix multiplication.

Answer $……………………...   [2]

16 The diagram shows three geometrically similar detergent bottles of the same brand.

4 litres	2.5 litres	800 ml
A: $15.50	*B*: $10.20	*C*: $4.50

(a) Explain which of the above bottles is the best value for money.
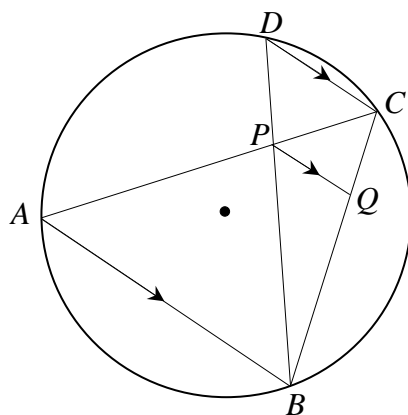
Answer

Bottle …………… is the best value for money.   [2]

(b) The bottles are all geometrically similar.
The height of the 800 ml bottle is 12.5 cm.
Calculate the height of the 2.5 litres bottle.

Answer ……………………... cm   [2]

17 *ABCD* is a cyclic quadrilateral and *AB*, *PQ* and *DC* are parallel.

Given that $PQ : DC = 2 : 3$ and area of triangle $BPQ = 25 \text{ cm}^2$.

(a) Show that triangle *BPQ* and triangle *BDC* are similar.

Answer

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

………………………………………………………………………………………… [2]

(b) Calculate the area of triangle *PCD*.

Answer …………………….... cm^2 [2]

(c) What type of triangle is *PAB*? Justify your answer.

Answer

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

……………………………………………………………………………………… [2]

18

Quarterly Household Electricity Tariff*
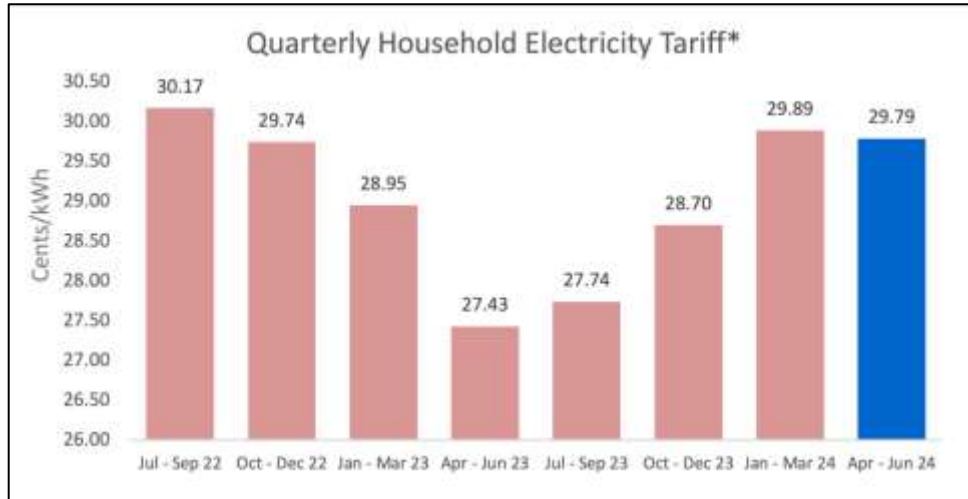
(a) Find the percentage decrease of the quarterly household electrical tariff between July – Sept 22 and April – June 2023.

Answer ……………... %   [2]

(b) Explain briefly why the bar chart might be considered misleading.

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………   [1]

19 The table below shows the heights of 40 students.

Height (H cm)	$90 < H \leq 100$	$100 < H \leq 110$	$110 < H \leq 120$	$120 < H \leq 130$	$130 < H \leq 140$
Frequency	2	9	13	10	6

(a) Calculate an estimate of the mean height of the students.

Answer ……………... cm   [1]

**(b)** Calculate an estimate of the standard deviation of the heights.

Answer ……………... cm   [1]

(c) It was discovered that the students were not instructed to remove their shoes during measurement. Assuming that the average height of the soles of the shoes is 2.5 cm, state the standard deviation.

