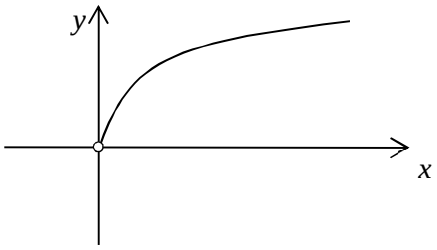
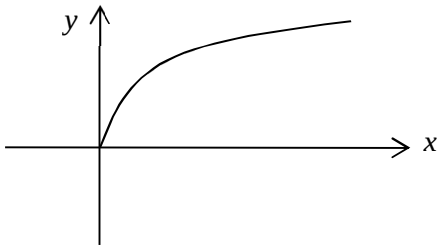
Chapter 5: Functions

Basics

Fill in the blanks.

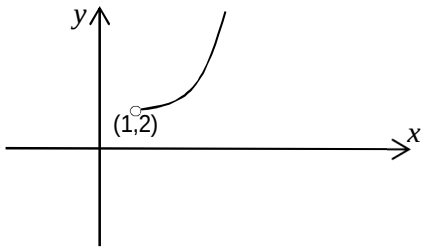
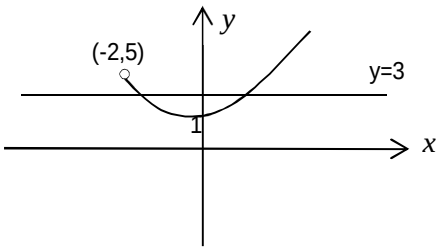
Part 1: Definitions of Functions

To test whether a relation is a function, we use the _____ _____ _____ It states that: If for each value a in _____, the vertical line $x = a$ intersects the graph _____, then the given graph is the graph of a function.	**Example of Relation that is a Function** $f(x) = \sqrt{x},\ x \in \mathbb{R}, x > 0$  f is a function because for each value $a \in D_f$, i.e $a > 0$, the line $x = a$ _____.	**Example of Relation that is not a Function** $f(x) = \sqrt{x},\ x \in \mathbb{R}$  f is not a function because _____ _____ _____

IMPT!: Note the difference between proving and disproving. Disproving only requires a counter-example.

Part 2: Inverse Functions

Existence of Inverse Function

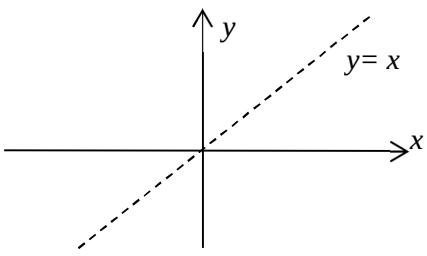
Only one-to-one function have an inverse. We use the _____ _____ to check if the function is one-to-one, and thus, if the function has an inverse. It states that: a function f is one-to-one if _____ horizontal line $y = k,\ k \in R_f$ intersects the graph of $y = f(x)$ exactly once.	**Example of Function with Inverse** $f(x) = x^2 + 1,\ x \in \mathbb{R}, x > 1$  	**Example of Function without Inverse** $f(x) = x^2 + 1,\ x \in \mathbb{R}, x > -2$ 

Finding Rule of Inverse Function and Sketching Graph of its Inverse

Consider the function: $\mathrm{f}(x)=(x+1)^2+2, x\in\mathbb{R}, x\le -1$.

Verify that the inverse exists by sketching the graph and using the Horizontal Line Test.	Sketch the graph of $y=\mathrm{f}(x)$ here.
To find the inverse function, let y = f(x) and make x the subject.	$y=(x+1)^2+2$ $(x+1)^2=y-2$ $x+1=\pm\sqrt{y-2}$ $x=-1\pm\sqrt{y-2}$ Since _____, $\mathrm{f}^{-1}(x)=-1-\sqrt{x-2}, x\ge 2$
Sketch the graph of the inverse function. The graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ are reflections of each other in the line $y=x$	Use a few points to help guide you. Choose a different point on the graph $y=\mathrm{f}(x)$ and write down the corresponding point on the inverse graph.

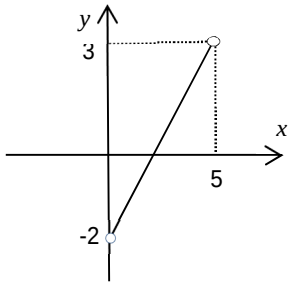
$y=\mathrm{f}(x)$	$y=\mathrm{f}^{-1}(x)$
(-1,2)	
(-2, 3)	



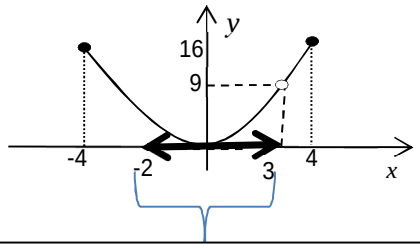
Part 3: Composite Functions

Finding Range of a Composite Function gf

$\mathrm{f}(x) = x - 2, 0 < x < 5$



$\mathrm{g}(x) = x^2, -4 \le x \le 4$



Restrict the domain of g to only

<u>Alternative Method (not recommended if question did not ask for rule of gf or if gf is a complex function to sketch)</u>

$\mathrm{gf}(x) = \mathrm{g}(x-2)$

$= (x-2)^2, 0 < x < 5$