CANDIDATE NAME: ……………………………………………………………

CLASS: ………………………   INDEX NUMBER: ……..........

ADDITIONAL MATHEMATICS   **4049/01**

Paper 1   **November 2024**

Secondary 4 Express   **2 hours 15 minutes**
Setter: itzpipey :)))

Candidates answer on the question paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

90

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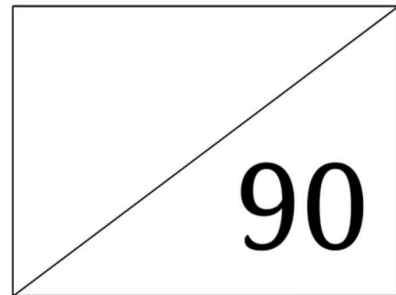
MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1   (a)   Find the equation of the normal to the curve $y = \tan(\frac{x}{2} - \pi)$ at the origin.   [3]

(b)   A particle moves along the curve. When $x = \frac{\pi}{3}$, y is decreasing at a rate of π $units/s$.
Find the rate of change of x at this instant.   [3]

2   Show that the curve $(m - 1)x^2 - 2y^2 = mxy + 1$ and the line $y = mx + 1$ will never intersect for all values of m. [5]

3 **(a)** Given that $log_3 p = q$ and $log_3 q = r$, express $log_{27}(pq^6)$ in terms of r in the **simplest form.** [4]

(b) Solve $lg(x^2 + 1) + ln(x^2 + 1) = e$. [5]

4 **(a)** Explain why all the powers of x in the binomial expansion of $(2x - \frac{1}{ax})^9$ are odd. [3]
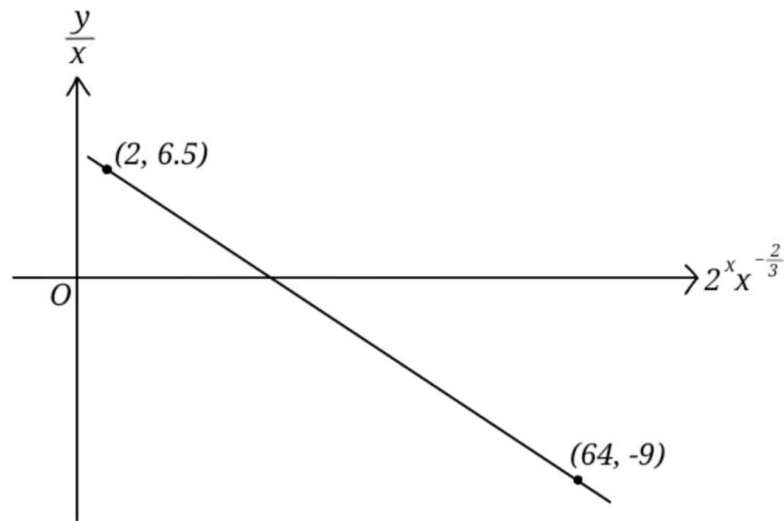
(b) Find the range of values of a such that the term independent of x in the expansion of $(\frac{a}{x^7} + \frac{1}{x^5})(2x - \frac{1}{ax})^9$ has a positive coefficient. [5]

5   For the function $f(x) = px^3 + p^2x^2 + 18x - 54$, where p is a constant, the remainder when $f(x)$ is divided by $x - 1$ is the same as when it is divided by $x + 2$.

(a)   Find the possible values of p.   [3]

(b)   Given further that $f(x)$ is divisible by $x + \sqrt{6}$, find $f(x)$.   [3]

6

(a) The variables x and y are connected by the equation $2^{3x-m}x = (nx - y)^3$ where m and n are constants. The diagram shows the straight line graph obtained by plotting $\dfrac{y}{x}$ against $2^x x^{-\frac{2}{3}}$, of which $(2,\ 6.5)$ and $(64, -9)$ are points on. Find the values of m and n and hence express y in terms of x. [6]

(b) V and t are connected by the equation $V = Ae^{kt+1}$. Explain how a straight line graph may be drawn using given values of V and t and how the values of A and k may be found using the straight line graph. [4]

7   A particle moves in a straight line such that its velocity, $v\ m/s$, at time t seconds for a few seconds is given by the equation $v = -\dfrac{2}{pt+3}$, where p is a negative constant.