Answer ……………... cm   [1]

20 A solid cone of height and radius r cm is placed inside a cylindrical container of the same radius and a height of $2r$ cm.

Water is poured into the empty container at a constant rate.
After 4 minutes, the depth of the water is r cm.

2r

r
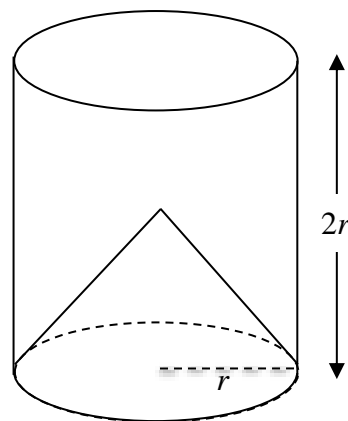
(a) Find the time taken to fill the container completely.

Answer ……………..... minutes   [2]

(b) On the axes in the answer space, sketch the graph showing how the height varies with time.

Depth (cm)

2r

r

0   2   4   6   8   10   12   Time (mins)   [2]
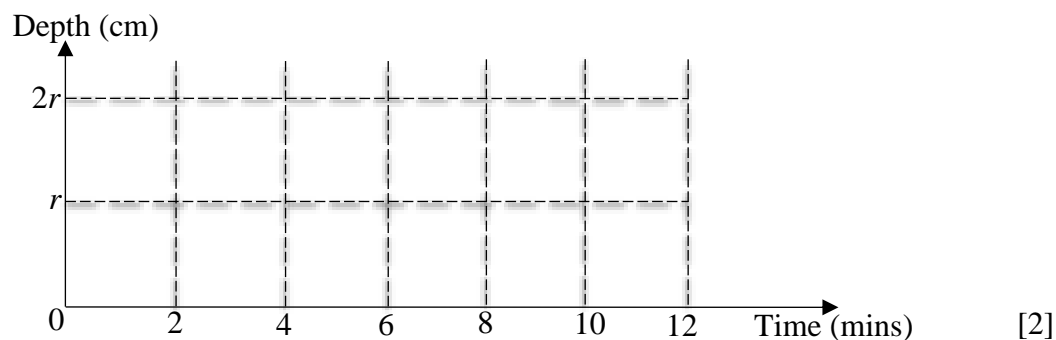
21 The diagram below is a map showing the positions of two buildings, *P* and *Q*.
Building *Q* is due south of *P*. The scale is 1 cm to 100 m.
Another building, *R* is on a bearing of 100° from *P*.
The distance between *Q* and *R* is 1050 m.

(a) Mark and label on the diagram the position of building *R*. [1]

(b) A convenience store is located at *T*, which is equidistant from lines *PQ* and *QR*
and is nearer to *P* than to *Q*.
Mark and label the possible position of the convenience store *T*. [3]
Answer
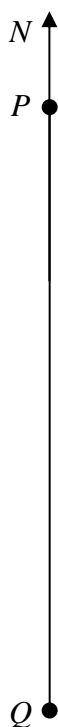
22 The equation of a line *L*, is given as $y = 2x + 5$.

(a) A point on the line *L* is thrice as far from *y*-axis as compared to the *x*-axis. Find the coordinates of the point.

Answer (………. , ……….) [2]

(b) The line *L* is reflected about the *x*-axis. Find the equation of this line.

Answer ………………………… [2]

23 A box contains red, blue and green balls. Each ball is labelled either 2, 3, 4 or 5.
The table below shows the probability of the colour and number drawn for the balls.