(a)   Express the particle's acceleration in terms of p and t. Explain what the expression says about the particle's movement. Show all workings.   [3]

(b)   (i)   Express the particle's displacement from a fixed point O in terms of p and t, with c as an arbitrary constant of integration.   [1]

(ii)   The particle starts with a displacement of $ln\ 243$ metres from O. Given now that $c = 0$, find p.   [1]

(c)   Using your value of p obtained in part **b(ii)**,

     (i)   find the time at which the particle crosses O,   [2]

     (ii)   show that the particle does not change directions from $t = 0$ to $t = 7$,   [2]

     (iii)   find the distance travelled by the particle in the first 7 seconds.   [2]

8   **(a)**   Show that $cosec^2(67.5°) = 4 - 2\sqrt{2}.$ [5]

(b)   Showing your workings fully, find the **exact** value of $7cot[sin^{-1}(-\frac{35}{37})].$ [2]

(c) Solve $5sin^2A + cosA = 2sin(90° - 2A)$, where $-\frac{\pi}{2} \leq A \leq \frac{\pi}{2}$. [5]

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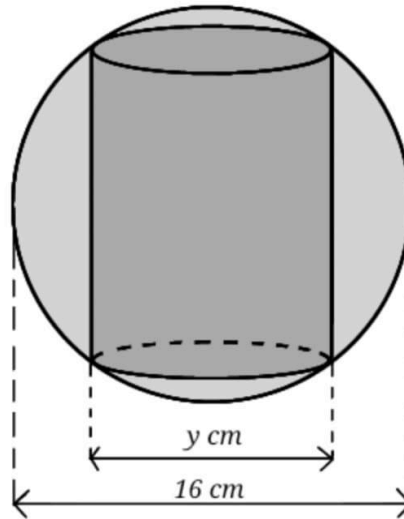
A cylinder of diameter $y\ cm$ is inscribed in and is to be cut out of a plastic sphere of diameter $16\ cm$.

(a)   After the cylinder is cut out, there will be some waste remains from the sphere. Show that the volume, V, of these remains can be expressed as

$$V = \pi(\frac{2048}{3} - \frac{\sqrt{256y^4 - y^6}}{4})$$

[4]

(b) Given that y can vary, find the optimal volume of the cylinder to be cut out such that minimal waste is produced. Explain why the value you obtain is optimal. [7]

10
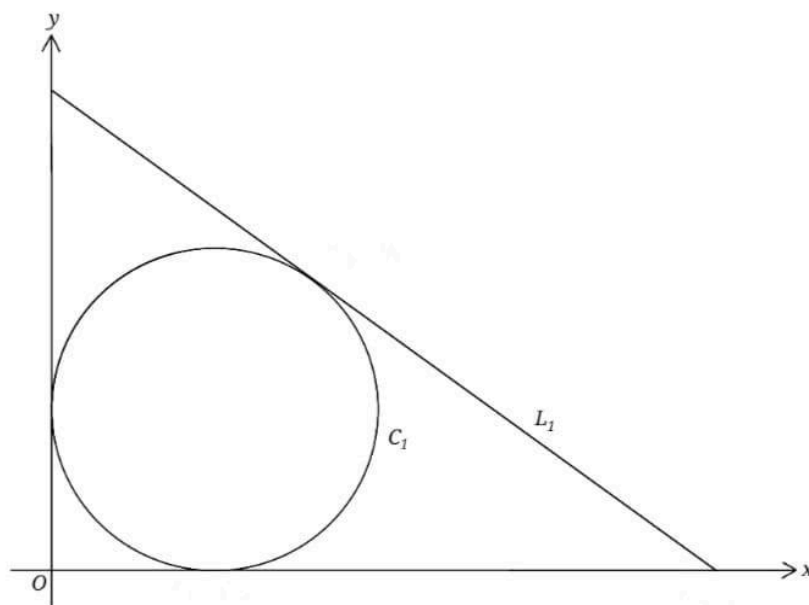
A cylindrical barrel rests on its side on the floor against a wall. These are represented in the diagram above by the circle C_1, the x-axis and the y-axis respectively. A straight rod is situated on the side of the cylinder such that it lies in the same plane as C_1 and can be represented by the line L_1. The barrel has a cross-sectional area of $100\pi\ cm^2$.

(a)   Find the equation of C_1.   [2]

(b) Find the angle at which the rod is lifted off the floor at. [4]

(c) Find the equation of L_1. [2]

(d) Find the **exact** coordinates of the point on C_1 that is the furthest away from the origin O. [4]

END OF PAPER

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