Colour	Number on the ball			
	2	3	4	5
Red	0.1	0	0.2	0.15
Blue	0.1	0.05	0	0.2
Green	0	0.1	0.1	0

(a) A ball is drawn at random from the box. Find the probability that
(i) it is blue or odd,

Answer ………………. [1]

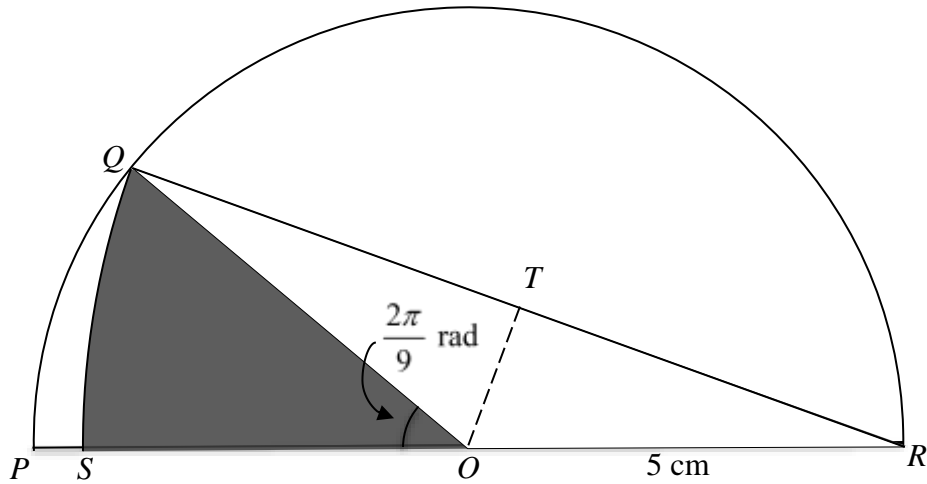**(ii)** it is not red and not even.

Answer ………………. [1]

(b) There are 135 red balls in the box. How many balls are there in the box altogether?

Answer ………………. [1]

24 The diagram shows a semicircle, centre *O*, of radius 5 cm.

The radius *OQ* makes an angle of $\dfrac{2\pi}{9}$ radians with the radius *OP*.

The arc *QS* of the circle has centre *R* and the point *S* lies on *OP*.

Find

(a) angle *QRO*,

Answer ……………..… rad   [1]

(b) *QR*,

Answer ……………..… cm   [2]

(c) the area of the shaded region.

Answer ……………..… cm²   [3]

END OF PAPER

Answers

1(a)	5.07533	15(a)	$\begin{pmatrix} 55 & 41 & 25 \\ 42 & x & 20 \end{pmatrix}$	
1(b)	5.075	15(b)	$\begin{pmatrix} 593.80 \\ 308.80 + 4.8x \end{pmatrix}$	
2(a)	42	15(c)	The elements show the total revenue of the fruit juice stall on Saturday and Sunday respectively	
2(b)	2 packets of cereals, 3 cans of baked beans, 4 packets of biscuits	15(d)(i)	47	
3(a)	$\dfrac{5}{12}$	15(d)(ii)	$1128.20	
3(b)	1200	16(a)	Bottle A	
4	525%	16(b)	18.3 cm	
5	No, interest from finance company A is higher.	17(b)	18.75 cm^2	
6(a)	$19x^2 - 15x + 4$	17(c)	Isosceles triangle	
6(b)	$(p-2)^2(p+2)$	18(a)	9.08%	
7(a)	$4 < x \le 13\dfrac{4}{7}$	18(b)	As the vertical axis does not start from zero, it might give the wrong impression that the percentage changes are much larger	
7(b)(i)	$\{4,5,10\}$	19(a)	117.25 cm	
7(b)(ii)	2	19(b)	11.1 cm	
8	3:2	19(c)	11.1 cm	
9(a)	6.875 km	20(a)	10 mins	
9(b)	1.5625 cm^2	20(b)		
10(a)	63	22(a)	$(-3, -1)$	
10(c)	$2n + 8$	22(b)	$y = -2x - 5$	
10(d)	Since n must be a positive integer, two consecutive terms in the sequence cannot have a difference of 6	23 (a)(i)	0.6	
11(a)	8	23(a)(ii)	0.35	
11(b)	When the interior angle is 110º, n is not an integer	23(b)	300	
12	$x = -3.\ y = 1$	24 (a)	$\dfrac{\pi}{9}$ rad	
13(a)	$y = -(x+3)^2 + 4$	24(b)	9.40 cm	
13(b)		21		
14	$76			